Generalised Soundness of Workflow Nets is Decidable

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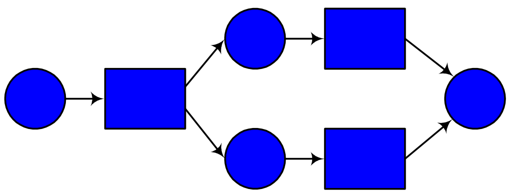
Workflow nets

A Petri net N is a Workflow net (WF-net) iff:

- N has two special places (or transitions): an initial place (transition) i: ${}^\bullet i = \emptyset$, and a final place (transition) f: $f^\bullet = \emptyset$.
- For any node $n \in (P \cup T)$ there exists a path from i to n and a path from n to f.



Applications: business process modelling, software engineering, ...

Soundness

Desired property: proper completion

Classical definition of soundness for WF-nets ([vdAalst]):
A WF-net N is sound iff:

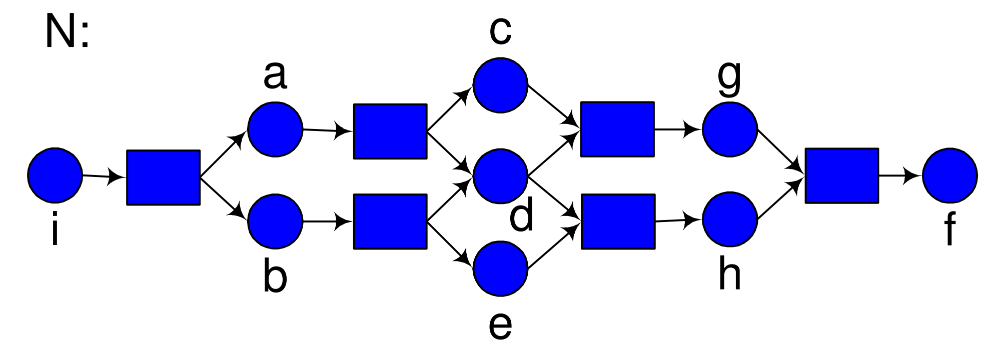
- For every marking M reachable from $[i]$, there exists a firing sequence leading to $[f]$.
- Marking $[f]$ is the only marking reachable from $[i]$ with at least one token in $[f]$.
- There are no dead transitions in $(N, [i])$.

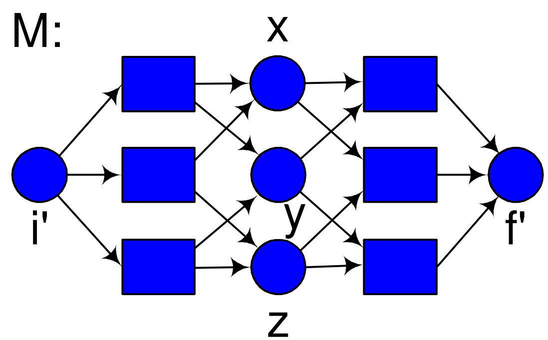
Refinement of Workflow Nets

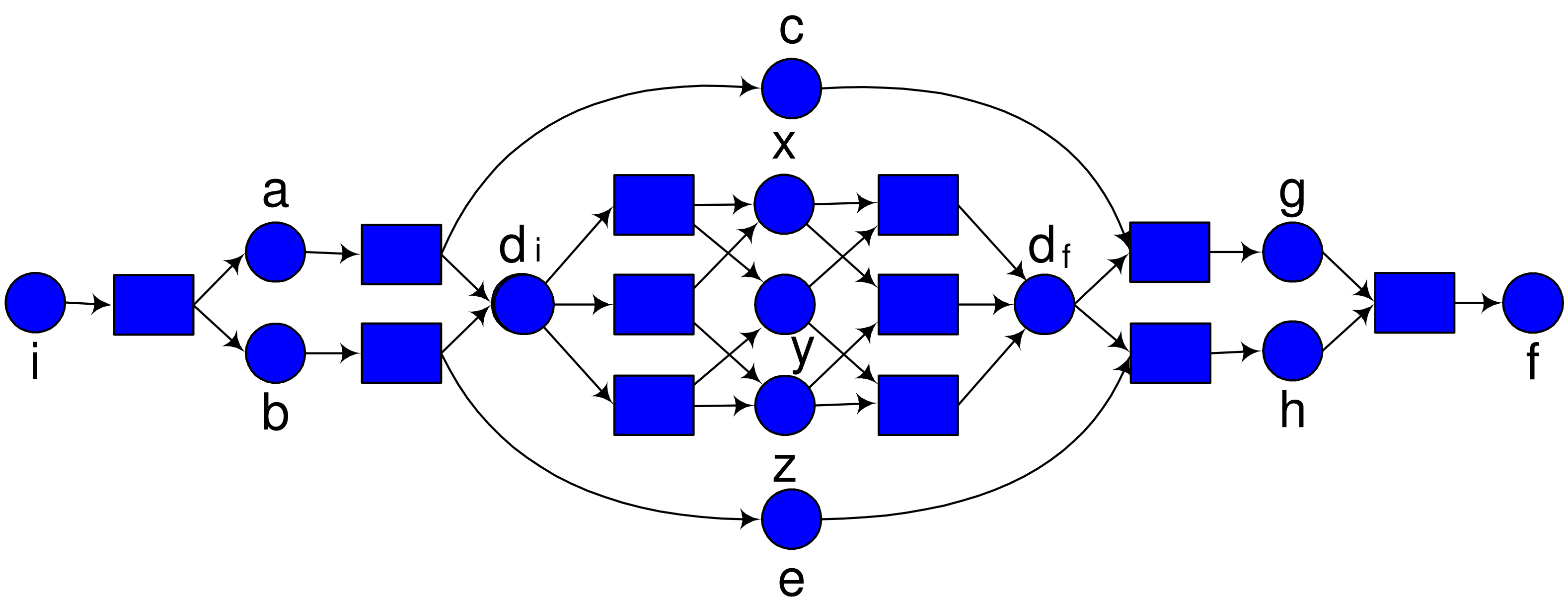
Place refinement: $N = L \otimes_p M$
Being at some location (place of the net) resources (tokens) undergo a number of operations.

Transition refinement: $N = L \otimes_t M$
A single task on a higher level becomes a sequence of subtasks also involving choice and parallelism.

Refinements and soundness







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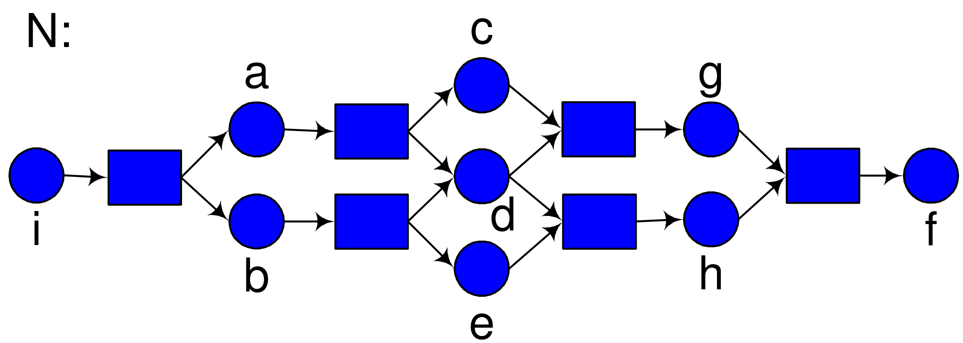

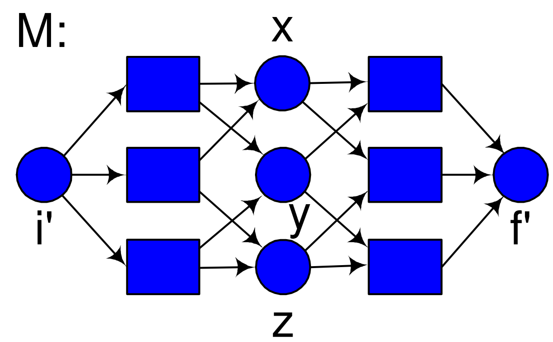

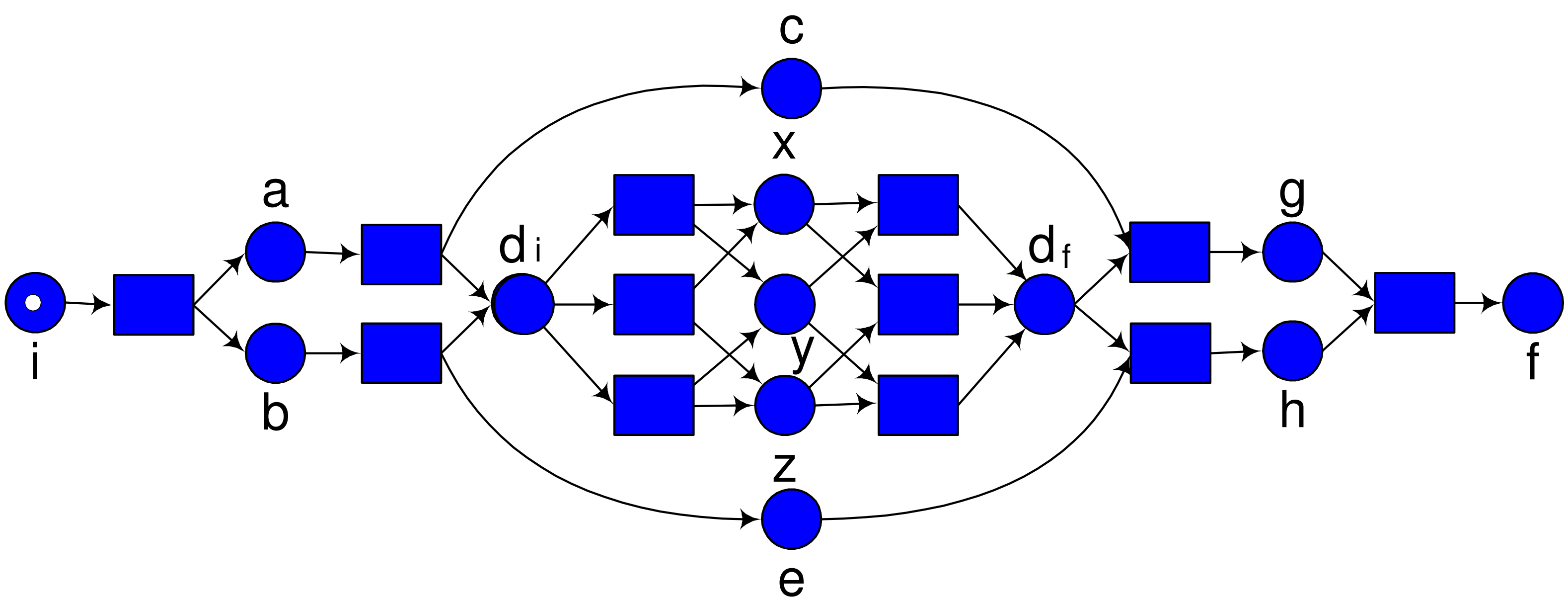

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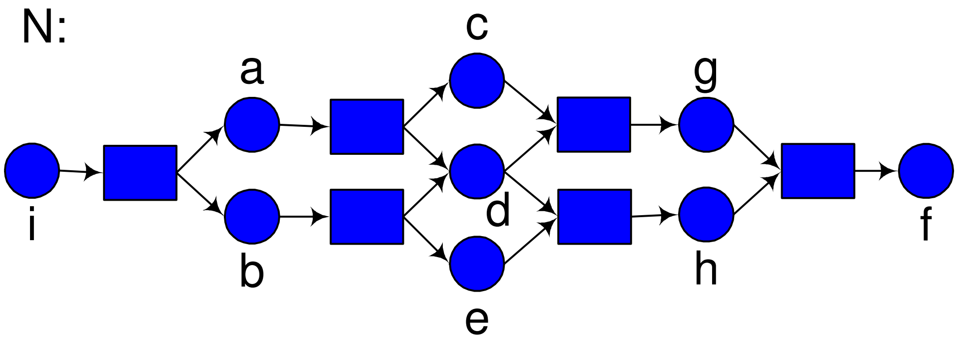

