2IV10/2IV60 Computer Graphics

Intermediate Examination, December 14 2015, 10:45 – 12:30

This examination consist of **three** questions with in total 9 subquestion. Each subquestion weighs equally. In all cases: **EXPLAIN YOUR ANSWER. Use sketches where needed to clarify your answer. Read first all questions completely.** If an algorithm is asked, then a description in steps or pseudo-code is expected, which is clear enough to be easily transferred to real code. Aim at compactness and clarity. Use additional functions and procedures if desired. Give for each function and procedure a short description of input and output. The use of the book, copies of slides, notes and other material is not allowed.

1 We use 2D transformations to transform a square, defined by $0 \le x \le 1$, and $0 \le y \le 1$ in local coordinates. New positions X are derived from local positions X' by $X=MX'$, where M is a homogeneous transformation matrix. Suppose that we have a set of functions to define basic transformation matrices, *i.e.*,

- $T(x,y)$ gives a translation matrix over a vector (x,y);
- $S(s_x, s_y)$ gives a scaling matrix with scale factors s_x and s_y for the axes; and
- $R(a)$ gives a counterclockwise rotation over a radians around the origin.

a) Give a matrix M, expressed in the basic transformation functions, to obtain a rectangle with width w and height h, centered around the origin, and rotated over an angle α, with $0 \le \alpha < \pi/2$.

b) Calculate a bounding box, defined by $x_{\min} \le x \le x_{\max}$ and $y_{\min} \le y \le y_{\max}$, around the rectangle of question 1 a). The values $x_{\min}$, $x_{\max}$, $y_{\min}$, and $y_{\max}$ must be chosen such that the bounding box fits tightly around the rectangle.

c) Suppose that the (rotated) rectangle of question 1 a) must be displayed in a viewport with width a pixels and height b pixels. We use a clipping window, defined by $wx_{\min} \le x \le wx_{\max}$ and $wy_{\min} \le y \le wy_{\max}$. Calculate the values of $wx_{\min}$, $wx_{\max}$, $wy_{\min}$, and $wy_{\max}$ such that the rectangle is displayed as large as possible, centered in the viewport and without distortion.

2 Given a triangle PQR in 3D space.

a) Give a formula for $N(P, Q, R)$: the normal vector on a triangle, and use this to calculate the equation $f(X) = 0$ of the plane in which the triangle is embedded, where X is a point in 3D space. Standard vector operations must be used, do not use vector coordinates explicitly.

b) Given a quadrilateral $PQRS$, give a formula to determine if the quadrilateral is planar or not.

c) Give a formula for an approximation of the normal vector on the quadrilateral, which gives a useful result even when the quadrilateral is not planar.

3 A standard cone is defined by $z = x^2 + y^2$, with $0 \le z \le 1$.

a) Give a parametric definition $P(u, v) = (P_x(u, v), P_y(u, v), P_z(u, v))$ with $0 \le u, v \le 1$ of this cone.

b) Give a unit normal N for an arbitrary point on the cone. Choose yourself if you assume the point to be given via coordinates (x, y, z) or parameter values (u, v).

c) We want to define a cone with height h, distorted such that the boundary of the cone at $z = h$ is an ellipse with diameter $2a$ in the x-direction and diameter $2b$ in the y-direction. Give an implicit definition of the form $f(x, y, z) = 0$, with $0 \le z \le h$.