2M050: Computer Graphics

Jack van Wijk

vanwijk@win.tue.nl

HG6.71

1

Aims

- Introduce basic graphics concepts and terminology
- Base for development of 2D interactive computer graphics programmes (OGO 2.3)

2

Literature

Computer Graphics - Principles and Practice
Foley - van Dam - Feiner - Hughes
2nd edition in C - Addison and Wesley

Computer Graphics - C Version
Donald Hearn - M. Pauline Baker
2nd edition - international edition
Prentice Hall

3

Overview

Introduction

Geometry

Interaction

Raster graphics

4

Introduction

What is Computer Graphics?

Applications

Computer Graphics in Eindhoven

Raster/vector graphics

Hardware

5

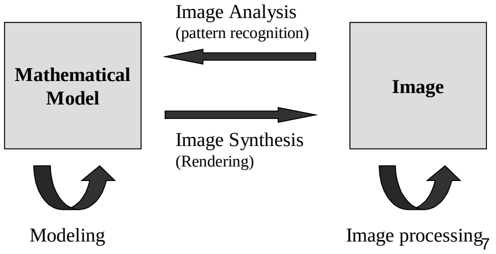
Computer Graphics

Computer Graphics is ubiquitous:

- Visual system is most important sense:
  - High bandwidth
  - Natural communication
- Fast developments in
  - Hardware
  - Software

6

Computer Graphics



Supporting Disciplines

- Computer science *(algorithms, data structures, software engineering, …)*
- Mathematics *(geometry, numerical, …)*
- Physics *(Optics, mechanics, …)*
- Psychology *(Colour, perception)*
- Art and design

Applications

- Computer Aided Design (CAD)
- Computer Aided Geometric Design (CAGD)
- Entertainment (animation, games, …)
- Geographic Information Systems (GIS)
- Visualization (Scientific Vis., Inform. Vis.)
- Medical Visualization
- …

Computer Graphics Eindhoven

Current:

•Information visualisation

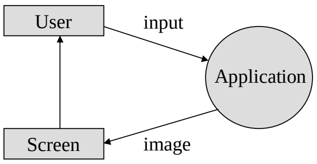
•Interactive 3D design

•Virtual reality

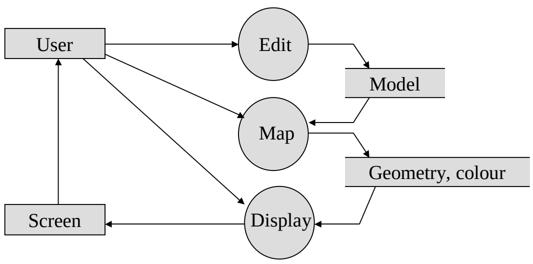
Jack van Wijk
Kees Huizing
Arjan Kok
Robert van Liere
Wim Nuij
Alex Telea
Huub van de Wetering

Past: Rasterization, Animation

Interactive Computer Graphics



Graphics pipeline



Representations in graphics

Vector Graphics
- *Image is represented by continuous geometric objects: lines, curves, etc.*

Raster Graphics
- *Image is represented as an rectangular grid of coloured squares*

13

Vector graphics

- Graphics objects: geometry + colour
- Complexity ~ O(number of objects)
- Geometric transformation possible without loss of information (zoom, rotate, …)
- Diagrams, schemes, ...
- Examples: PowerPoint, CorelDraw, ...

14

Raster graphics

- Generic
- Image processing techniques
- Geometric Transformation: loss of information
- Complexity ~ O(number of pixels)
- Jagged edges, anti-aliasing
- Realistic images, textures, ...
- Examples: Paint, PhotoShop, ...

15

Conversion

Vector graphics

↓ *Rasterization, Scan conversion* ↓ ↑ *Pattern recognition* ↑

Raster graphics

16

Hardware

- Vector graphics
- Raster graphics
- Colour lookup table
- 3D rendering hardware