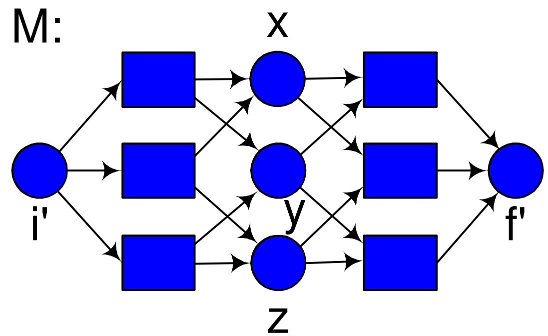

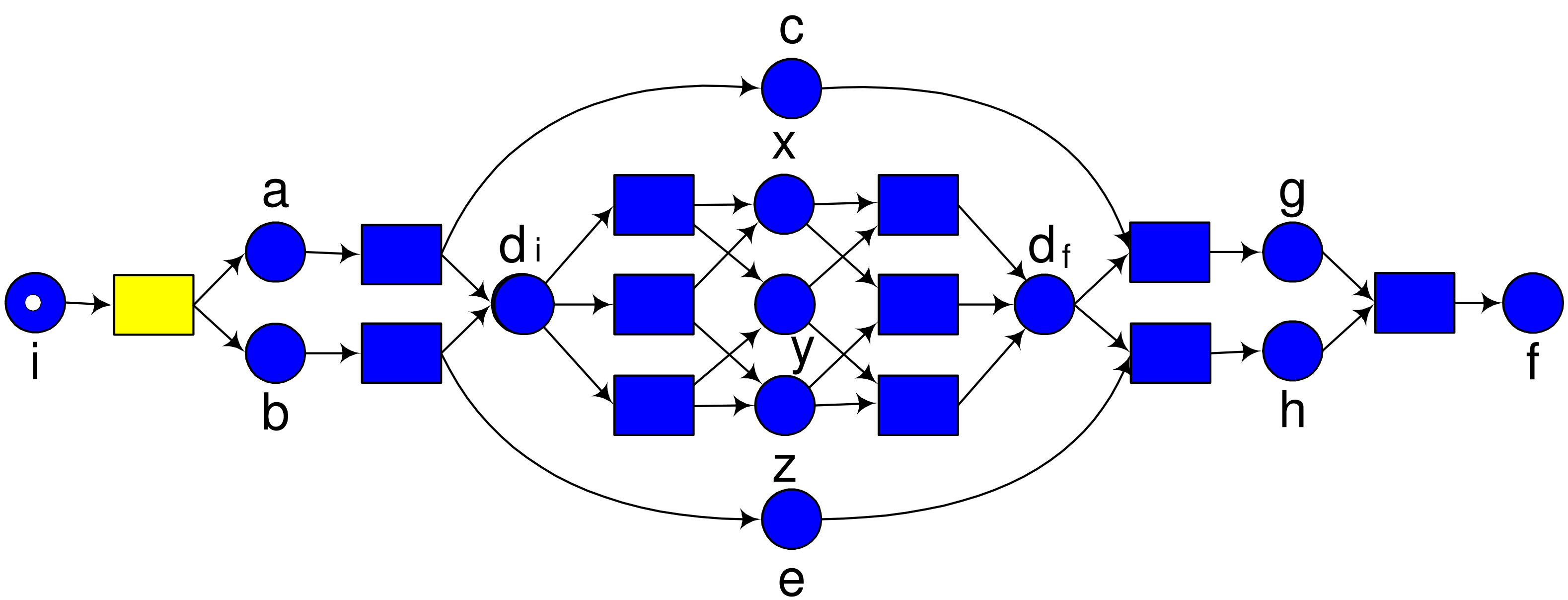

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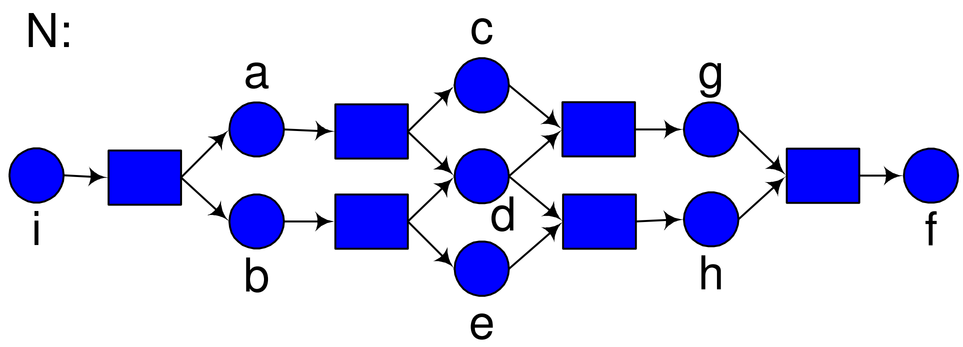

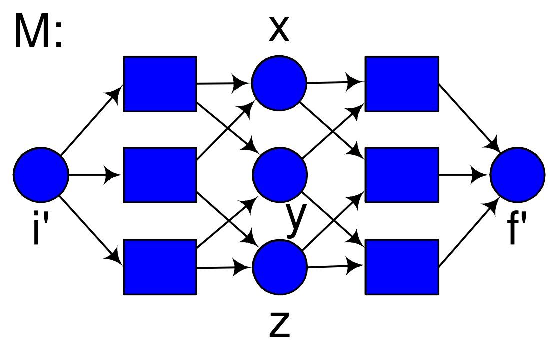

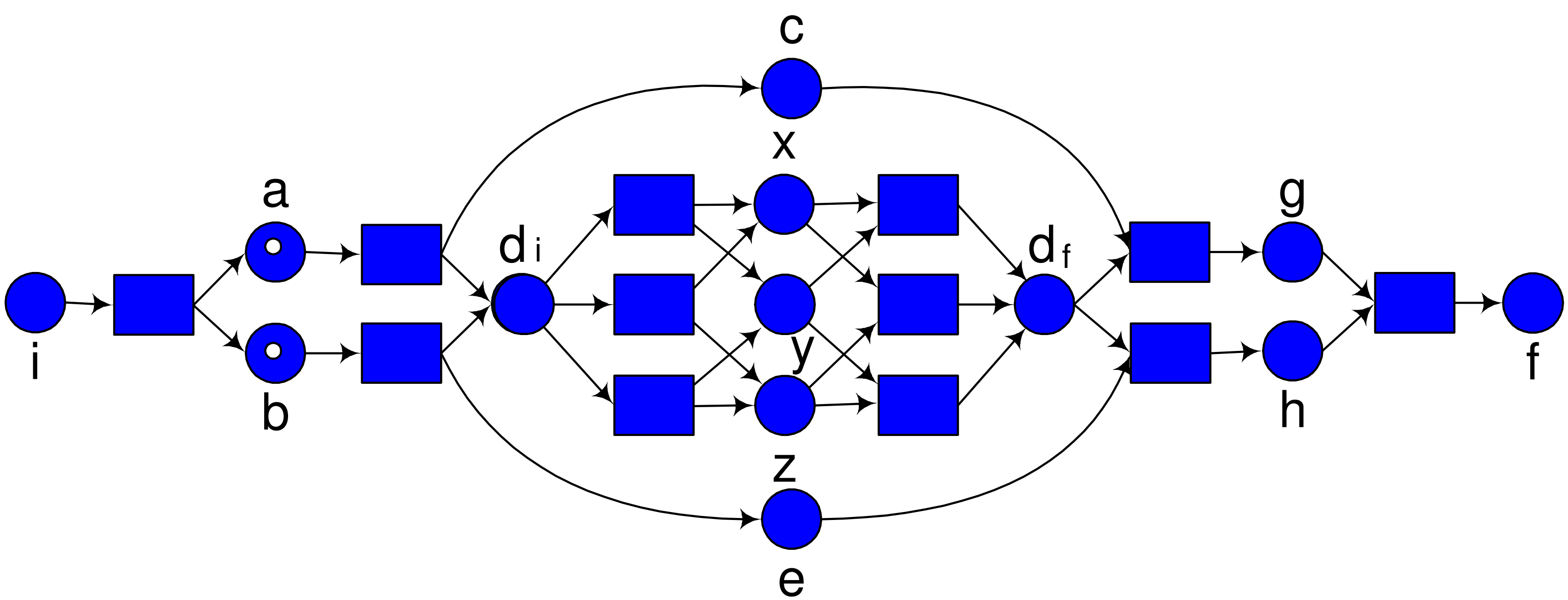

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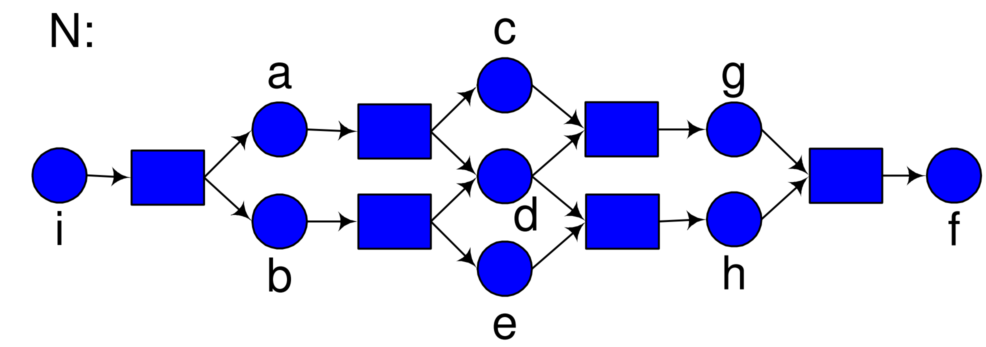

