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Composable Markov Building Blocks

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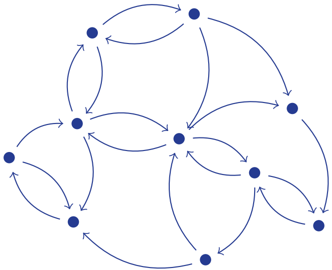
Problem (abstract)



- state space with known transitions
- goal: make Markov model
- problem: only local observations

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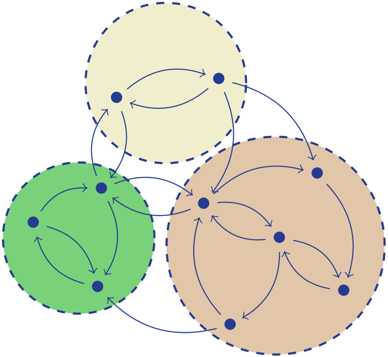

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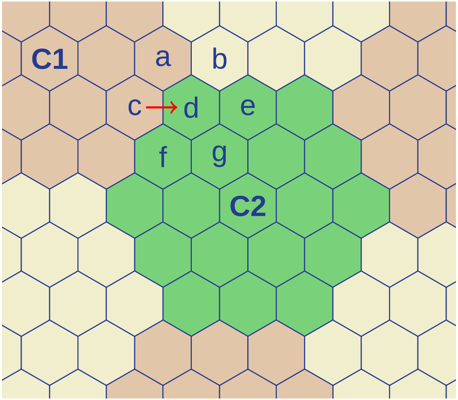
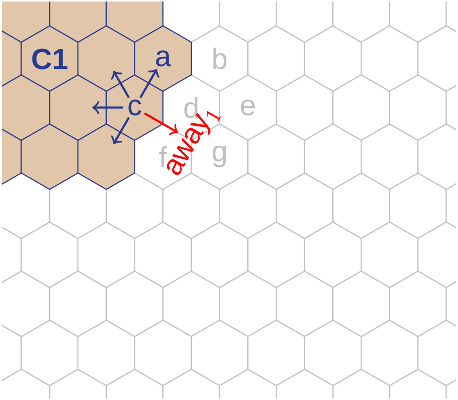
Problem (more concrete)



- two-dimensional area
- *granules* (a-g)
- *clusters* (C1,C2)
- per cluster: local Markov model (or frequencies) of transitions

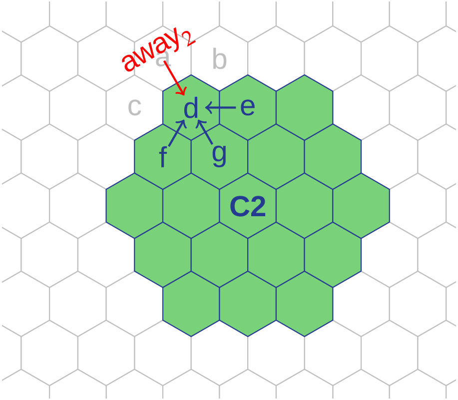
Question



We know:

- P(X_{t+1} = away$_1$ | X_t = c)

Question



We know:

- $\mathrm{P}(X_{t+1} = \text{away}_1 \mid X_t = \text{c})$
- $\mathrm{P}(X_{t+1} = \text{d} \mid X_t = \text{away}_2)$
- . . . etc.

Question

$\mathrm{P}(X_{t+1} = \text{d} \mid X_t = \text{c})$?

Step 1. Rewrite conditional probabilities to absolute frequencies (if necessary).

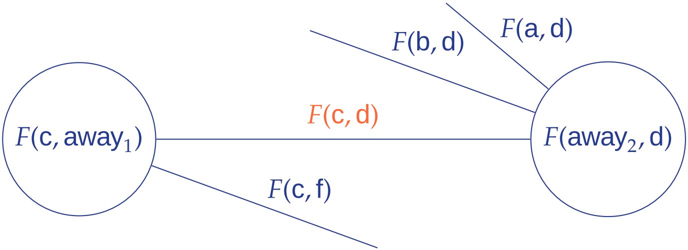
$$\mathrm{P}(X_{t+1} = \mathrm{d} \mid X_t = \mathrm{c}) = \frac{\text{\# of c} \longrightarrow \text{d transitions}}{\text{\# of transitions from c}}$$

$$F(\mathrm{c}, \mathrm{d}) = \frac{\text{\# of c} \longrightarrow \text{d transitions}}{\text{total \# of transitions}}$$

$$F(\mathrm{c}, \mathrm{d}) = \pi_{\mathrm{C1}}(\mathrm{c}) \cdot \mathrm{P}(X_{t+1} = \mathrm{d} \mid X_t = \mathrm{c})$$