17

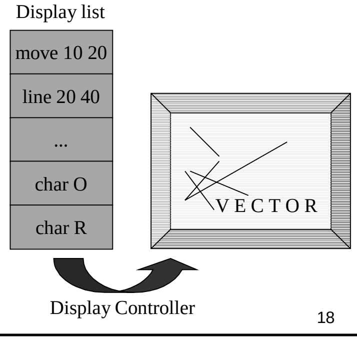
Vector Graphics Hardware

continuous & smooth lines

no filled objects

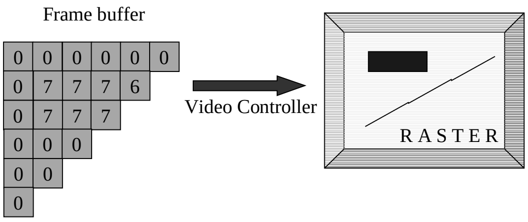
random scan

refresh speed depends on complexity of the scene



18

Raster Graphics Hardware

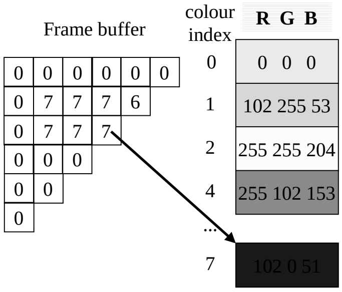


jaggies (stair casing)
filled objects
(anti)aliasing

refresh speed independent of scene complexity
pixel

scan conversion
resolution
bit planes

Colour Lookup Table



CLUT:
pixel = code

True colour:
pixel = R,G,B

3D rendering hardware

Geometric representation: *Triangles*

Viewing: *Transformation*

Hidden surface removal: *z-buffer*

Lighting and illumination: *Gouraud shading*

Realism: *texture mapping*

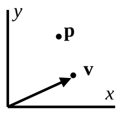
Special effects: *transparency, antialiasing*

2D geometric modelling

- Coordinates
- Transformations
- Parametric and implicit representations
- Algorithms

Coordinates

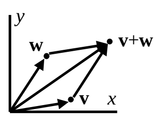
- Point: position on plane
  $\mathbf{p} = (p_x , p_y)$
  $\mathbf{x} = (x, y)$
  $\mathbf{x} = (x_1 , x_2)$
  $\mathbf{x} = x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2$, $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = (0, 1)$



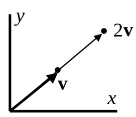
- Vector: direction and magnitude
  $\mathbf{v} = (v_x , v_y)$, etc.

Vector arithmetic

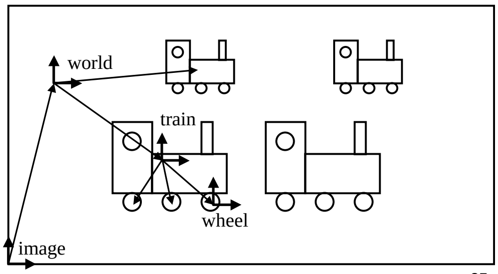
- Addition of two vectors:
  $\mathbf{v} + \mathbf{w} = (v_x + w_x , v_y + w_y)$



- Multiplication vector-scalar:
  $\alpha\mathbf{v} = (\alpha v_x , \alpha v_y)$



Coordinate systems



25

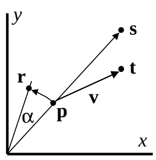
Why transformations?

- Model of objects
  - world coordinates: *km, mm, etc.*
  - hierarchical models:
  - *human = torso + arm + arm + head + leg + leg*
  - *arm = upperarm + lowerarm + hand …*
- Viewing
  - zoom in, move drawing, etc.

26

Transformation types

- Translate according to vector **v**:
  - **t** = **p** + **v**
- Scale with factor *s*:
  - **s** = *s***p**
- Rotate over angle α:
  - $r_x = \cos(\alpha)p_x - \sin(\alpha)p_y$
  - $r_y = \sin(\alpha)p_x + \cos(\alpha)p_y$



27

Homogeneous coordinates

- Unified representation of rotation, scaling, translation
- Unified representation of points and vectors
- Compact representation for sequences of transformations
- Here: convenient notation, much more to it

28

Homogeneous coordinates

- Extra coordinate added:
  - $\mathbf{p} = (p_x, p_y, p_w)$ or
  - $\mathbf{x} = (x, y, w)$
- Cartesian coordinates: divide by *w*
  - $\mathbf{x} = (x/w, y/w)$
- Here: for a point $w = 1$, for a vector $w = 0$

29

Matrices for transformation

$\mathbf{x}' = \mathrm{M}\mathbf{x}$, or

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}, \text{ or}$$

$$x' = m_{11}x + m_{12}y + m_{13}w$$

$$y' = m_{21}x + m_{22}y + m_{23}w$$

$$w' = m_{31}x + m_{32}y + m_{33}w$$

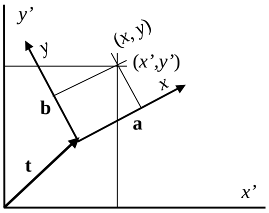
30

Direct interpretation

$\mathbf{x}' = \mathrm{M}\,\mathbf{x}$, or

$\mathbf{x}' = (\mathbf{a}\ \ \mathbf{b}\ \ \mathbf{t})\mathbf{x}$, or

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$



Translation matrix

Translation :