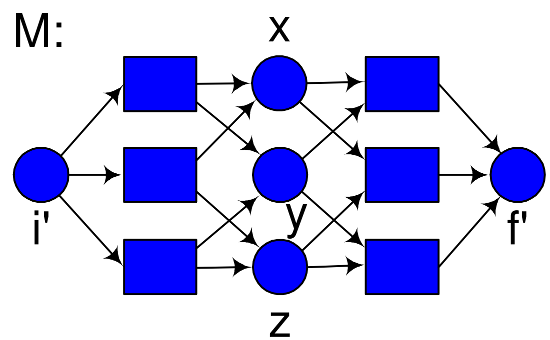

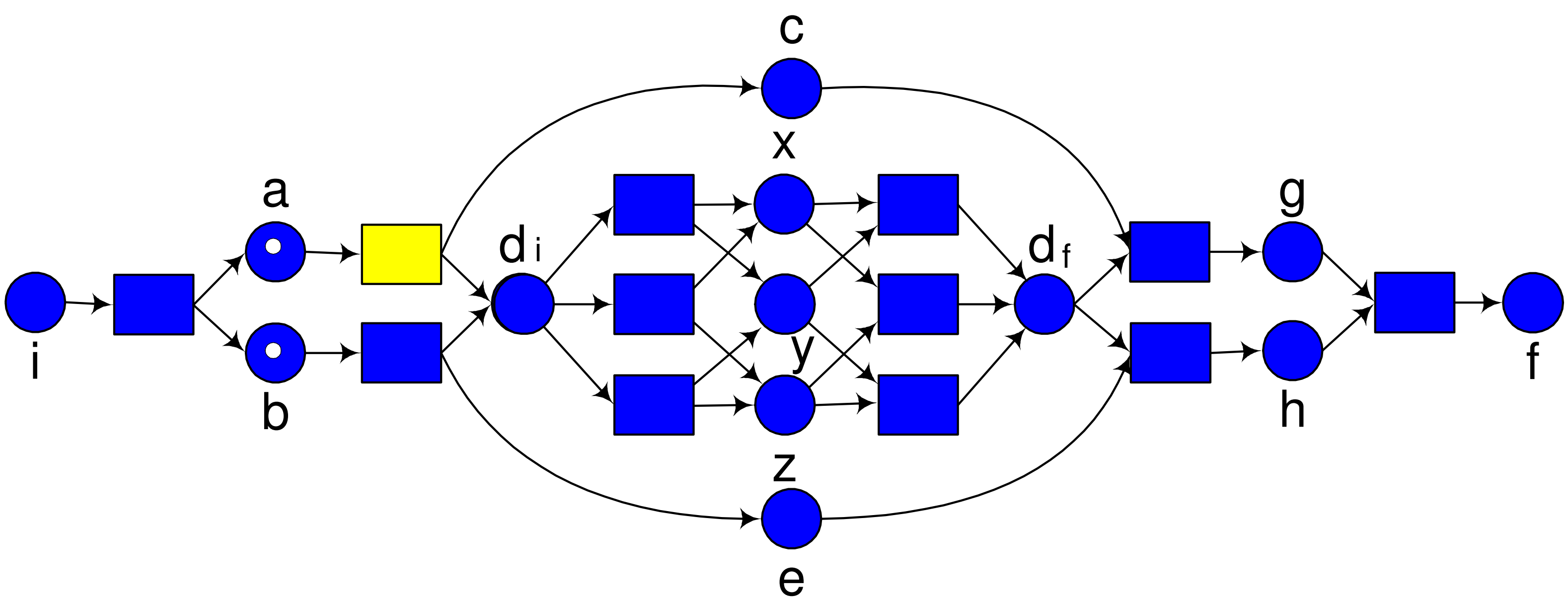

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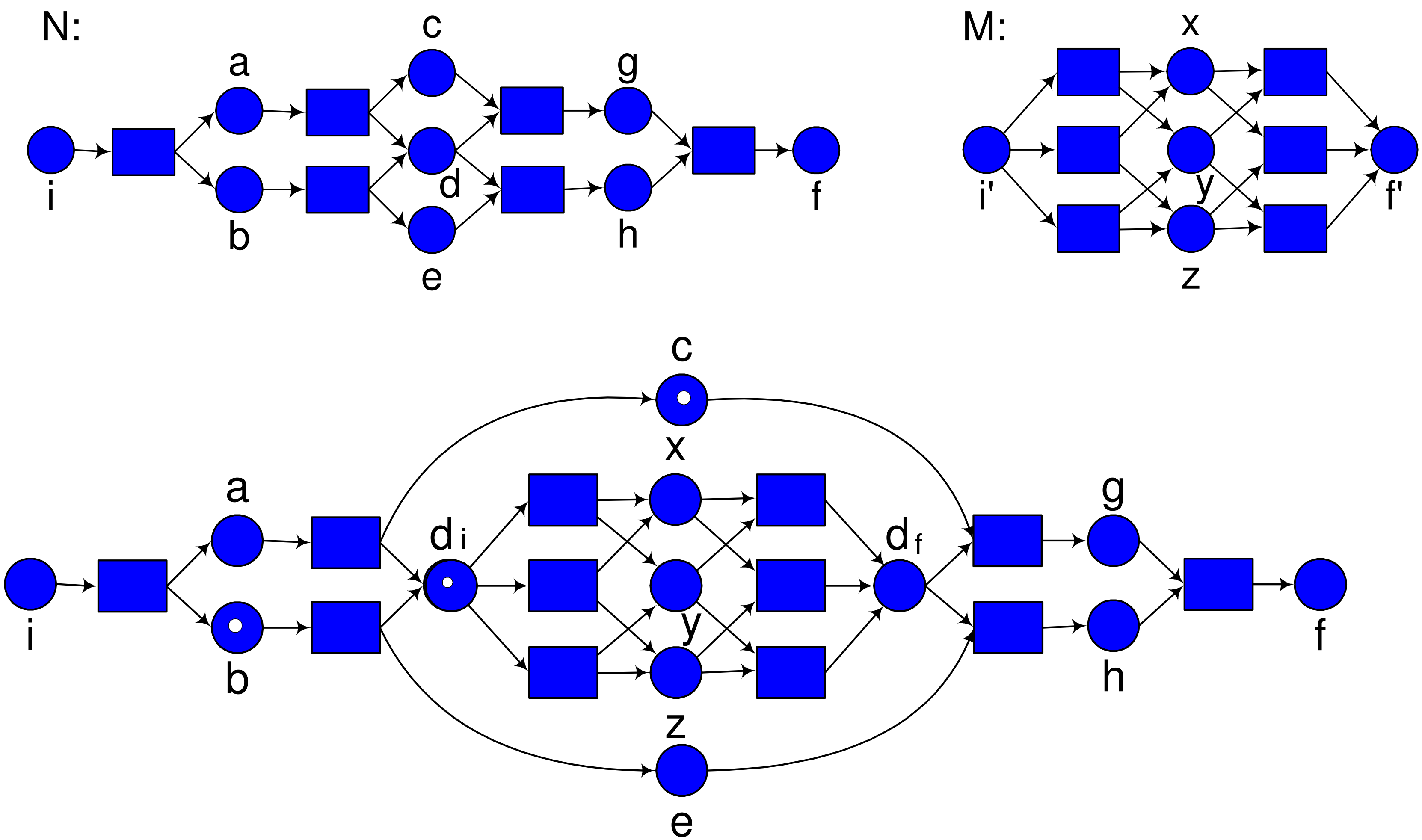

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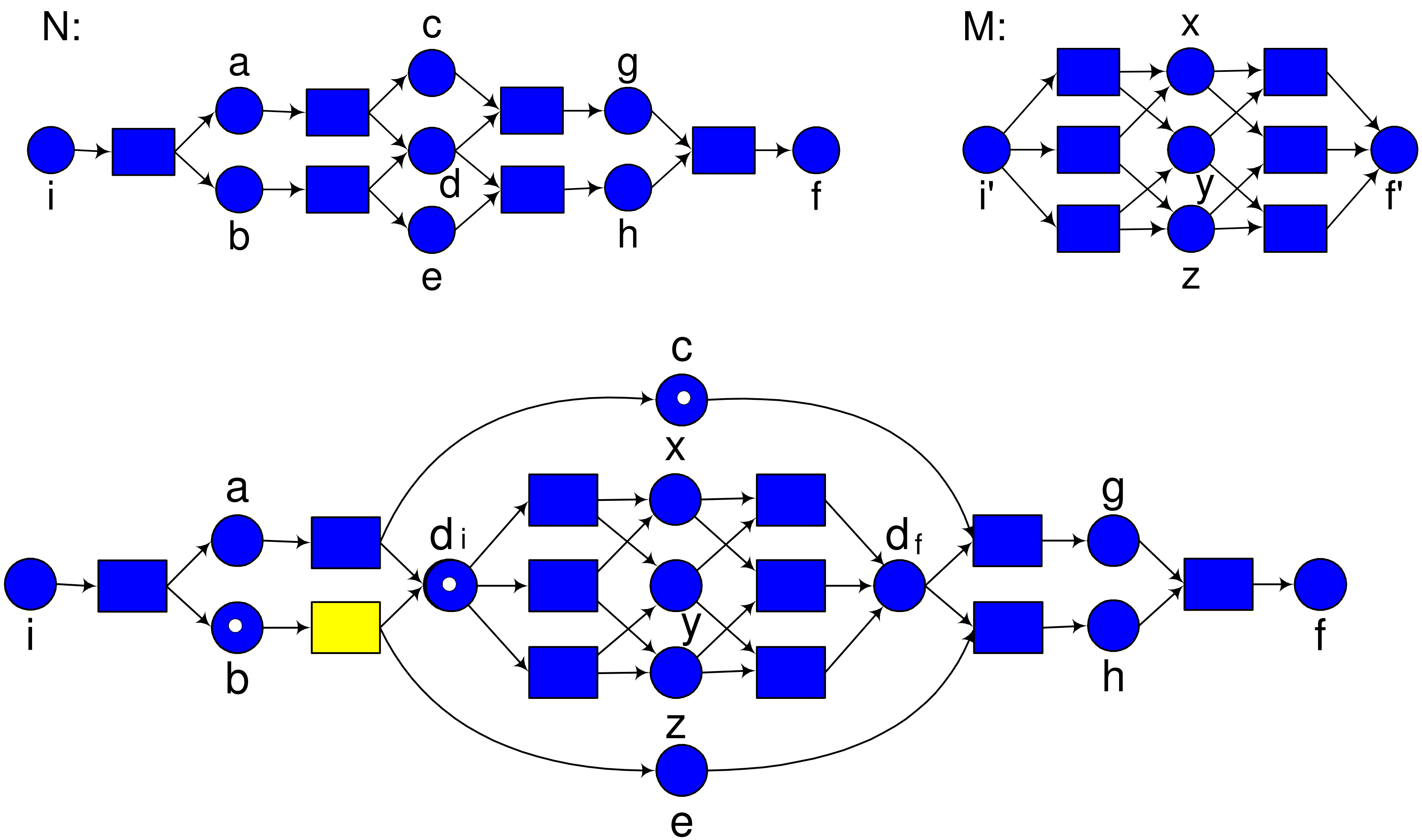

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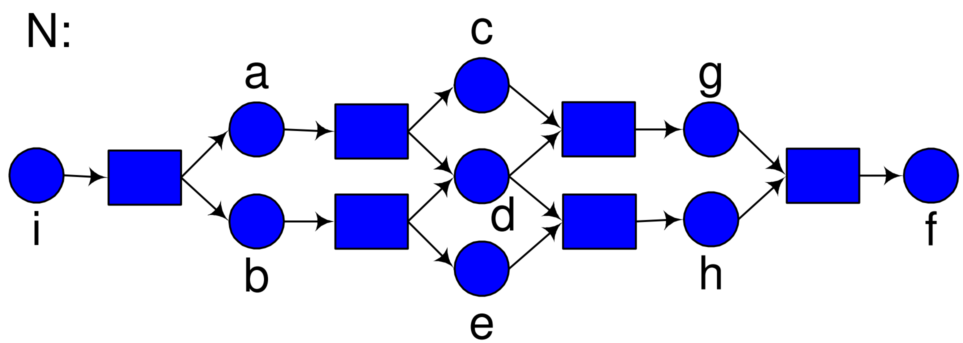

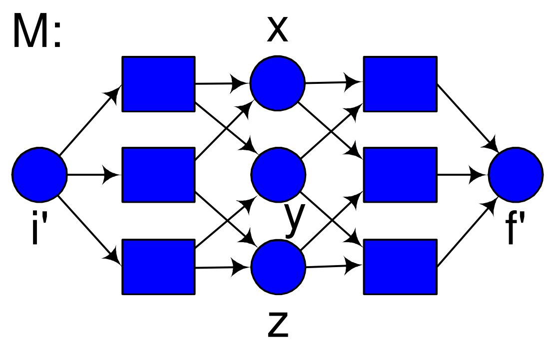

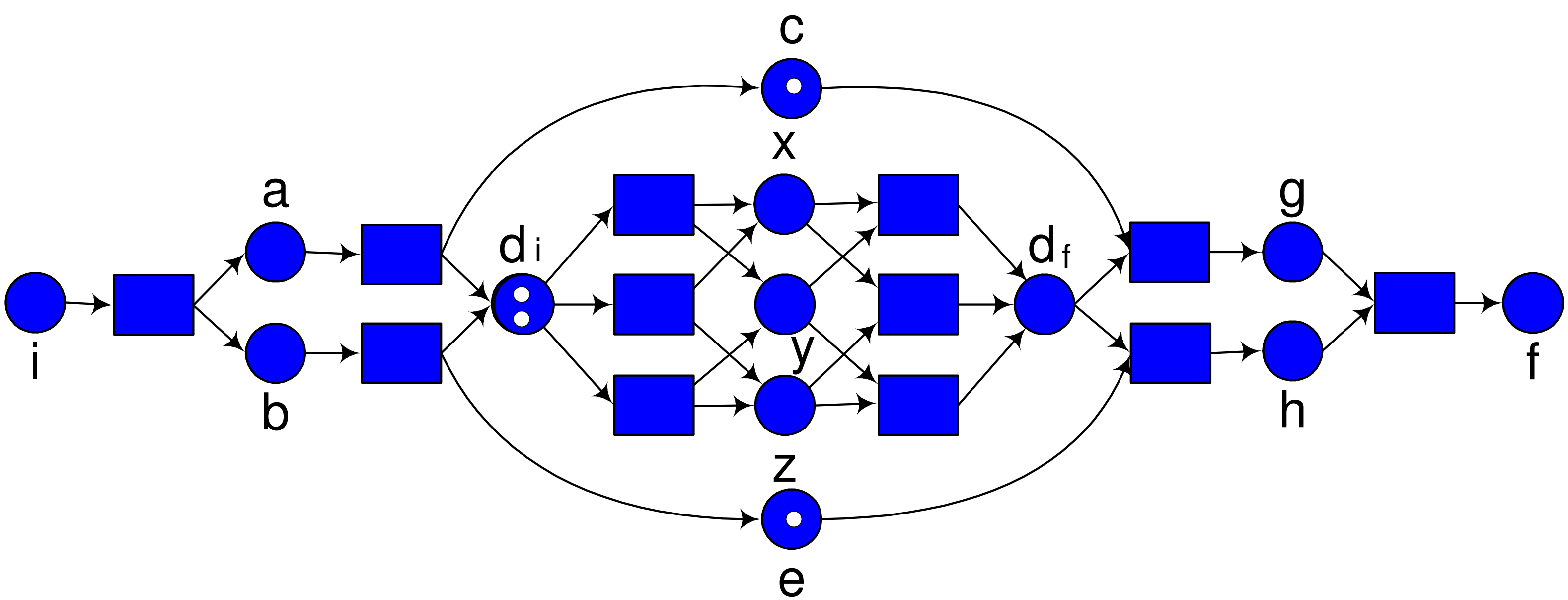

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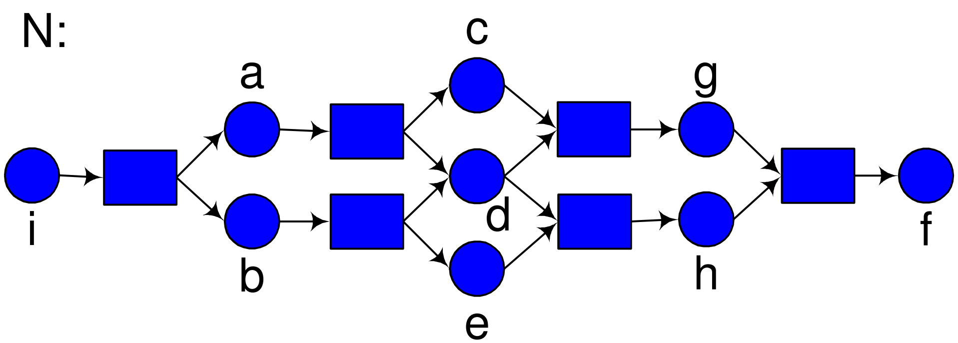

