Algorithmic Methods in Queueing Theory (AIQT)
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Assignment 2

Deadline: April 1 2017

General information: This is the second assignment for the course Algorithmic Methods in Queueing Theory (AIQT). The assignment requires the implementation of some of the notions and approaches discussed during the course. The assignment can be made in groups of at most two persons, and it is also required to submit a report with your findings.

The report of the assignment should be sent by email to s.kapodistria@tue.nl with the subject "Assignment 2 for course AIQT" before 23:59 on April 1 2017. To this purpose, you need to create a single pdf file with the answers to the assignment. Since, the assignment requires programming, you can use your favourite programming language or mathematical/statistical software, however all program code should be included as an appendix to the pdf file of the report and the original source code, in original format, should be submitted together with the report, i.e. in the same email as the solutions of the assignment. Zip all files together (pdf file and code files) and submit one single zipped file by email to s.kapodistria@tue.nl.

The findings reported in the assignment should be presented in a clear, concise way and the code should be documented and should provide sufficient information for confirmation and replication of the results. Moreover, all graphs/tables should have a caption explaining what is depicted and all axes should be labeled.

Lastly, but most importantly, create a cover for the report and include the names and student ID numbers of all the members of the group, the course name and the assignment number.

Assignment description:

Consider the symmetric join the shortest queue model: Assume a queueing system consisting of two identical exponential servers, in which each server has its own dedicated (infinite) queue and serves the customers in that queue according to FCFS. The duration of a service time is exponentially distributed at rate μ. Customers arrive to the system according to a Poisson process at rate λ and join the shortest queue. In case of a tie, the arriving customers join either queue with the same probability (1/2).

We can model the above described system as a two-dimensional stochastic process by considering the corresponding two queue lengths, i.e. $\{(X_1(t), X_2(t)),\ t \geq 0\}$, with $X_i(t)$ the number of customers in the system of the i-th server, $i = 1, 2$. Note that the two-dimensional stochastic process $\{(X_1(t), X_2(t)),\ t \geq 0\}$ is a Markov chain defined on $\mathbb{N}_0 \times \mathbb{N}_0$. This modelling renders the Markov chain into an inhomogeneous random walk in the quadrant. In order, to convert the Markov chain into a homogeneous random walk in the quadrant, we propose the following transformation

$$N(t) = \min\{X_1(t), X_2(t)\} \text{ and } M(t) = |X_1(t) - X_2(t)|.$$

The system as a whole is stable if $\lambda < 2\mu$ or equivalently if $\rho = \frac{\lambda}{2\mu} < 1$.

Assignment questions:

1) For $\rho = 0.1, 0.5, 0.9$, write a numerical routine based on the compensation approach and calculate the probability of an empty system.

2) For $\rho = 0.1, 0.5, 0.9$, write a numerical routine based on the power series algorithm and calculate the probability of an empty system.

3) For $\rho = 0.1, 0.5, 0.9$, truncate the minimum queue length variable, $N(t) = \min\{X_1(t), X_2(t)\}$, and derive the probability of an empty system as a function of the truncation value. Considering increased values of the truncation, does the solution converge to the desired value? Motivate intuitively your answer.

4) Compare the above approaches in terms of accuracy, computation time and memory space.

5) Describe the procedure for the calculation of the covariance of the two queue lengths in stationarity, i.e. $\mathrm{Cov}(X_1, X_2)$, by writing a pseudo-code, based on your favourite algorithmic procedure treated in classroom.