$\mathbf{x}' = \mathrm{T}(t_x, t_y)\mathbf{x}$, with

$$\mathrm{T}(t_x, t_y) = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

Scaling matrix

Scaling :

$\mathbf{x}' = \mathrm{S}(s_x, s_y)\mathbf{x}$, with

$$\mathrm{S}(s_x, s_y) = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

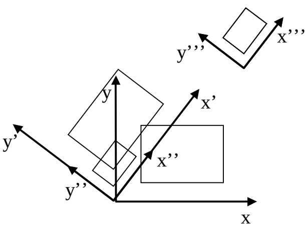
Rotation matrix

Rotation :

$\mathbf{x}' = \mathrm{R}(\alpha)\,\mathbf{x}$, with

$$\mathrm{R}(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Sequences of transformations



$\mathbf{x}' = \mathrm{R}(\pi/2)\mathbf{x}$

$\mathbf{x}'' = \mathrm{S}(1/2)\mathbf{x}'$

$\mathbf{x}''' = \mathrm{T}(5,4)\mathbf{x}''$

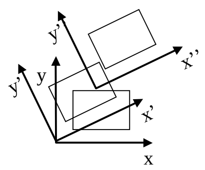
or

$\mathbf{x}''' = \mathrm{M}\mathbf{x}$, with

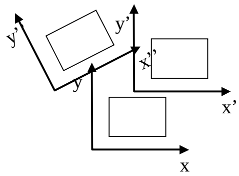
$\mathrm{M} = \mathrm{T}(5,4)\mathrm{S}(1/2)\mathrm{R}(\pi/2)$

Sequences of transformations can be described with a single transformation matrix, which is the result of concatenation of all transformations.

Order of transformations



$\mathbf{x}'' = \mathrm{T}(2,3)\mathrm{R}(30)\mathbf{x}$



$\mathbf{x}'' = \mathrm{R}(30)\mathrm{T}(2,3)\mathbf{x}$

Matrix multiplication is not commutative. Different orders of multiplication give different results.

Order of transformations

- Pre-multiplication:
  $\mathbf{x'} = M_n M_{n-1} \ldots M_2 M_1 \mathbf{x}$
  Transformation M_n in global coordinates
- Post-multiplication:
  $\mathbf{x'} = M_1 M_2 \ldots M_{n-1} M_n \mathbf{x}$
  Transformation M_n in local coordinates, i.e., the coordinate system that results from application of $M_1 M_2 \ldots M_{n-1}$

37

Window and viewport

Viewport:

Area on screen to be used for drawing.

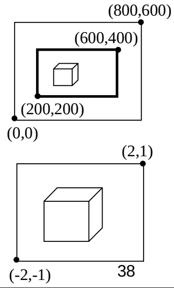
Unit: pixels (screen coordinates)

Note: *y*-axis often points down

Window:

Virtual area to be used by application

Unit: km, mm,… (world coordinates)



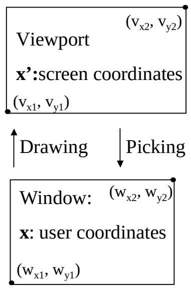
38

Window/viewport transform

- Determine a matrix M, such that the *window* (w_{x1}, w_{x2}, w_{y1}, w_{y2}) is mapped on the *viewport* (v_{x1}, v_{x2}, v_{y1}, v_{y2}):
- $A = T(-w_{x1}, -w_{y1})$
- $B = S(1/(w_{x2}-w_{x1}), 1/(w_{y2}-w_{y1}))\ A$
- $C = S(v_{x2}-v_{x1}, v_{y2}-v_{y1})B$
- $M = T(v_{x1}, v_{y1})\ C$

39

Forward and backward



Drawing**:** (meters to pixels)

Use $\mathbf{x'} = M\mathbf{x}$

Picking**:**(pixels to meters)

Use $\mathbf{x} = M^{-1}\mathbf{x'}$

40

Implementation example

Suppose, basic library supports two functions:

- MoveTo(x, y: integer);
- LineTo(x, y: integer);
- x and y in pixels.

How to make life easier?

