Sandy2x: New Curve25519 Speed Records

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X25519 and Ed25519

X25519

- ECDH scheme
- public keys and shared secrets are points on the Montgomery curve

$$y^2 = x^3 + 486662x^2 + x$$

  over $\mathbb{F}_{2^{255}-19}$
- by Bernstein, 2006

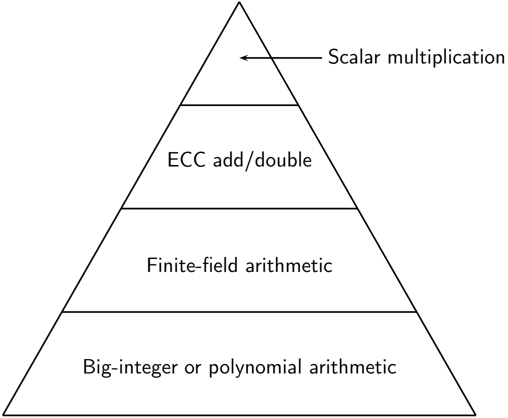
Ed25519

- signature scheme
- public keys and (part of) signatures are points on the twisted Edwards curve

$$-x^2 + y^2 = 1 - 121665/121666x^2y^2$$

  over $\mathbb{F}_{2^{255}-19}$
- by Bernstein, Duif, Lange, Schwabe, and Yang, 2011

The ECC implementation pyramid



(slide credit: Peter Schwabe)

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- (This talk focuses on Sandy Bridge/Ivy Bridge)

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\begin{array}{rrrrrr}
h_0 = & f_0 g_0 + & 19 f_1 g_4 + & 19 f_2 g_3 + & 19 f_3 g_2 + & 19 f_4 g_1 \\
h_1 = & f_0 g_1 + & f_1 g_0 + & 19 f_2 g_4 + & 19 f_3 g_3 + & 19 f_4 g_2 \\
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- 25 multiplication instructions + overhead.

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- usage:

$$(a_0 b_0, a_1 b_1) = (a_0, a_1) \times (b_0, b_1)$$

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h_1 &= f_0g_1 + f_1g_0 + 19f_2g_9 + 19f_3g_8 + 19f_4g_7 + 19f_5g_6 + 19f_6g_5 + 19f_7g_4 + 19f_8g_3 + 19f_9g_2 \\
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h_5 &= f_0g_5 + f_1g_4 + f_2g_3 + f_3g_2 + f_4g_1 + f_5g_0 + 19f_6g_9 + 19f_7g_8 + 19f_8g_7 + 19f_9g_6 \\
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h_7 &= f_0g_7 + f_1g_6 + f_2g_5 + f_3g_4 + f_4g_3 + f_5g_2 + f_6g_1 + f_7g_0 + 19f_8g_9 + 19f_9g_8 \\
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- 100 multiplication instructions + overhead; 50 per multiplication.
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Sandy2x sets new speed records by using the vectorized multiplier.

Performance results

	SB cycles	IB cycles	reference
X25519 public-key generation	54 346	52 169	**Sandy2x**
	61 828	57 612	[A. Moon]
	194 165	182 876	[Ed25519]
X25519 shared secret computation	156 995	159 128	**Sandy2x**
	194 036	182 708	[Ed25519]
Ed25519 public-key generation	57 164	54 901	**Sandy2x**
	63 712	59 332	[A. Moon]
	64 015	61 099	[Ed25519]
Ed25519 sign	63 526	59 949	**Sandy2x**
	67 692	62 624	[A. Moon]
	72 444	67 284	[Ed25519]
Ed25519 verification	205 741	198 406	**Sandy2x**
	227 628	204 376	[A. Moon]
	222 564	209 060	[Ed25519]

- Andrew Moon "floodyberry", https://github.com/floodyberry/ed25519-donna

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  - `vpaddq`: 1 μop, handled by either Port 1 or Port 5.
- port utilization gives a lower bound of cycle count

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Using the vectorized multiplier

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- 25 `mul` + 4 `imul` + 20 `add` + 20 `adc`

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- lower bound: $(25 \cdot 2 + 4 + 20 + 20 \cdot 2)/3 = 38$

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- lower bound: $(25 \cdot 2 + 4 + 20 + 20 \cdot 2)/3 = 38$
- actual cycle count is much larger: 52 cycles
- `perf-stat` shows that the core fails to distribute the μops equally over the ports

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	$\mathbf{M}^{-}$	$\mathbf{M}$
serial	52	68
vectorized	56	69.5

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- instruction interleaving hides cost for addition/subtraction
- constant-time table lookups are faster with vector instructions

Ending Remarks

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Code (for X25519 shared secret computation)

- https://sites.google.com/a/crypto.tw/blueprint/