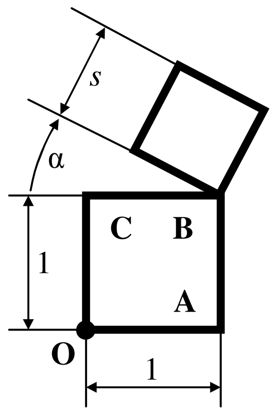
# 2IV60 – Exercise 2: Transformations and viewing

1. Given a square with width and height 1, with the lower left corner in the origin:




   a. Give a transformation matrix to transform this square to a square with width $s$, rotated over an angle α, such that the lower right corner coincides with the upper right corner of the original square (see figure).
   b. We consider a more generic version. Give a recipe for a transformation matrix for a rotated and scaled unit square **OABC**, such that a new corner **O**', **A**', **B**' or **C**' coincides with one of the original corners **O**, **A**, **B** of **C**.

2. We consider 3D rotations.
   a. Give 3×3 rotation matrices for a rotation $\mathbf{R}_x(\alpha)$ around the $x$-as and a rotation $\mathbf{R}_z(\alpha)$ around the $z$-axis.
   b. Calculate (explicitly) the composite rotation matrix for a rotation over 90 degrees (π/2 radians) around the $x$-axis followed by a rotation over 90 degrees over the (global) $z$-axis.
   c. What is the effect of these two rotations? Elucidate your answer.
   d. Do we get the same effect if we change the order of the rotations?

3. We consider a distorted cube. In the original state the cube is described by $0 \le x, y, z \le 1$. After distortion the following applies:
   - The lower face (in the XOY-plane) remains fixed;
   - The upper face is horizontal, but moves: the point (0,0,1) moves to ($a$, $b$, $c$) ;
   - all edges remain straight and all faces remain flat.

   a. Give a transformation matrix for this distortion.

4. Given a digital map. It is desired to show a part of this map in a viewport on the screen. In the center of the viewport the point **C** (in map coordinates) has to be shown, the width of the part to be displayed is $w$ (again in map coordinates). Obviously, distortion is not allowed. The viewport is specified in pixels. For the pixel coordinates it holds that the origin (the point (0,0)) is located in the upper left corner and the point (Nx−1, Ny−1) in the lower right corner. The upper left corner of the viewport has coordinates ($\mathbf{X}_{mi}$, $\mathbf{Y}_{mi}$), the lower right corner ($\mathbf{X}_{ma}$, $\mathbf{Y}_{ma}$).
   a. Give a transformation to map a point $Q$ in map coordinates to a point $Q'$ in pixel coordinates.

b. A user indicates a point **R***'* (pixel coordinates). Give a transformation to determine the corresponding point *R* in map coordinates.

5. Given a 3D scene that is viewed with a virtual camera. The camera has position **P**, points in the direction **W**, and lines in the direction of a vector **V** are shown vertically on the screen. It may be assumed that **V** and **W** are unit vectors and that they are orthogonal. In the following questions the effect of a given camera movement on **P**, **W** and **V** is asked for.
   a. Give recipes to move the camera forward and backward, up and down, and to the left and to the right.
   b. Give recipes to rotate the camera: *pitch* (left/right), *yaw (*up/down), *roll* (rotation around center of viewing axis).
   c. Suppose that the camera is centered at a point **C**. Give again recipes for the rotation, such that the camera remains fixed on this point.

For students attracted by challenges:

6. Consider question 3, where we looked at displaying a digital map. Suppose that a user has centered the map on a point **C** (for instance Eindhoven) with a width *w* (10 km). Next he indicates that he wants to center the map on a point **C***'* (say, Amsterdam) with a width *w'* (20 km). An animation **C***(t), w(t),* is desired, such that a smooth transition is showed.
   a. What are the requirements?
   b. How to define such an animation?