2IV10/2IV60 Computer Graphics

Intermediate Examination, December 15 2014, 10:45 – 12:30

This examination consist of **three** questions with in total 9 subquestion. Each subquestion weighs equally. In all cases: **EXPLAIN YOUR ANSWER. Use sketches where needed to clarify your answer. Read first all questions completely.** If an algorithm is asked, then a description in steps or pseudo-code is expected, which is clear enough to be easily transferred to real code. Aim at compactness and clarity. Use additional functions and procedures if desired. Give from each function and procedure a short description of input and output. The use of the book, copies of slides, notes and other material is not allowed.

1 We use 2D transformations to transform a square, defined by $0 \le x \le 1$, and $0 \le y \le 1$ in local coordinates. New positions X are derived from local positions X' by $X=MX'$, where M is a homogeneous transformation matrix. Suppose that we have a set of functions to define basic transformation matrices, *i.e.*,

- $T(x,y)$ gives a translation matrix over a vector (x,y);
- $S(s_x, s_y)$ gives a scaling matrix with scale factors s_x and s_y for the axes; and
- $R(a)$ gives a counterclockwise rotation over a radians around the origin.

a) Give a matrix M, expressed in the basic transformation functions, to obtain an axis-aligned rectangle with width w and height h, centered around the point (a, b).

b) Give a matrix M^{-1} to transform a position X into a local position X' for the case of the previous question. Again, express M^{-1} in the basic transformation functions.

c) Give a matrix M, expressed in the basic transformation functions, which transforms the initial square in a diamond shape (◊), centered around the origin: a parallelogram with vertices (1, 0), (0, 2), (−1, 0), and (0, −2).

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2 We consider projection on a plane. Suppose, a camera is located in the origin and pointing in the direction of the positive z-axis. The projection plane is a plane $z = d$. Answer the following questions.

a) Given a point $P=(P_x, P_y, P_z)$, calculate its projection P' on the projection plane.

b) Given a line segment PQ in a plane $z = a$ with length s. Does the length s' of the projected line segment $P'Q'$ depend on the position and orientation of the line segment? Motivate your answer.

c) Suppose that we want to display a wire frame of a cube. The cube is centered around the point $(0, 0, d)$, and all its edges are aligned with the coordinate axes. After projection, we want the lengths of the projected edges of the front face to be twice as long as those of the back face. What should the length r of the edges be?

Question 3: See the other side

3 We consider a surface patch defined by $z^2 = x^2 - y^2 + 4$ with $0 \le x \le 2$, $0 \le y \le 2$, and $z \ge 0$.

a) Give a parametric definition $P(u, v)$ with $0 \le u, v \le 1$ of this surface; and an implicit definition $f(x, y, z) = 0$ of the unbounded surface where this patch is part of.

b) What is the shape of the boundary of the patch for $x = 0$?

c) Give a unit length normal N for an arbitrary point on this surface patch. Choose yourself if you assume the point to be given via coordinates (x, y, z) or parameter values (u, v).