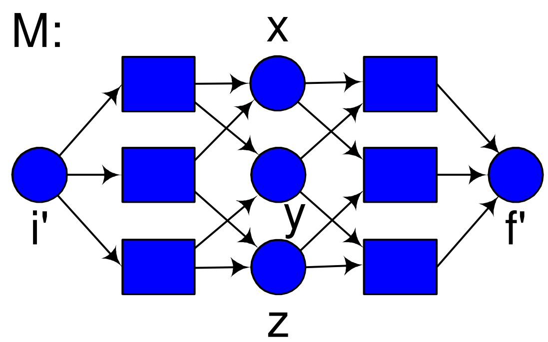

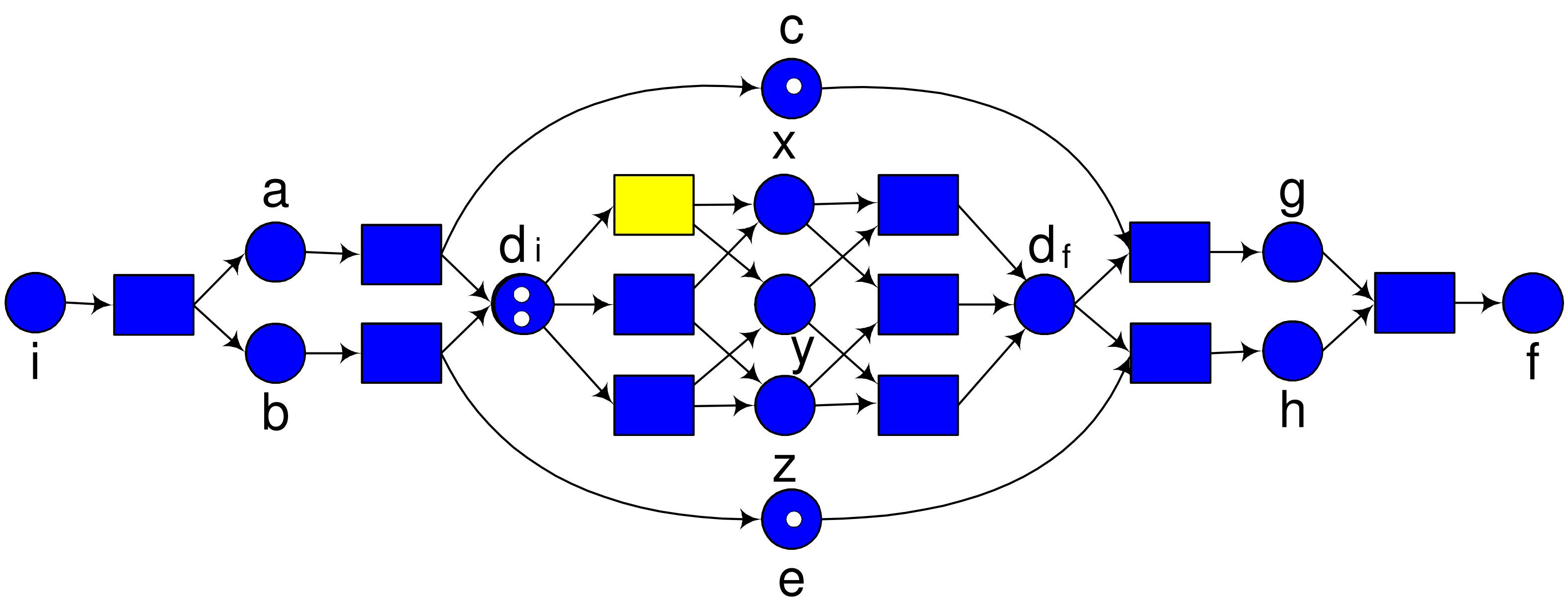

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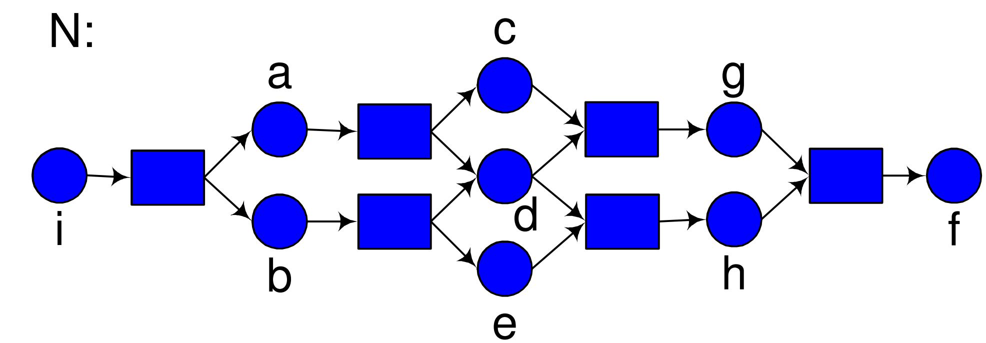

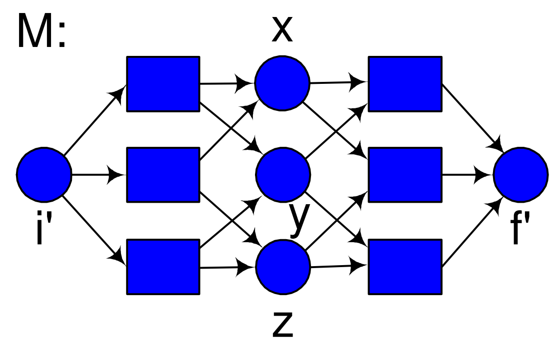

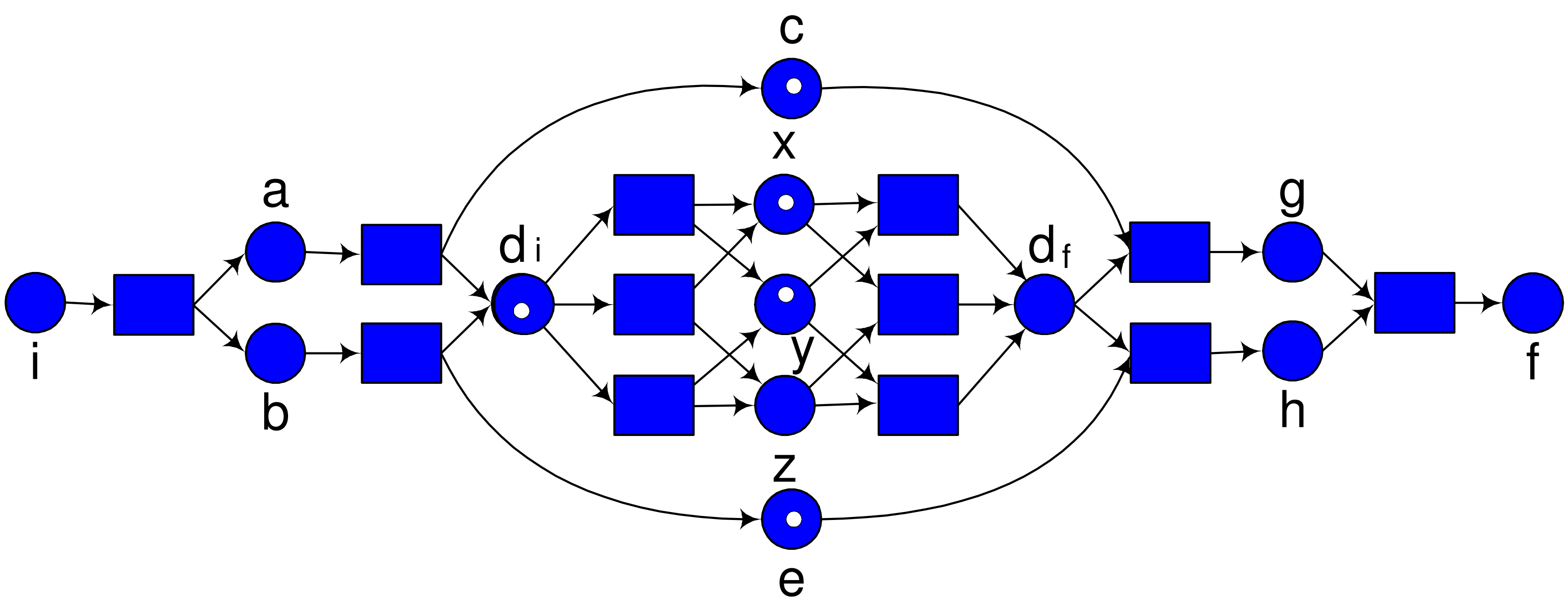

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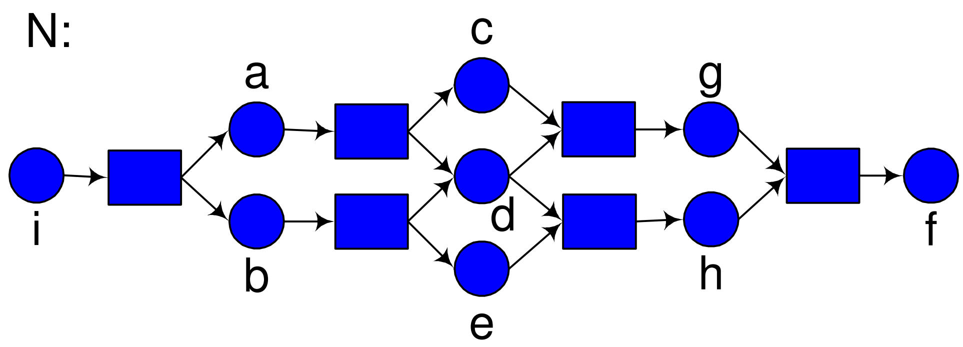