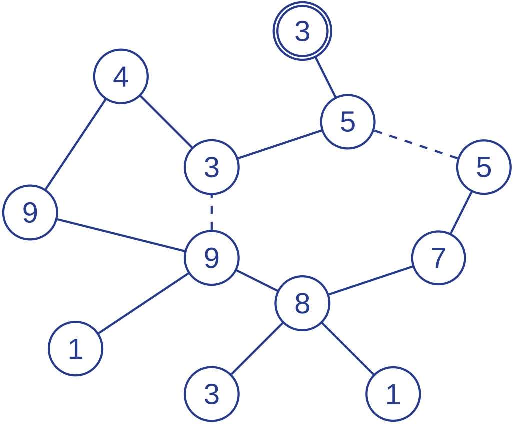
Step 2. Formulate system of linear equations.

$$
\begin{aligned}
F(\mathrm{c}, \mathrm{away}_1) &= F(\mathrm{c}, \mathrm{d}) + F(\mathrm{c}, \mathrm{f}) \\
F(\mathrm{away}_2, \mathrm{d}) &= F(\mathrm{a}, \mathrm{d}) + F(\mathrm{b}, \mathrm{d}) + F(\mathrm{c}, \mathrm{d}) \\
F(\mathrm{c}, \mathrm{d}) &\geq 0
\end{aligned}
$$



Step 3. Solve system using graph.

- every solution can be written as $\mathbf{p} + \alpha_1 \mathbf{c}_1 + \alpha_2 \mathbf{c}_2 + \ldots + \alpha_q \mathbf{c}_q$.



- $\mathbf{p}$ is any *particular* solution
- $\mathbf{c}_i$'s are *fundamental cycles* of the graph
- Both can be found using the spanning tree.

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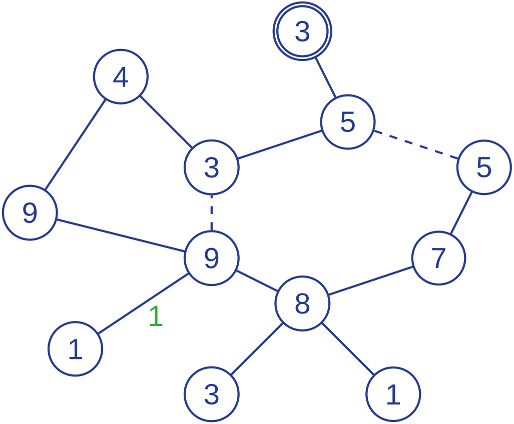

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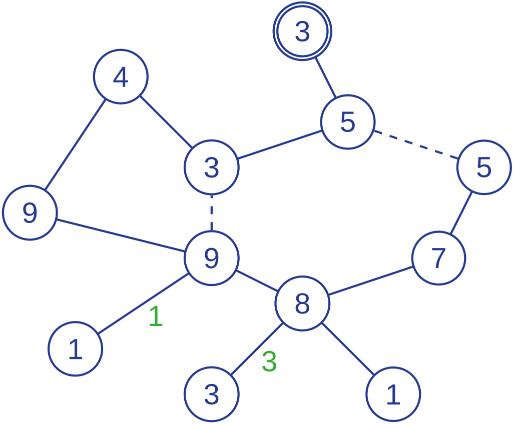

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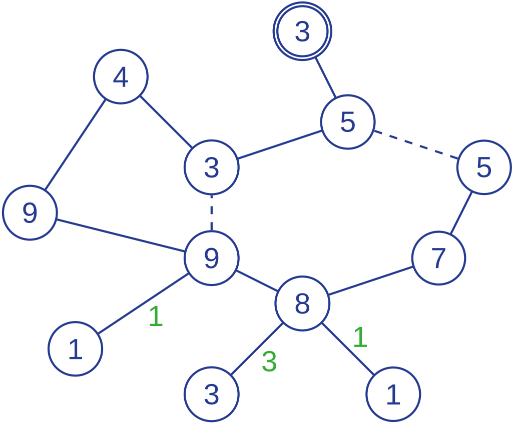

