2IV10/2IV60 Computer Graphics

Examination, April 16 2013, 14:00 – 17:00

This examination consist of **four** questions with in total 16 subquestion. Each subquestion weighs equally. In all cases: **EXPLAIN YOUR ANSWER. Use sketches where needed to clarify your answer. Read first all questions completely.** If an algorithm is asked, then a description in steps or pseudo-code is expected, which is clear enough to be easily transferred to real code. Aim at compactness and clarity. Use additional functions and procedures if desired. Give from each function and procedure a short description of input and output. The use of the book, copies of slides, notes and other material is not allowed.

1 We consider some basic techniques for computer graphics.

a) What is a viewport in computer graphics terminology?

b) Give a criterion to distinguish convex and concave polygons.

c) In the simplest model for transparency the color $I_{surface}$ of the surface and the color I_{back} of the background are blended into a perceived color I. Give a formula for I, assuming a transparency coefficient α in the range from 0 (opaque) to 1 (fully transparent).

d) What is the difference in the computation of light intensities between Phong shading and Gouraud shading?

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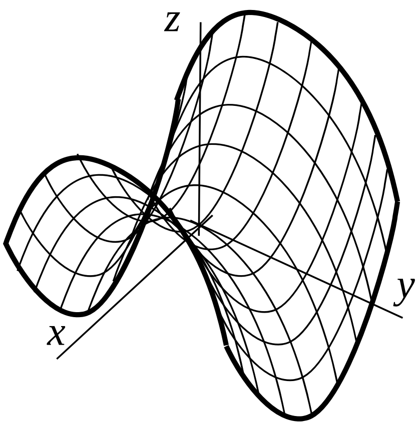
2 We consider a part of a hyperbolic surface, described by $z = x^2 - y^2$ with $-1 \le x \le 1$ and $-1 \le y \le 1$.



a) Give a parametric description $S(u, v)$ and an implicit description $F(x, y, z)=0$ of this surface.

b) Derive a formula for a normal vector for a point on this surface, either using a parametric or an implicit description.

c) Calculate all intersection points of a line $P(t) = C + Vt$ with this surface, with $P = (P_x, P_y, P_z)$, $C = (C_x, C_y, C_z)$, and $V = (V_x, V_y, V_z)$.

d) Give a procedure to draw this surface, assuming a procedure $DrawTriangle(A, B, C)$ is available.

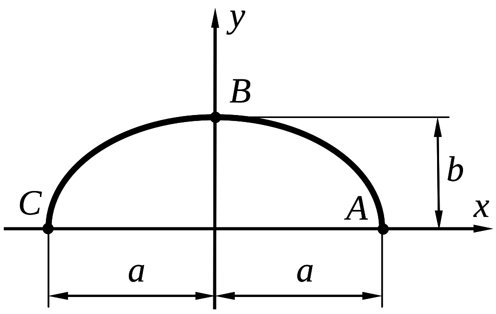
3 We want to draw an arc as shown in the figure. The arc starts in point $A = (a, 0)$, passes through point $B = (0, b)$, and ends in point $C = (-a, 0)$. At point A and C the arc is perpendicular to the x-axis, at point B the arc is perpendicular to the y-axis. We want to define the curve *parametrically* as $P(t)$, with $t \in [0, 1]$. We explore different options to define this arc. Indicate for each option if it is possible to define an arc that meets the requirements, and if not, explain why not; if yes, explain how this can be done and define $P(t)$ exactly. We consider:

a) Use of an ellipse (scaled circle);

b) Use of a curve based on a cubic function $y = a_3x^3 + a_2x^2 + a_1x + a_0$;

c) Use of a single quadratic Bézier segment $P(t) = (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2$; and

d) Use of a single cubic Bézier segment $P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3$.



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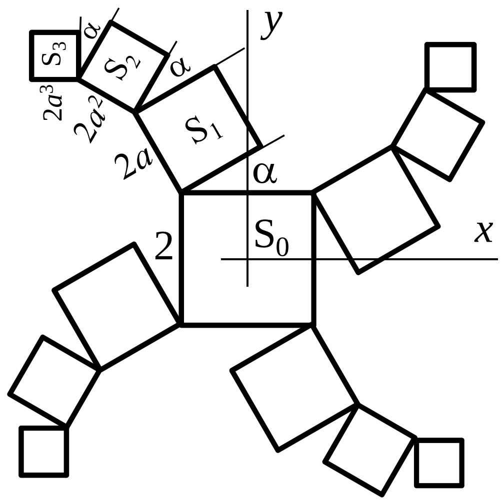
4 We aim to draw the figure shown. In the center is a square S_0, centered on the origin, with size 2. The square S_1 has size $2a$, $a < 1$, its lower left corner coincides with the upper left corner of S_0, and S_1 is rotated over α degrees. This pattern is repeated, the size of a square S_{i+1} is a times the size of square S_i. On top of the other edges squares are positioned similarly.



We use a 3×3 homogenous transformation matrix M, such that a position A in global coordinates is related to a position B in local coordinates via $A=MB$. It may be assumed that $T(x,y)$ gives a translation matrix along the vector (x,y); that $R(\varphi)$ gives a rotation matrix of φ degrees around the origin; and that $S(s)$ gives a uniform scaling matrix with a scale factor s. The routine *DrawSquare*() draws a square in the local coordinate frame that is implicitly defined by the matrix M. In these local coordinates, the square that is drawn has size 2 and is centered on the origin.

a) Set M such that a call to *DrawSquare*() draws S_1, exactly according to the specification given and the figure.

b) Suppose M has been set such that S_i has just been drawn with a call *DrawSquare*(). Update M to draw S_{i+1} with yet another call to *DrawSquare*().

c) It is desired that square S_n has size p and is rotated over β degrees in total. How to set a and α to get this effect?

d) Give a procedure to draw the complete figure, including all four arms, where each arm consists of n squares.