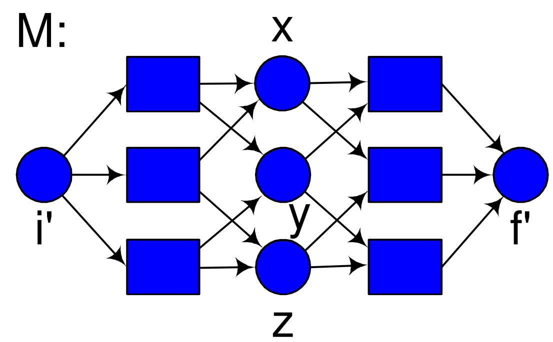

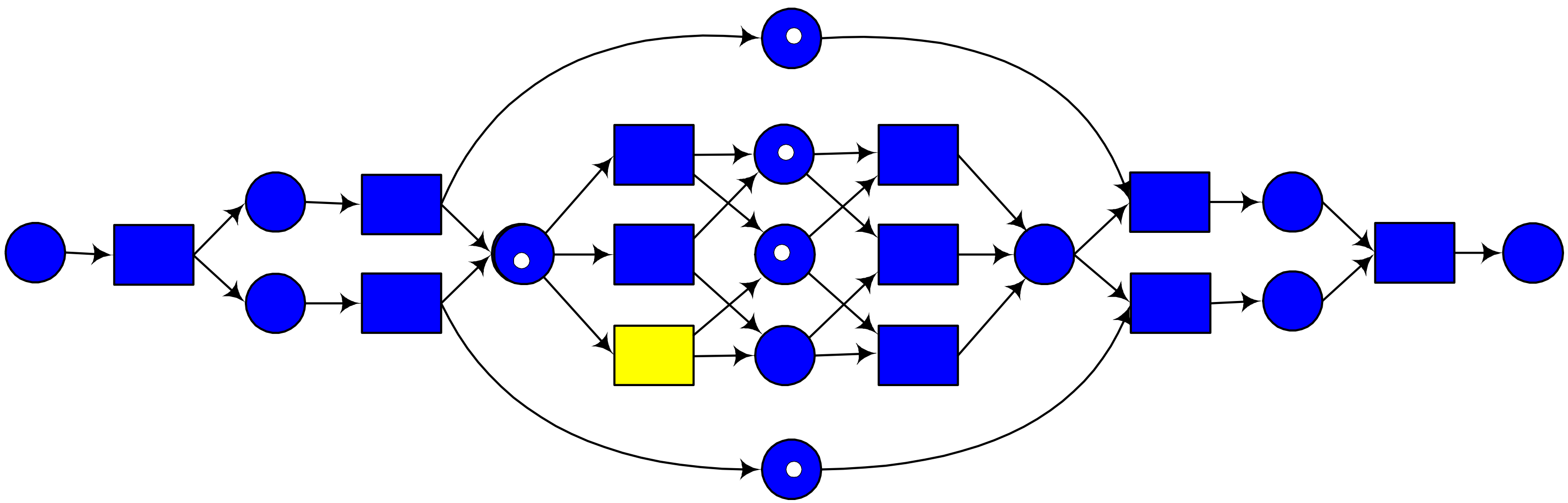

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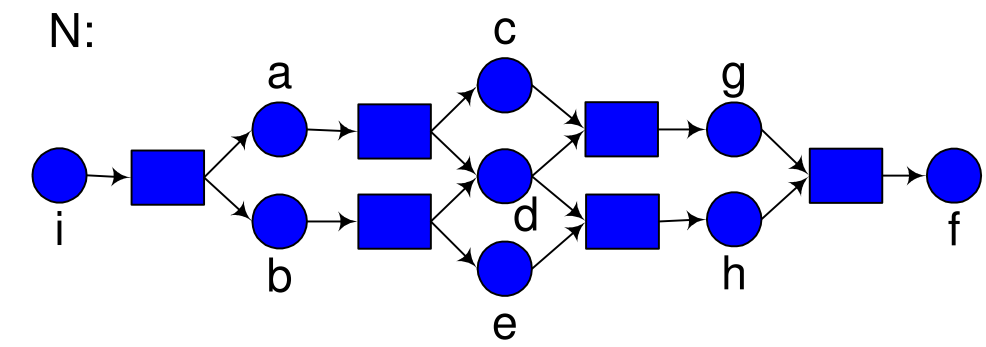

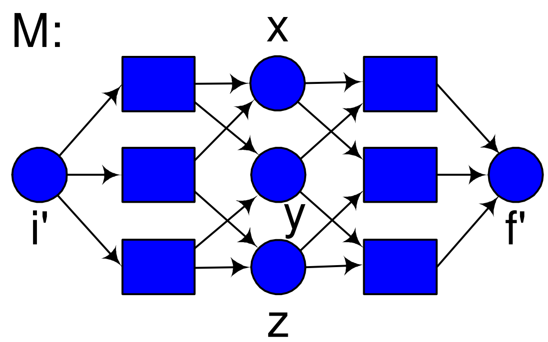

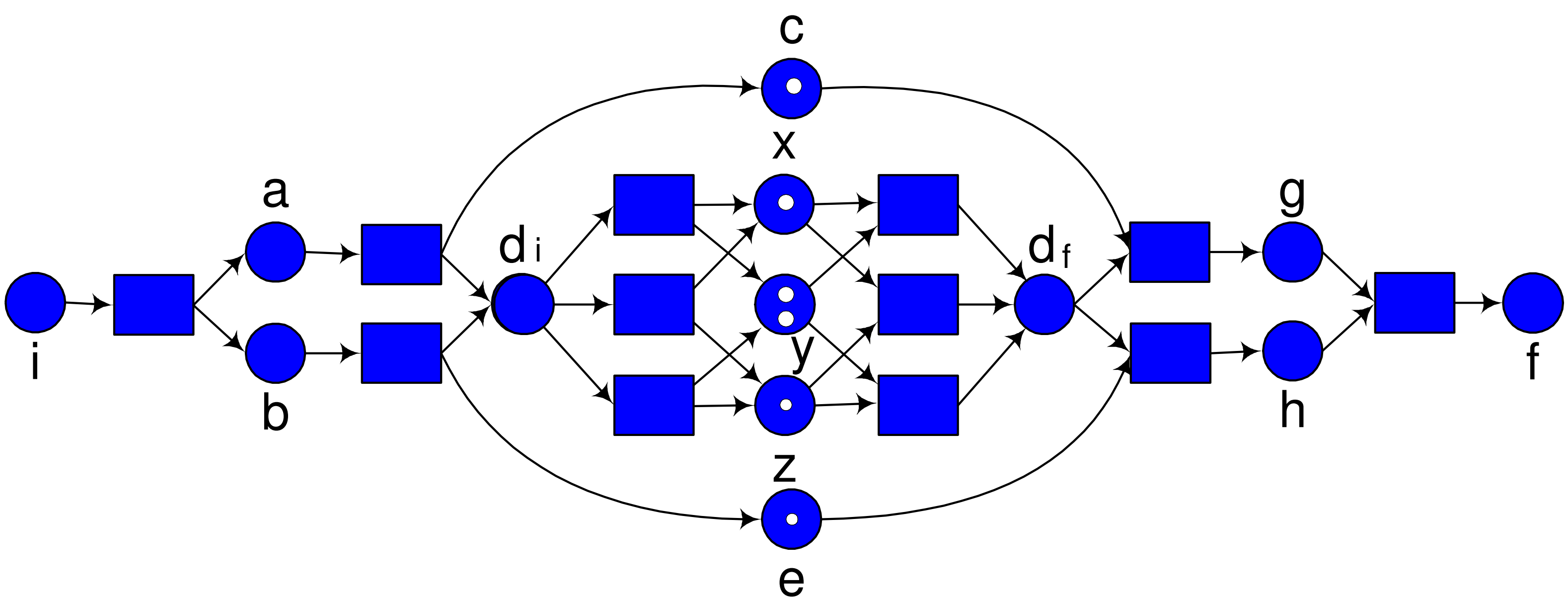

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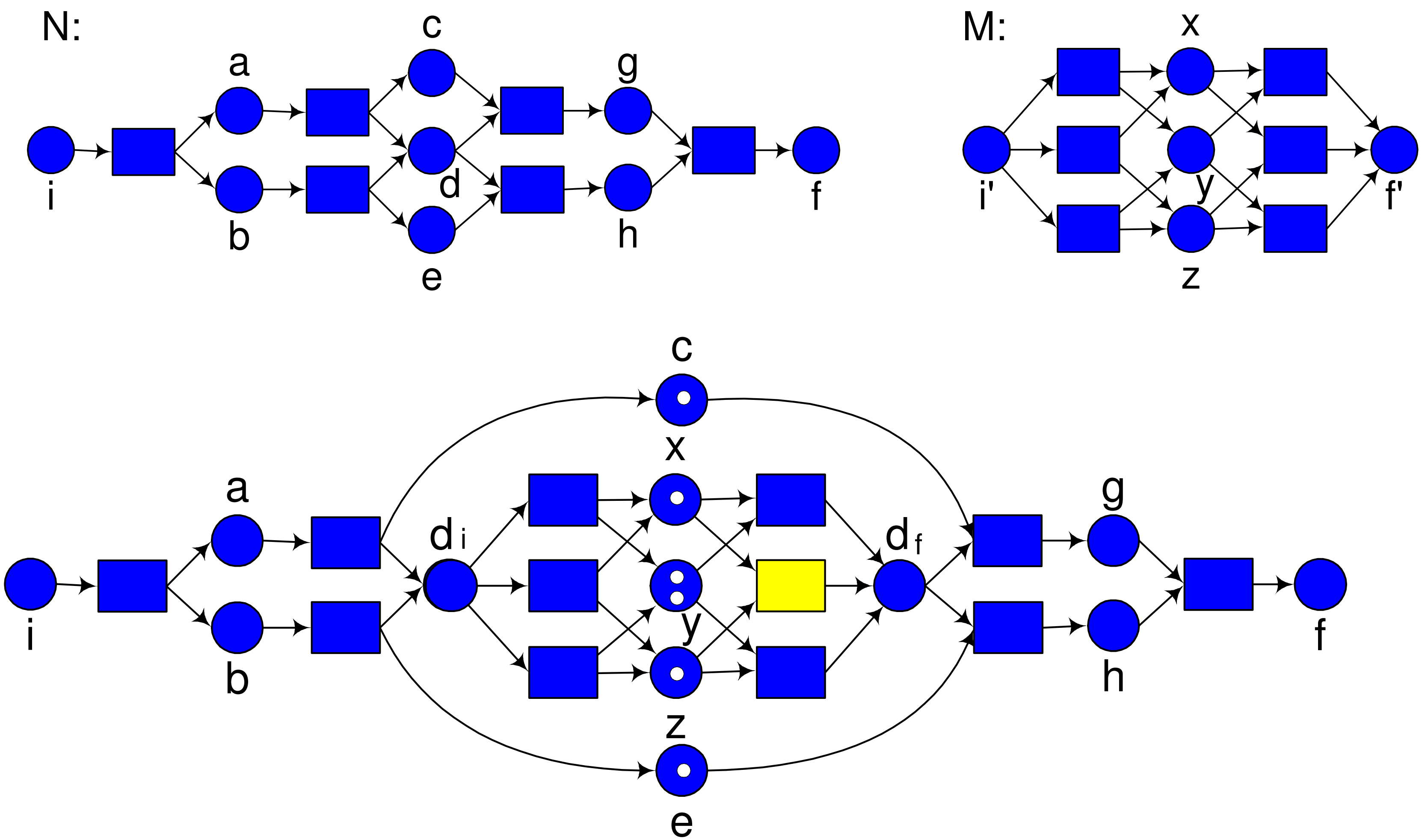

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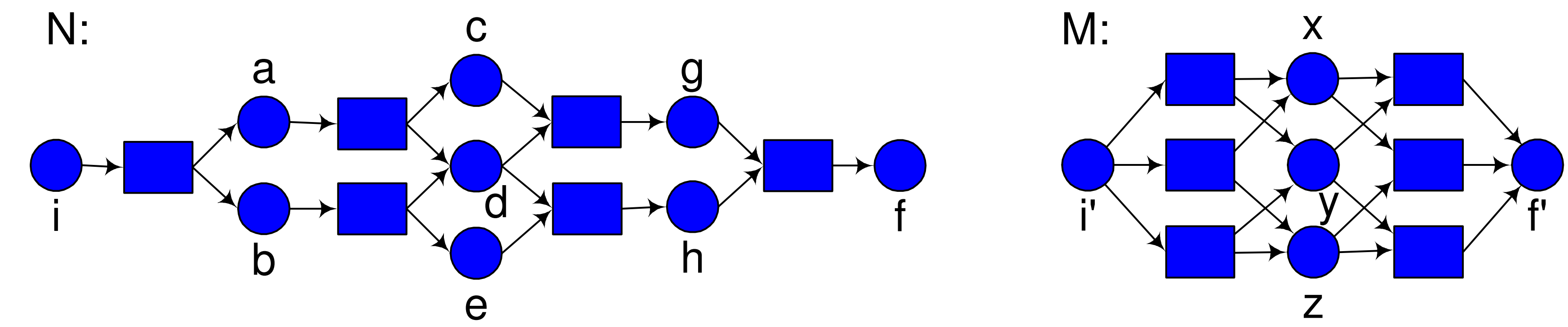