41

State variables

- Define state variables:
  Viewport: array[1..2, 1..2] of integer;
  Window: array:[1..2, 1..2] of real;
  Mwv, Mobject: array[1..3, 1..3] of real;

  Mwv: transformation from world to view
  Mobject: extra object transformation

42

Procedures

- Define coordinate system:
  SetViewPort(x1, x2, y1, y2):
    Update Viewport and Mwv
  SetWindow(x1, x2, y1, y2):
    Update Window and Mwv

43

Procedures (continued)

- Define object transformation:
  ResetTrans:
    Mobject := IdentityMatrix
  Translate(tx, ty):
    Mobject := T(tx,ty)* Mobject
  Rotate(alpha):
    Mobject := R(tx,ty)* Mobject
  Scale(sx, sy):
    Mobject := S(sx, sy)* Mobject

44

Procedures (continued)

- Handling hierarchical models:
  – PushMatrix();
    Push an object transformation on a stack;
  – PopMatrix()
    Pop an object transformation from the stack.

Or:
  – GetMatrix(M);
  – SetMatrix(M);

45

Procedures (continued)

- Drawing procedures:
  MyMoveTo(x, y):
    (x', y') = Mwv*Mobject*(x,y);
    MoveTo(x', y')
  MyLineTo(x,y):
    (x', y') = Mwv*Mobject*(x,y);
    LineTo(x', y')

46

Application

```
DrawUnitSquare:
  MyMoveTo(0, 0);
  MyLineTo(1, 0);
  MyLineTo(1, 1);
  MyLineTo(0, 1);
  MyLineTo(0, 0);

Initialize:
  SetViewPort(0, 100, 0, 100);
  SetWindow(0, 1, 0, 1);
```

```
Main program:
  Initialize;
  Translate(-0.5, -0.5);
  for i := 1 to 10 do
  begin
     Rotate(pi/20);
     Scale(0.9, 0.9);
     DrawUnitSquare;
  end;
```

47

Puzzles

- Modify the window/viewport transform for a display y-axis pointing downwards.
- How to maintain aspect-ratio world->view? Which state variables?
- Define a transformation that transforms a unit square into a "wybertje", centred around the origin with width *w* and height *h*.

48

Geometry

- Dot product, determinant
- Representations
- Line
- Ellipse
- Polygon

Good and bad

- Good: symmetric in *x* and *y*
- Good: matrices, vectors
- Bad: $y = f(x)$

- Good: dot product, determinant
- Bad: arcsin, arccos

Dot product

Notation : $\mathbf{v} \cdot \mathbf{w}$ (sometimes (v, w))

Definition :

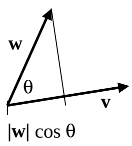
$$\mathbf{v} \cdot \mathbf{w} = v_x w_x + v_y w_y$$

Also :

$$\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}||\mathbf{w}| \cos\theta \quad (0 \le \theta \le \pi)$$

with θ angle between $\mathbf{v}$ and $\mathbf{w}$,

and $|\mathbf{v}|$ is the length of vector $\mathbf{v}$



Dot product properties

$$\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$$

$$(\mathbf{v} + \mathbf{w}) \cdot \mathbf{u} = \mathbf{v} \cdot \mathbf{u} + \mathbf{w} \cdot \mathbf{u}$$

$$(\lambda \mathbf{v}) \cdot \mathbf{w} = \lambda \ \mathbf{v} \cdot \mathbf{w}$$

$$\mathbf{v} \cdot \mathbf{v} = |\mathbf{v}|^2$$

$\mathbf{v} \cdot \mathbf{w} = 0$ iff $\mathbf{v}$ and $\mathbf{w}$ are perpendicular

Determinant

$$Det(\mathbf{v}, \mathbf{w}) = v_x w_y - v_y w_x$$

$$= |\mathbf{v}||\mathbf{w}| \sin\theta$$

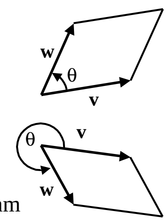
θ is angle from $\mathbf{v}$ to $\mathbf{w}$

$0 < \theta < \pi : Det(\mathbf{v}, \mathbf{w}) > 0$

$\pi < \theta < 2\pi : Det(\mathbf{v}, \mathbf{w}) < 0$

Det(**v**, **w**): signed area of parallellogram

Det(**v**, **w**) = 0 iff **v** and **w** are parallel



Curve representations

- Parametric: **x**(*t*) = (*x*(*t*), *y*(*t*))