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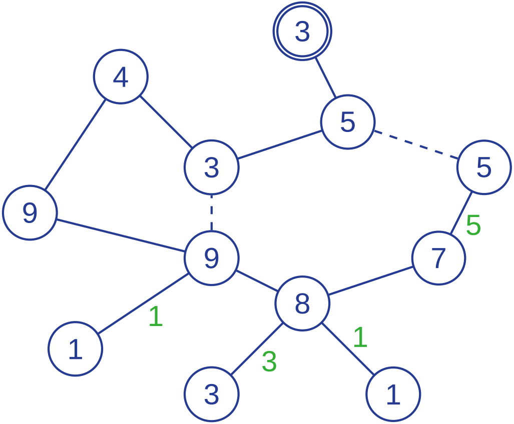

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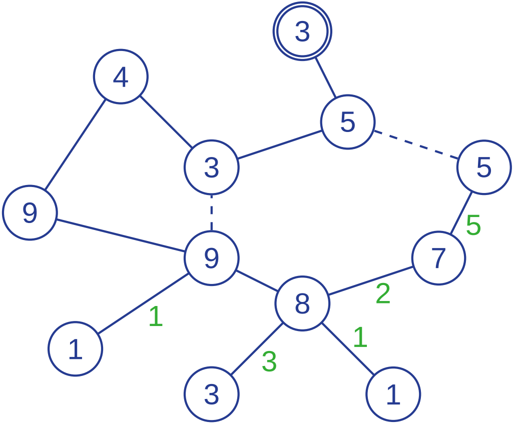

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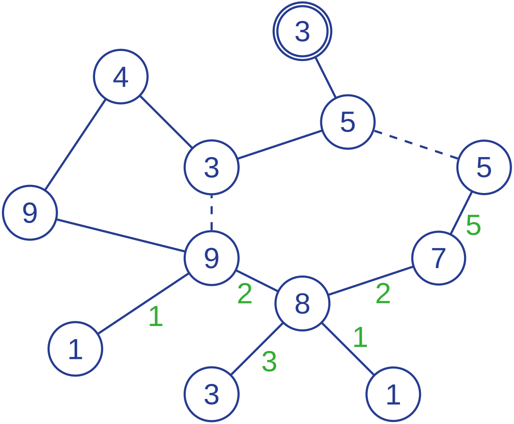

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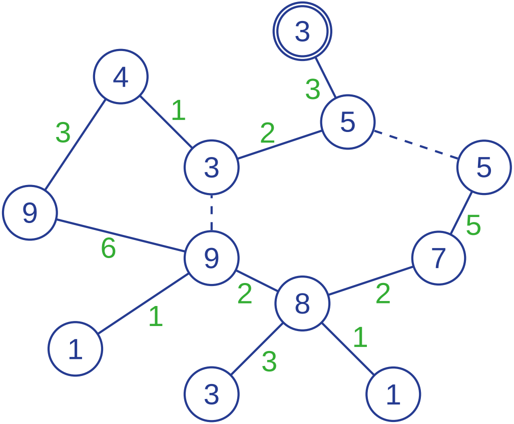

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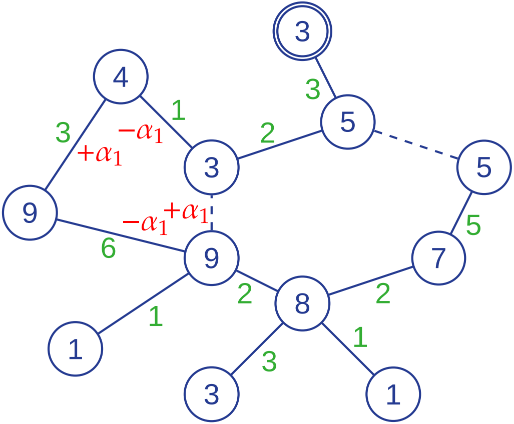

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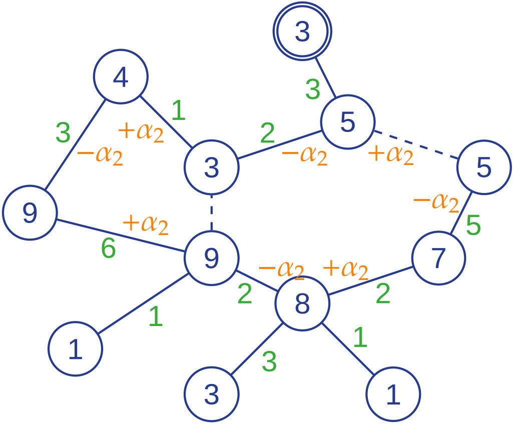

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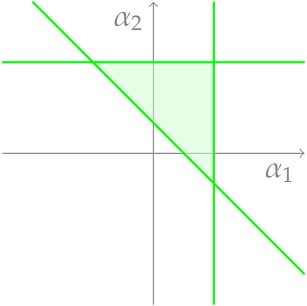
Step 4. Confine the solution space using $F(x, y) \geq 0$ inequalities.