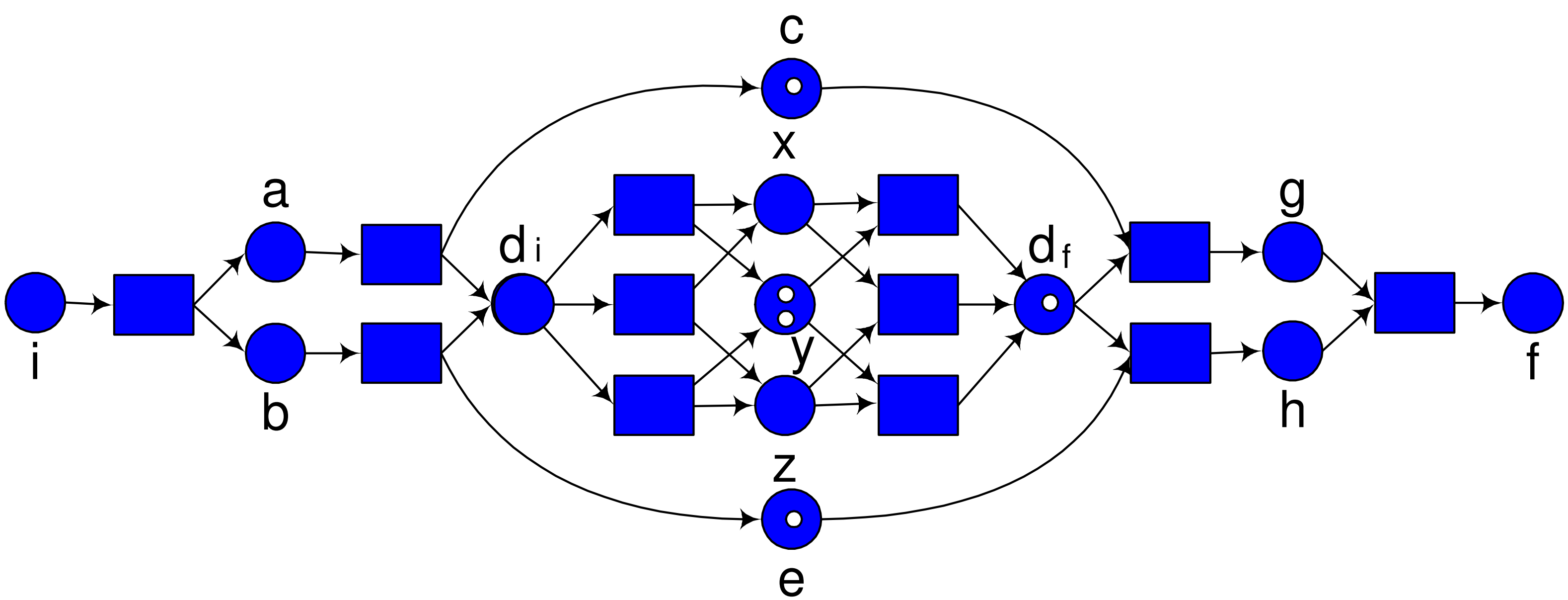

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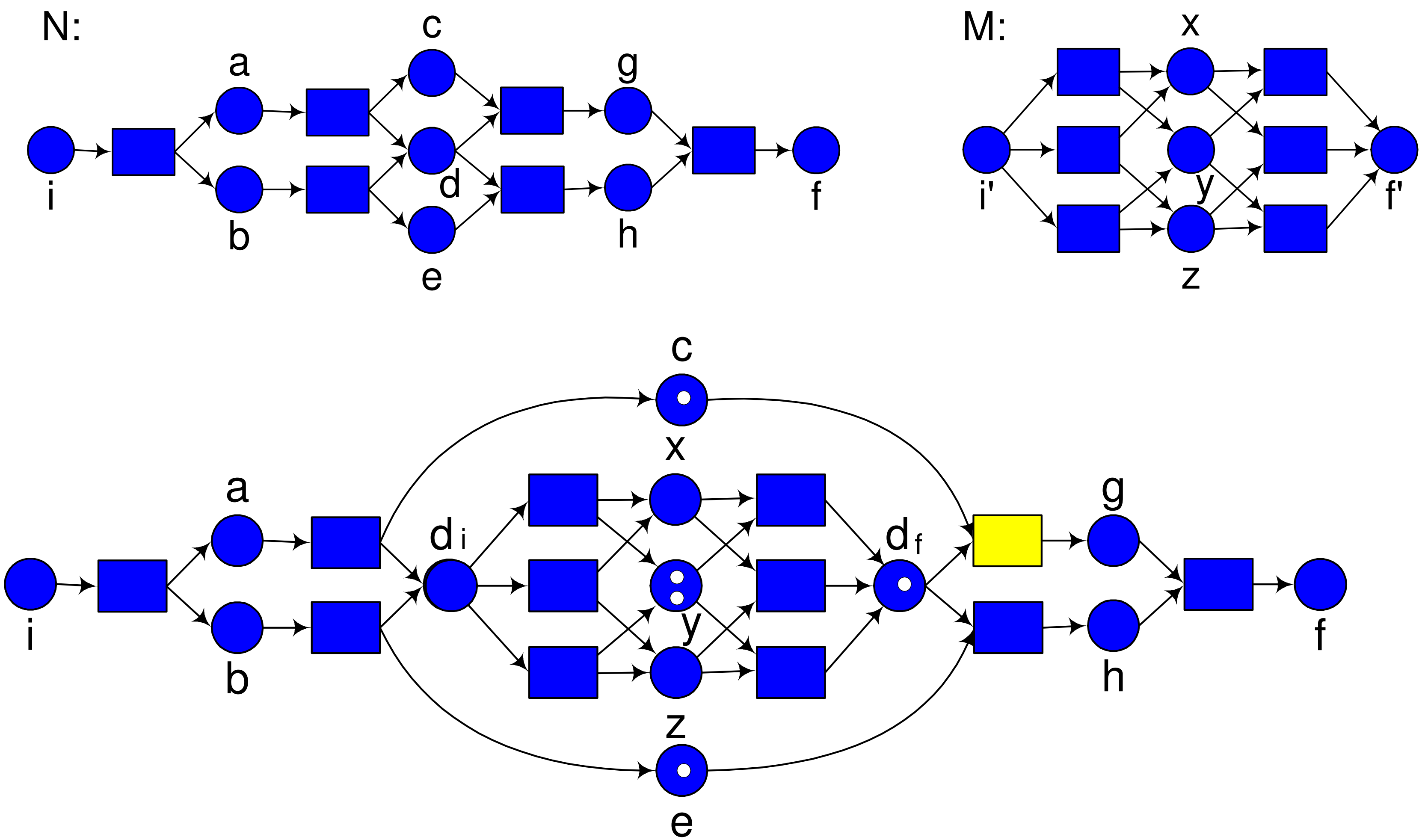

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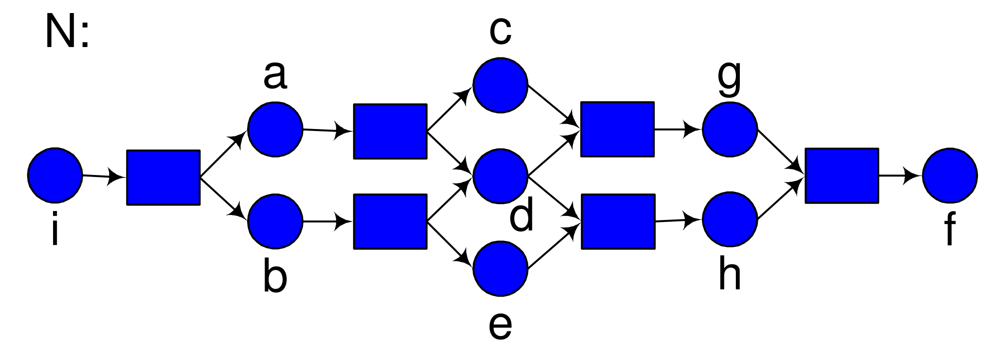

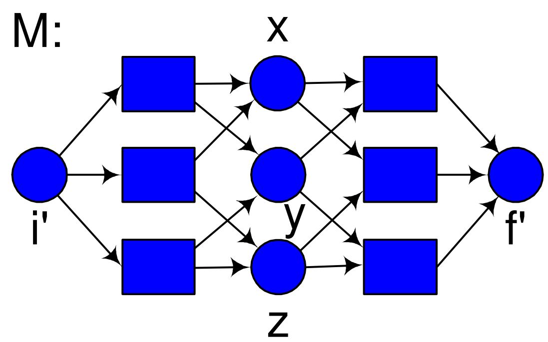

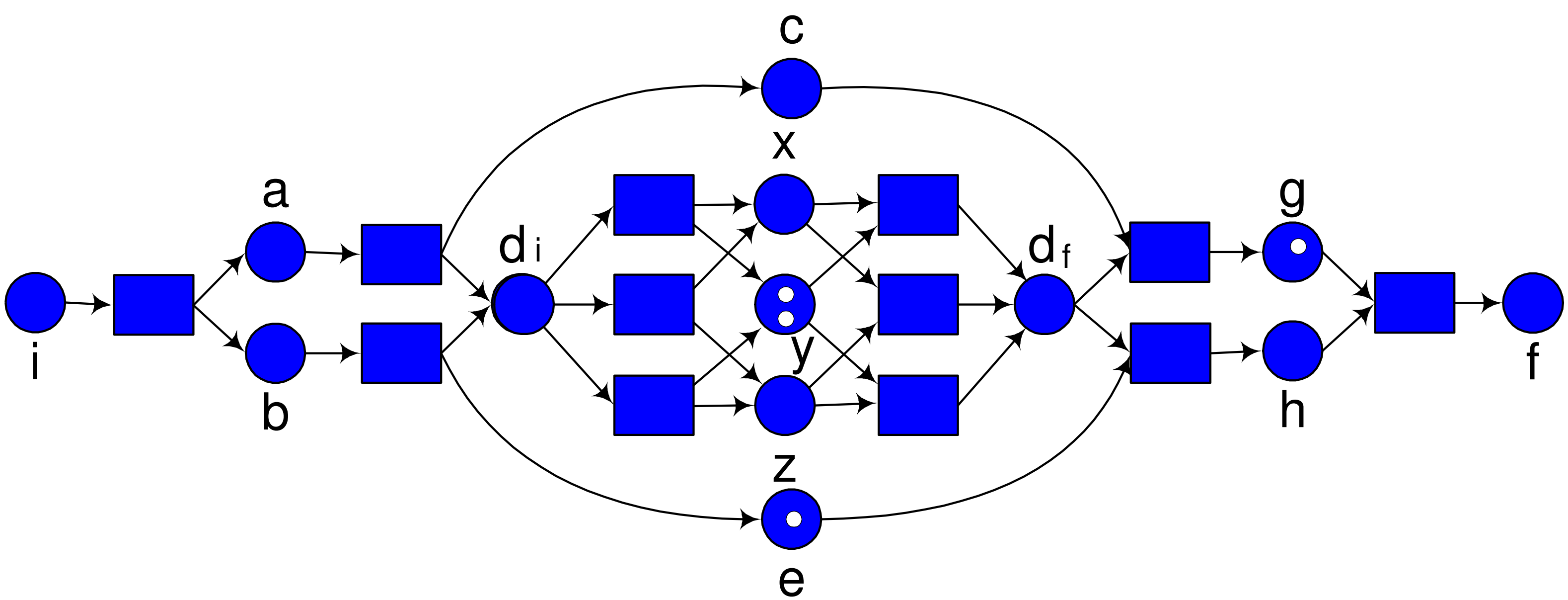

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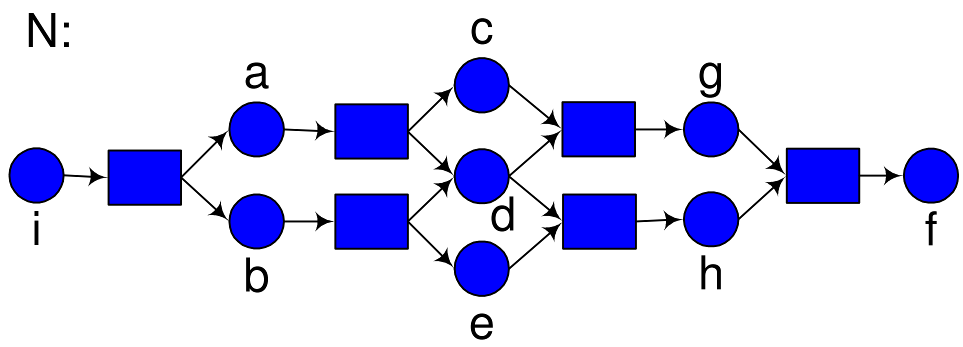

