1. a. Een normaal aan $S = 0$ is ∇S, dus hier $\nabla S = (2xz, -2yz, x^2 - y^2 + 4z^3) = (2, -2, 4)$.

   b. Het raakvlak staat loodrecht op de normaal, zodat een vergelijking van het raakvlak is $(x-1, y-1, z-1) \cdot \nabla S = 2(x-1) - 2(y-1) + 4(z-1) = 0$, of $z = 1 - \frac{1}{2}x + \frac{1}{2}y$.

   c. In $x = y = 1$ geldt $z^4 - 1 = 0$, dus $z = \pm 1$. Impliciet differentiëren van S levert $2xz + (x^2 - y^2)z_x + 4z^3 z_x = z(2 + 4z^2 z_x) = \pm(2 + 4z_x) = 0$, dus $z_x = -\frac{1}{2}$. Evenzo $-2yz + (x^2 - y^2)z_y + 4z^3 z_y = z(-2 + 4z^2 z_y) = \pm(-2 + 4z_y) = 0$, dus $z_y = \frac{1}{2}$.

2. a. $f_1(\mathbf{0}) = 1$, $f_2(\mathbf{0}) = -1$, $f_{11}(\mathbf{0}) = 4$, $f_{12}(\mathbf{0}) = 1$, $f_{22}(\mathbf{0}) = 0$, zodat
   $g_x(\mathbf{0}) = a(f_1 + f_2) = 0$,
   $g_y(\mathbf{0}) = b(f_1 - f_2) = 2b$,
   $g_{xx}(\mathbf{0}) = a^2(f_{11} + 2f_{12} + f_{22}) = 6a^2$,
   $g_{yy}(\mathbf{0}) = b^2(f_{11} - 2f_{12} + f_{22}) = 2b^2$,
   $g_{xy}(\mathbf{0}) = ab(f_{11} - f_{22}) = 4ab$.

   b. $D_{\boldsymbol{u}} g(\mathbf{0}) = (1g_x + 2g_y)/\sqrt{5} = \frac{4}{5}\sqrt{5}b$.

   c. $g(\mathbf{0}) + xg_x(\mathbf{0}) + yg_y(\mathbf{0}) + \frac{1}{2}(x^2 g_{xx}(\mathbf{0}) + 2xyg_{xy}(\mathbf{0}) + y^2 g_{yy}(\mathbf{0})) = 2by + 3a^2x^2 + 4abxy + b^2y^2$.

3. Verwissel integratievolgorde tot

$$\int_0^2 \int_0^{y^2} \frac{5x}{\sqrt{49 + y^5}}\, \mathrm{d}x\mathrm{d}y = \int_0^2 \frac{\frac{5}{2}y^4}{\sqrt{49 + y^5}}\, \mathrm{d}y = \sqrt{49 + y^5}\Big|_0^2 = \sqrt{81} - \sqrt{49} = 9 - 7 = 2.$$

4. a. $\boldsymbol{v} = \dot{\boldsymbol{x}} = -2\sin(2t)\,\boldsymbol{i} + 2\cos(2t)\,\boldsymbol{j} + 2t\,\boldsymbol{k}$,
   $v = |\boldsymbol{v}| = 2\sqrt{1 + t^2}$,
   $\boldsymbol{a} = \ddot{\boldsymbol{x}} = -4\cos(2t)\,\boldsymbol{i} - 4\sin(2t)\,\boldsymbol{j} + 2\,\boldsymbol{k}$.

   b.
$$\int_0^{\frac{3}{4}} |\boldsymbol{v}|\, \mathrm{d}t = \Big[t\sqrt{1 + t^2} + \ln\big(t + \sqrt{1 + t^2}\,\big)\Big]_0^{\frac{3}{4}} = \tfrac{15}{16} + \ln 2.$$

5. a. Een noodzakelijke voorwaarde is:
   $\nabla \times \boldsymbol{F} = (3 - b)y^2\,\boldsymbol{i} + (a - 2)xz^{-1}\,\boldsymbol{j} + 0\,\boldsymbol{k} = \mathbf{0}$,
   dus $a = 2$ en $b = 3$.

   b. In dit geval heeft $\boldsymbol{F}$ een potentiaal, namelijk $\phi = x^2 \ln z + y^3 z$, zodat de integraal gelijk is aan het verschil van de potentiaalwaarden, dus $\phi(2, 0, \mathrm{e}) - \phi(1, 1, 1) = 4 - 1 = 3$.

6.
$$S:\ \boldsymbol{r}(\theta, \lambda) = (3\cos\theta, 3\sin\theta, \lambda), \quad 0 \leqslant \theta \leqslant 2\pi,\ 0 \leqslant \lambda \leqslant 1,$$
$$\boldsymbol{n} = \frac{\partial \boldsymbol{r}}{\partial \theta} \times \frac{\partial \boldsymbol{r}}{\partial \lambda} = (3\cos\theta, 3\sin\theta, 0).$$
$$\boldsymbol{F}(\boldsymbol{r}) = \big(3\lambda^2\cos\theta, \sin\theta, 9\lambda\cos\theta\sin\theta\big),$$
$$(\boldsymbol{F}\cdot\boldsymbol{n})\,\mathrm{d}S = (9\lambda^2\cos^2\theta + 3\sin^2\theta)\,\mathrm{d}\theta\mathrm{d}\lambda.$$
$$\iint_{\mathcal{S}} (\boldsymbol{F}\cdot\boldsymbol{n})\,\mathrm{d}S = \int_0^{2\pi}\int_0^1 (9\lambda^2\cos^2\theta + 3\sin^2\theta)\,\mathrm{d}\lambda\mathrm{d}\theta = 6\pi.$$