- # dimensions = # fundamental cycles
- # inequalities = # border transitions (unknowns)



Step 5. Convert back to probabilities.

$$\mathrm{P}(X_{t+1} = \mathrm{d} \mid X_t = \mathrm{c}) = \frac{F(\mathrm{c}, \mathrm{d})}{\sum_x F(\mathrm{c}, x)}$$

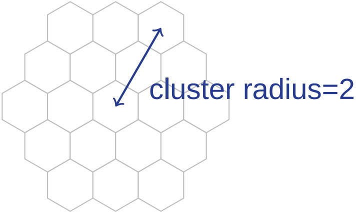
Is this 'valid'?

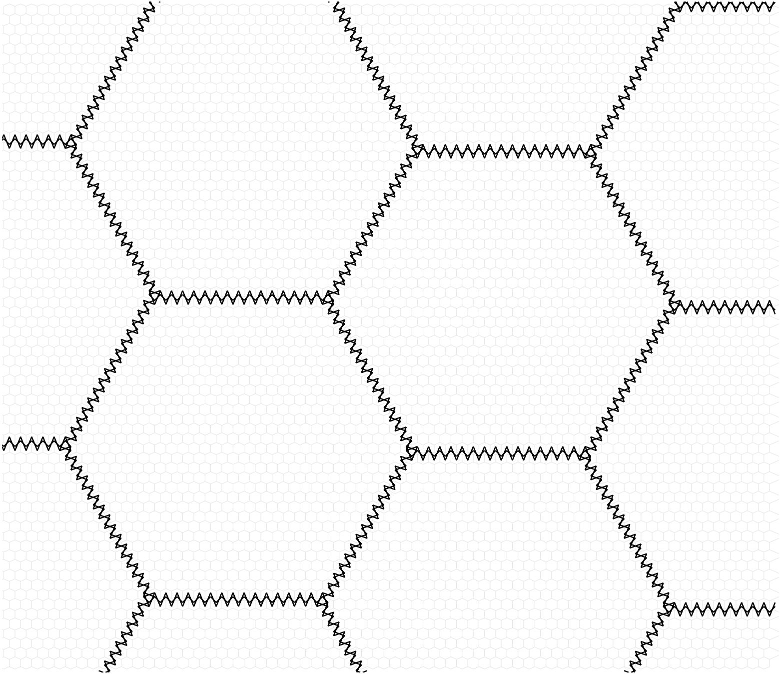
- Yes, concerning just the frequencies.
- Yes, conditional probabilities match local models.
- No, Markov property does not hold anymore for local models.

. . . see article

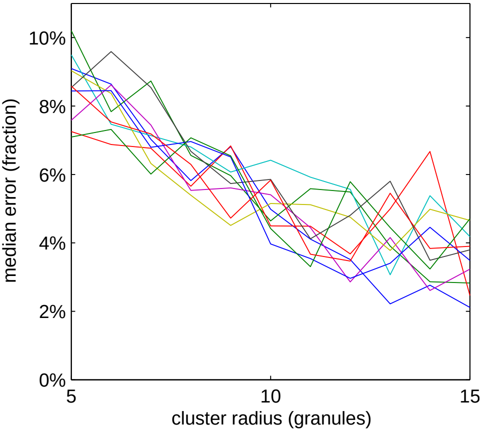
Experiment

- 70×70 grid
- Generate random transition frequencies.
- Sum border frequencies $\Rightarrow$ input to problem.
- Solve (pick from solution space).
- Solution near original frequencies?





Results


10%
8%
6%
4%
2%
0%
median error (fraction)
5
10
15
cluster radius (granules)

Future work

- Find nice applications!

- Deal with approximate transition counts.
- Use other optimality criteria?
- Find faster algorithm for inequalities.
- Find distributed algorithms?