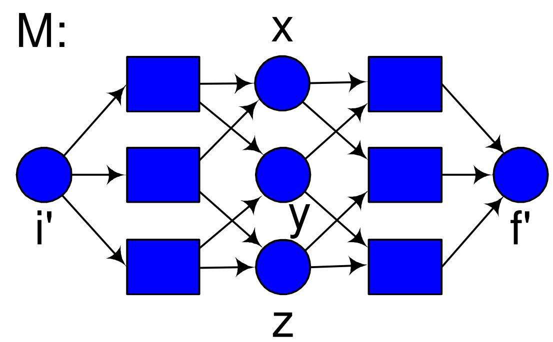

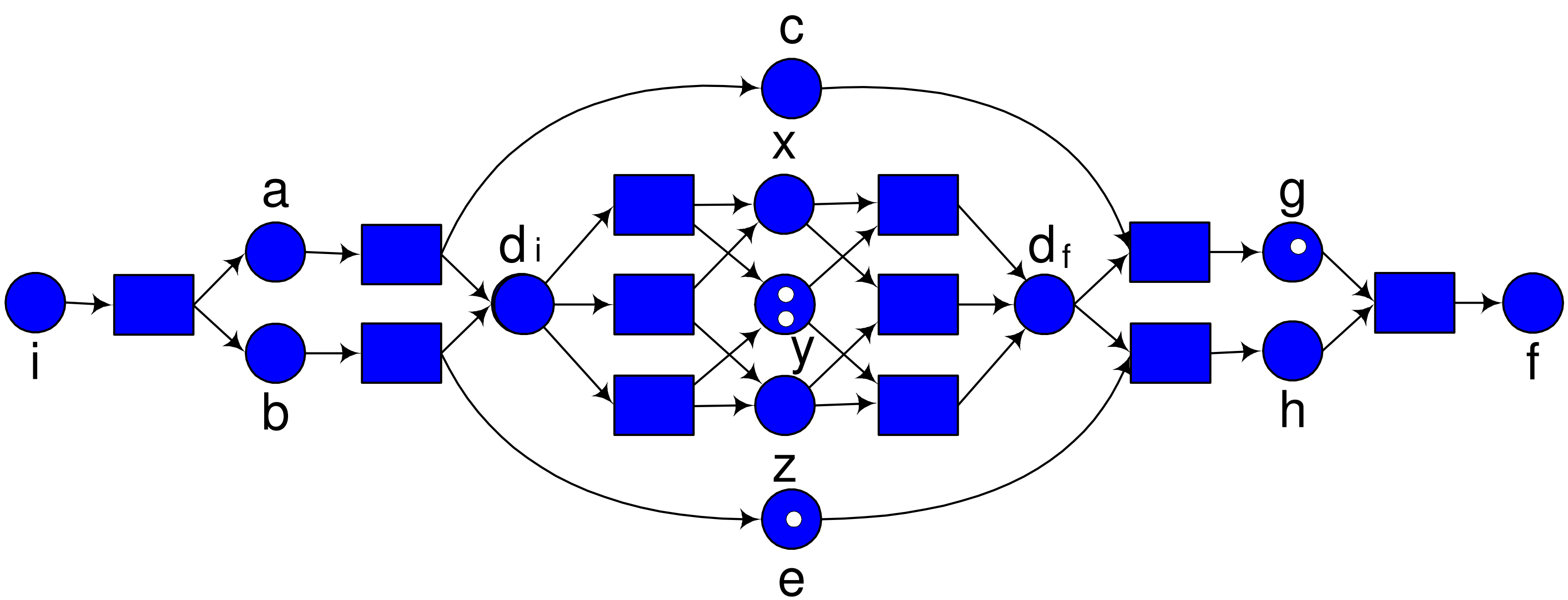

N and M are “sound”, but $N \otimes_d M$ is not!

New definition of soundness

A WF-net N with initial and final places i and f resp. is *k-sound* for $k \in \mathbb{N}$ iff $[f^k]$ is reachable from all markings m from $\mathcal{M}(N, [i^k])$.

A WF-net is *sound* iff it is k-sound for every natural k.

Old vs. new soundness

A WF-net N is sound iff:

- $[f]$ is reachable from any marking m from $\mathcal{M}(N, [i])$.
- Marking $[f]$ is the only marking reachable from $[i]$ with at least one token in $[f]$.
- There are no dead transitions in $(N, [i])$.

A WF-net N is (generalised) sound iff $[f^k]$ is reachable from all markings m from $\mathcal{M}(N, [i^k])$, for any for $k \in \mathbb{N}$.

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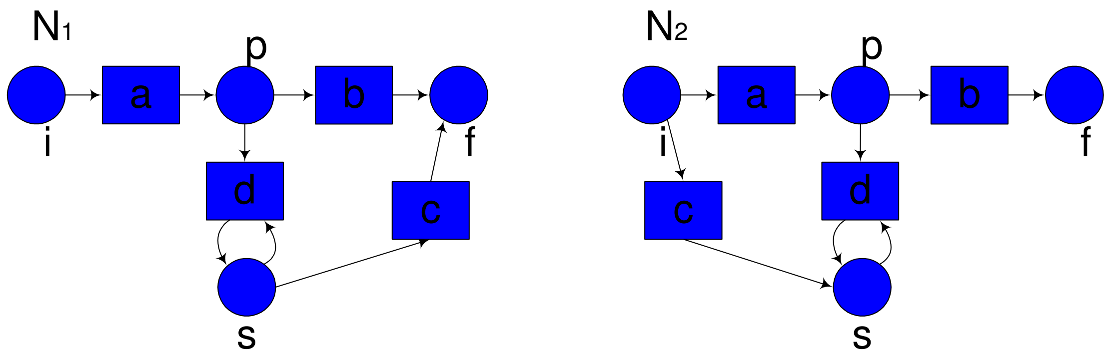
Structural non-redundancy



- Non-redundancy: every transition can potentially fire and every place can potentially obtain tokens, provided that there are enough tokens on the initial place.
- Persistency: it should be possible for every place (except for f) to become unmarked again—otherwise the net is guaranteed to be not sound.

Siphons

A set R of places is a siphon if ${}^{\bullet}R \subseteq R^{\bullet}$.
A siphon is a proper siphon if it is not empty.

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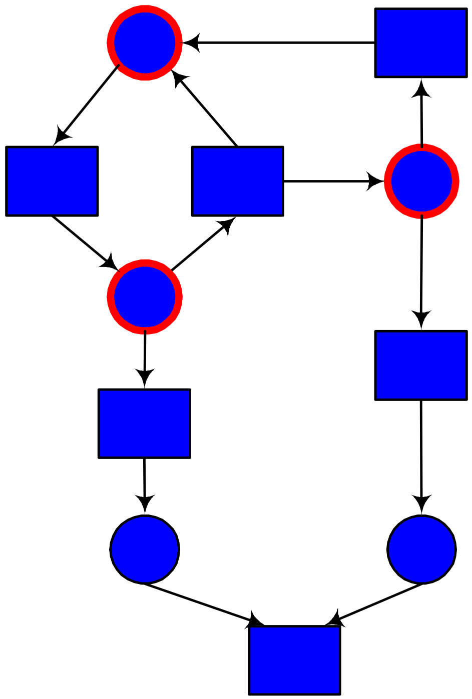

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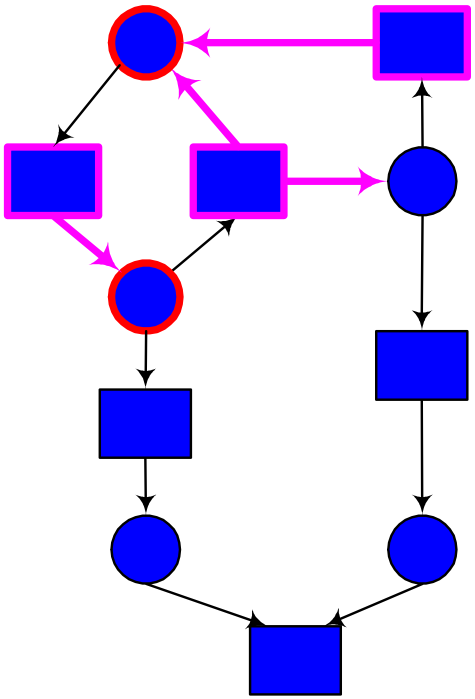

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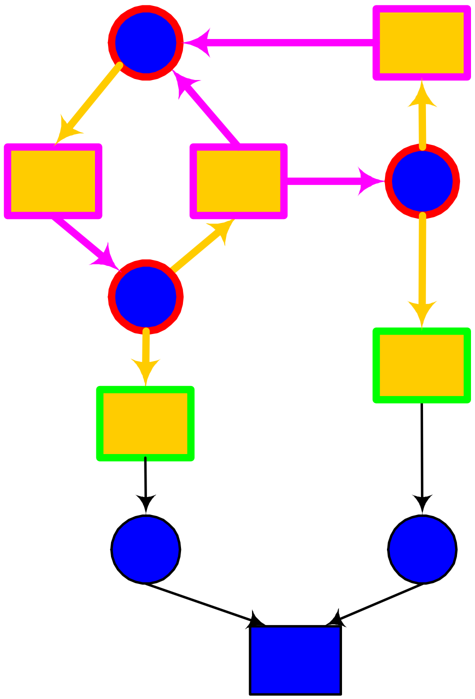

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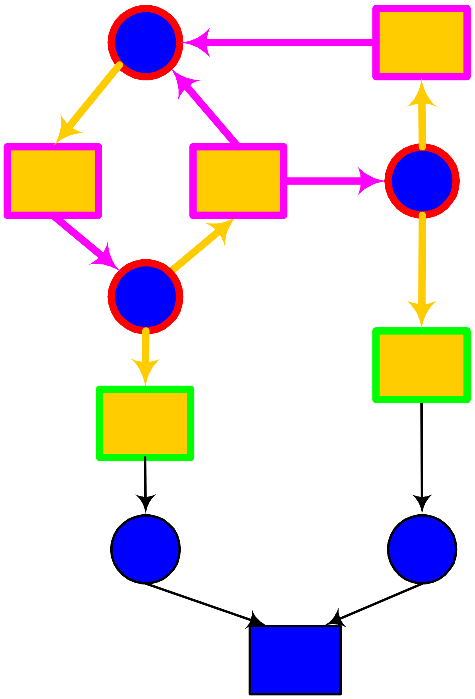
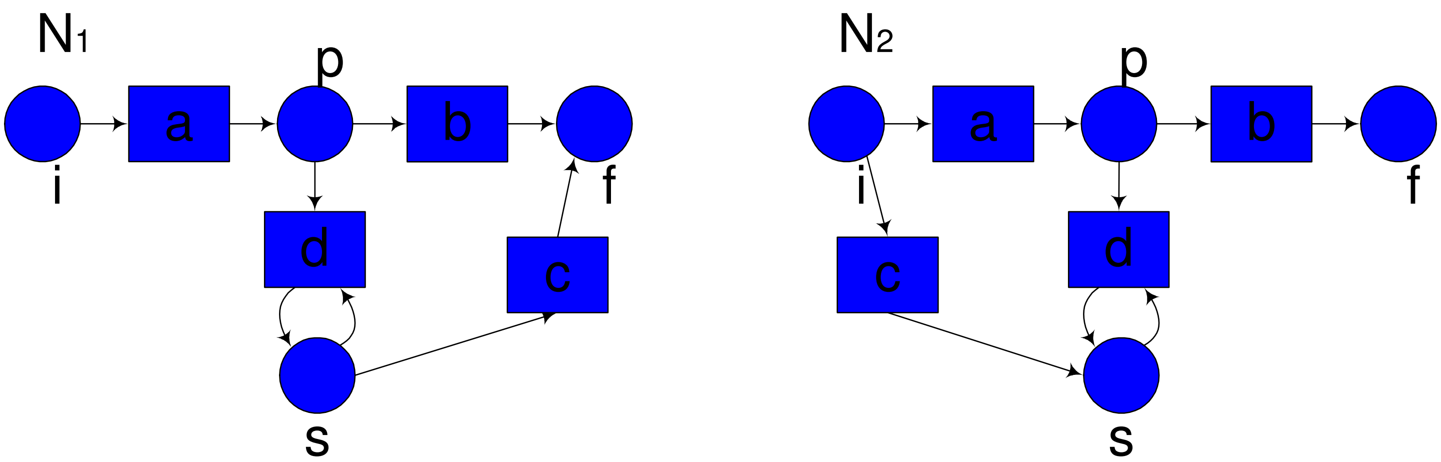


Unmarked siphons remain unmarked

Non-redundancy criterion

- A WF-net has no redundant places iff $P \setminus \{i\}$ contains no proper siphon.
- A WF-net has no redundant places iff it has no redundant transitions.



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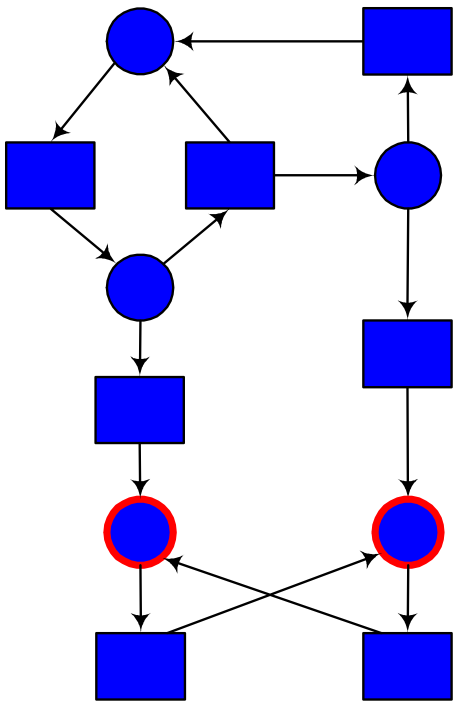

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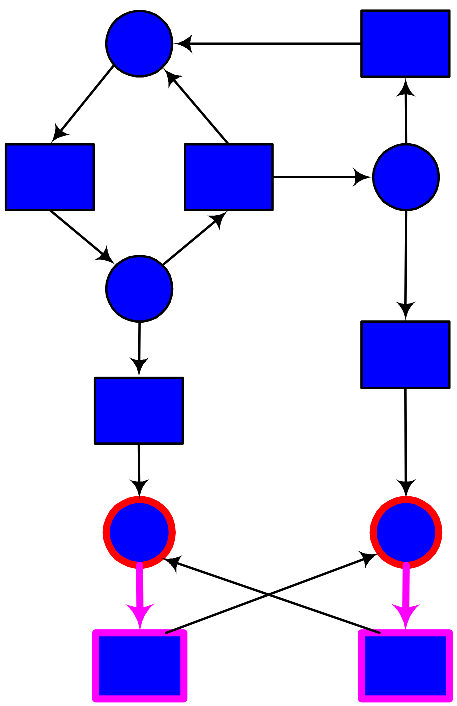

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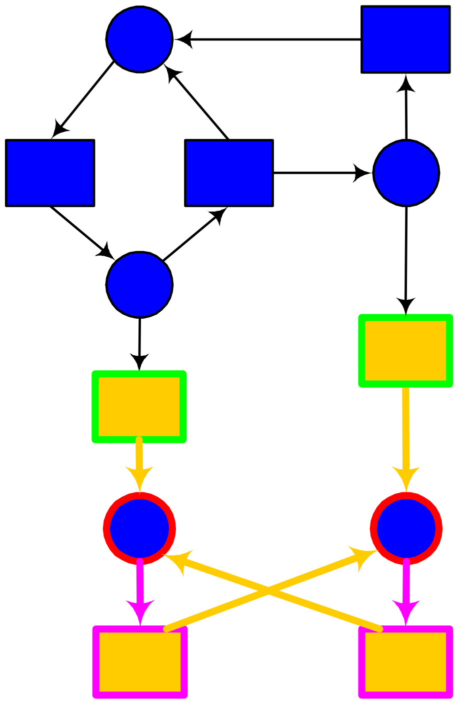

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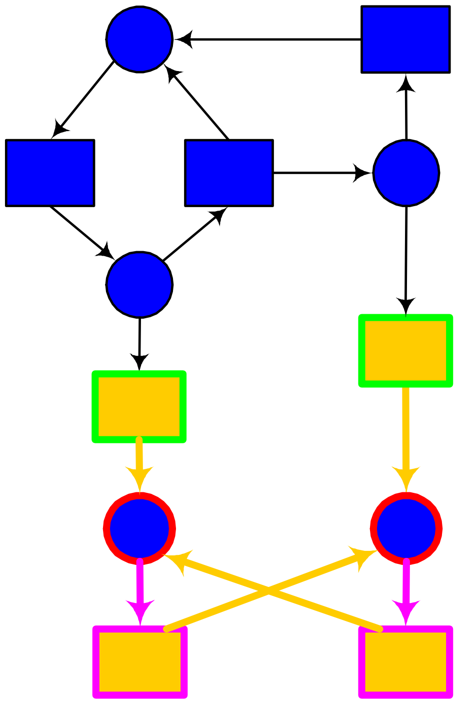
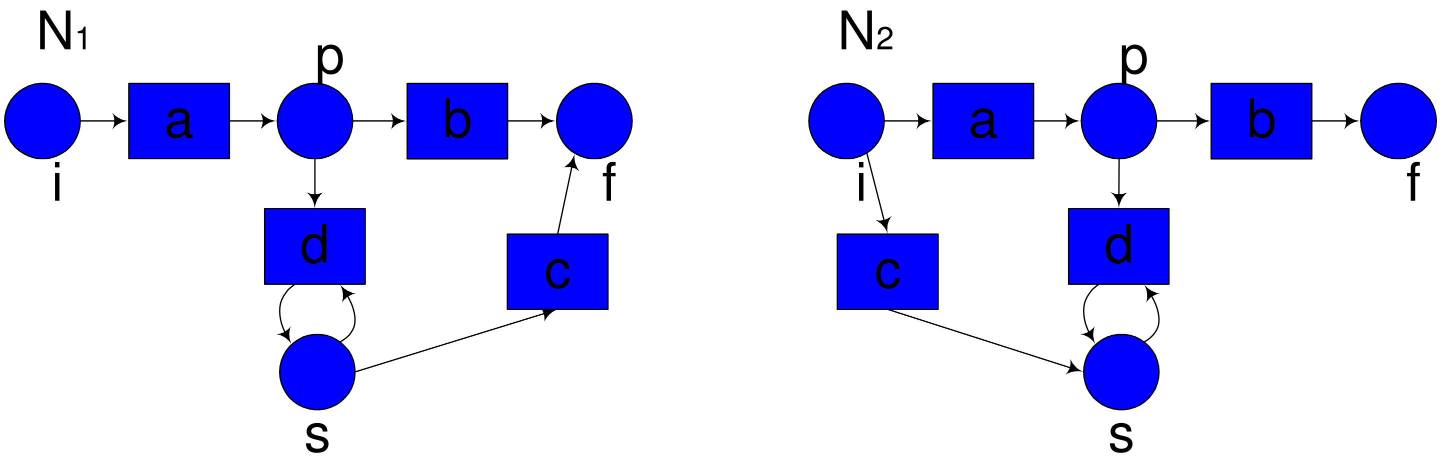


Marked traps remain marked.

Non-persistency criterion

- A WF-net has no persistent places iff $P \setminus \{f\}$ contains no proper trap.



A free check for the path property

Let N be a Petri net with

- a single source place i,
- a single sink place f,
- every transition of N has at least one input and one output place,
- $P \setminus \{i\}$ contains no proper siphon, and
- $P \setminus \{f\}$ contains no trap.

Then N is a WF-net (the path property holds).

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Batch workflow nets

A Batch Workflow net (BWF-net) N is a Petri net that has the following properties:

- N has a single source place i and a single sink place f;
- every transition of N has at least one input and one output place;
- every siphon of N contains i;
- every trap of N contains f.

WF-nets $\rightsquigarrow$ BWF-nets

Given a WF-net N,

- Find a maximal siphon X in $P \setminus \{i\}$.
  All places from X are redundant. $\Rightarrow$
  Transitions from $X^\bullet$ are redundant as well. $\Rightarrow$
- Construct N_1 by removing places from X and transitions from $X^\bullet$.
  - N_1 is either not a WF-net any more and so N was ill-designed,
  - or N_1 is a WF-net such that $(N_1, k[i])$ is WF-bisimilar to $(N, k[i])$ for any k.
- Find a maximal trap Y in $P \setminus \{f\}$.
  - If $Y \neq \emptyset$, N_1 has persistent places and is not sound.
  - Otherwise, N_1 is a BWF-net.

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Problem

Decidability of generalised soundness
for Batch workflow nets

Some facts

Marking Equation Lemma

Given a finite firing sequence σ of a net N: $m \stackrel{\sigma}{\longrightarrow} m'$, the following equation holds:
$m' = m + F^+ \cdot \overrightarrow{\sigma} - F^- \cdot \overrightarrow{\sigma}$, or in other words,
$m' = m + F \cdot \overrightarrow{\sigma}$.

The set of all markings reachable from $k[i]$ in N $\quad \mathcal{R}(k \cdot \mathbf{i})$ is a subset of $\mathcal{G}_k = \{k \cdot \mathbf{i} + F \cdot v \mid v \in \mathbb{Z}^T\} \cap \mathbb{N}^P$

The reverse is not true: not every marking $m' = m + F \cdot v$, $v \in \mathbb{N}^T$, is reachable from the marking m,
i.e. $\mathcal{G}_k \not\subseteq \mathcal{R}(k \cdot \mathbf{i})$.

Fundamental lemmas

- Let N be a sound BWF-net and $m \in \mathcal{G}_k$ for some $k \in \mathbb{N}$. Then there exists $\ell \in \mathbb{N}$ such that $(k+\ell) \cdot \mathbf{i} \xrightarrow{*} m + \ell \cdot \mathbf{f}$.
- Let N be a sound BWF-net and $m \in \mathcal{G}_k$. Then $m \xrightarrow{*} k \cdot \mathbf{f}$.
- N is sound iff all markings from $\mathcal{G} = \bigcup_{k \in \mathbb{N}} \mathcal{G}_k$, i.e. $\mathcal{G} = \{k \cdot \mathbf{i} + F \cdot v \mid k \in \mathbb{N} \wedge v \in \mathbb{Z}^T\} \cap \mathbb{N}^P$, terminate properly in N.

Fundamental lemmas

- Let N be a sound BWF-net and $m \in \mathcal{G}_k$ for some $k \in \mathbb{N}$. Then there exists $\ell \in \mathbb{N}$ such that $(k+\ell)\cdot \mathbf{i} \xrightarrow{*} m + \ell \cdot \mathbf{f}$.
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Next: Use the regularity of $\mathcal{G}$ to reduce the problem of proper termination of markings of $\mathcal{G}$ to the problem of proper termination of some finite subset Γ of $\mathcal{G}$.

Fundamental lemmas

$$\mathcal{G} = \{k \cdot \mathbf{i} + F \cdot v \mid k \in \mathbb{N} \wedge v \in \mathbb{Z}^T\} \cap \mathbb{N}^P$$

Let $m_1, m_2 \in \mathcal{G}$ be markings that terminate properly and $m = \lambda_1 m_1 + \lambda_2 m_2$ for some $\lambda_1, \lambda_2 \in \mathbb{N}$. Then $m \in \mathcal{G}$ and it terminates properly.

$$\mathcal{H} = \{a \cdot \mathbf{i} + F \cdot v \mid a \in \mathbb{Q}^+ \wedge v \in \mathbb{Q}^T\} \cap (\mathbb{Q}^+)^P$$

is a convex polyhedral cone and has a finite set of generators such that $e_1, \ldots, e_n \in \mathcal{G}$.

$$\Gamma = \{\sum_i \alpha_i \cdot e_i \mid 0 \leq \alpha_i \leq 1\} \cap \mathcal{G}$$

Conclusion

The generalised soundness is decidable

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Redundant and persistent places can be easily found as siphons and traps

Future work

- Optimise the algorithm we have now.
- Develop soundness preserving Petri net reduction techniques that can be employed prior to the use of the soundness decision procedure to speed up the check.

Proof

Let $m \in \mathcal{G}_k$, i.e. $m = k \cdot \mathbf{i} + F \cdot v$ for some $v \in \mathbb{Z}^T$.
Then there are $v_1, v_2 \in \mathbb{N}^T$ such that $v = v_1 - v_2$.
Note that $F = F^+ - F^-$. So
$m = k \cdot \mathbf{i} + F^+ \cdot v_1 + F^- \cdot v_2 - F^- \cdot v_1 - F^+ \cdot v_2$.

Since there are no redundant places,
there exist $a, b \in \mathbb{N}$ and markings A, B such that
$a \cdot \mathbf{i} \xrightarrow{*} A + F^+ \cdot v_1$ and $b \cdot \mathbf{i} \xrightarrow{*} B + F^- \cdot v_2$.

Then $(k + a + b) \cdot \mathbf{i} \xrightarrow{*} k \cdot \mathbf{i} + A + F^+ \cdot v_1 + B + F^- \cdot v_2 =$
$m + A + F^- \cdot v_1 + B + F^+ \cdot v_2$.

Now we need to prove that $A + F^- \cdot v_1 \xrightarrow{*} a \cdot \mathbf{f}$ and
$B + F^+ \cdot v_2 \xrightarrow{*} b \cdot \mathbf{f}$.

Proof (2)

Let γ_2 be an arbitrary firing sequence with $\overrightarrow{\gamma_2} = v_2$.
Then $b \cdot \mathbf{i} \xrightarrow{*} B + F^- \cdot v_2 \xrightarrow{\gamma_2} B + F^+ \cdot v_2$,
and since N is sound, $B + F^+ \cdot v_2 \xrightarrow{*} b \cdot \mathbf{f}$.

Now consider a marking $A + F^- \cdot v_1$.
For an arbitrary firing sequence γ_1 with $\overrightarrow{\gamma_1} = v_1$,
$A + F^- \cdot v_1 \xrightarrow{\gamma_1} A + F^+ \cdot v_1$.
Moreover, we have $a \cdot \mathbf{i} \xrightarrow{*} A + F^+ \cdot v_1$,
and since N is sound, $A + F^- \cdot v_1 \xrightarrow{*} A + F^+ \cdot v_1 \xrightarrow{*} a \cdot \mathbf{f}$.

Thus we obtain
$m + A + F^- \cdot v_1 + B + F^+ \cdot v_2 \xrightarrow{*} m + (a + b) \cdot \mathbf{f}$.
So with $\ell = a + b$ the lemma holds.