- Implicit: f(**x**) = 0

Parametric line representation

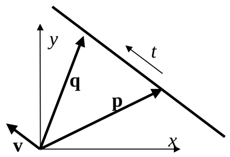
Given point **p** and vector **v:**

x(t) **= p** + **v**t

Given two points **p** and **q:**

x(t) = **p** + (**q-p**)t , or

= **p**t + **q**(1-t)



55

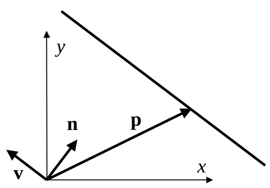
Parametric representation

- **x**(t) = (x(t), y(t))
- Trace out curve:
  MoveTo(**x**(0));
  for i := 1 to N do LineTo(**x**(i*Δt));
- Define segment: $t_{min} \leq t \leq t_{max}$

56

Implicit line representation

- (**x-p**).**n** = 0
  with **n.v** = 0
  n is normal vector:
  n = [-v_y , v_x]
- Also:
  $ax+by+c=0$



57

Implicit representation

$f(\mathbf{x}) = 0$: curve

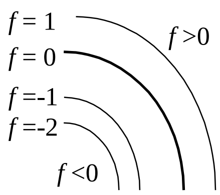
$f(\mathbf{x}) = C$: contours

$f = 0$ divides plane in

two areas : $f > 0$ and $f < 0$

$|f(\mathbf{x})|$: measure of distance

of **x** to curve



58

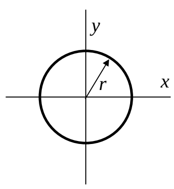
Circle

Parametric :

$(x, y) = (r\cos\alpha, r\sin\alpha)$

Implicit :

$x^2 + y^2 - r^2 = 0$



59

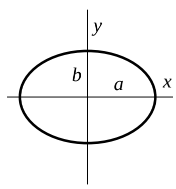
Ellipse

Parametric :

$(x, y) = (a\cos\alpha, b\sin\alpha)$

Implicit :

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 - 1 = 0$$

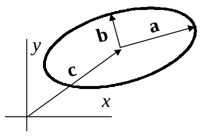


60

Generic ellipse

Parametric:

$\mathbf{x}(\alpha) = \mathbf{c} + \mathbf{a}\cos\alpha + \mathbf{b}\sin\alpha$



Implicit:

$|M\mathbf{x}| = 1$, with $M = (\mathbf{a} \quad \mathbf{b} \quad \mathbf{c})^{-1}$

61

Some standard puzzles

- Conversion of line representation
- Projection of point on line
- Line/Line intersection
- Position points/line
- Line/Circle intersection

62

Conversion line representations

Given line:

$\mathbf{p}(s) = \mathbf{a} + \mathbf{u}s;$

Find implicit representation:

$\mathbf{n} \cdot \mathbf{x} + c = 0.$

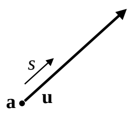
First, determine normal **n**.

n must be $\perp$ on **u**, hence we set:

$\mathbf{n} = (-u_y, u_x)$

a must be on the line, hence:

$c = -\mathbf{n} \cdot \mathbf{a}$



63

Projection point on line

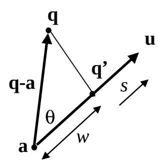
Project point **q** on line $\mathbf{p}(s) = \mathbf{a} + \mathbf{u}s$:

$\mathbf{q}' = \mathbf{a} + \cos\theta\, |\mathbf{q} - \mathbf{a}| \dfrac{\mathbf{u}}{|\mathbf{u}|}$

Use $(\mathbf{q} - \mathbf{a}) \cdot \mathbf{u} = |\mathbf{q} - \mathbf{a}||\mathbf{u}| \cos\theta$:

$\mathbf{q}' = \mathbf{a} + \dfrac{(\mathbf{q} - \mathbf{a}) \cdot \mathbf{u}}{|\mathbf{u}||\mathbf{u}|}\mathbf{u}$, or

$\mathbf{q}' = \mathbf{a} + \dfrac{(\mathbf{q} - \mathbf{a}) \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u}$



$\cos\theta\, |\mathbf{q} - \mathbf{a}|$: length w

$\dfrac{\mathbf{u}}{|\mathbf{u}|}$: unit vector along **u**

64

Intersection of line segments

Find intersection of line segments:

$\mathbf{p}(s) = \mathbf{a} + \mathbf{u}s,\ 0 \le s \le 1$ and

$\mathbf{q}(t) = \mathbf{b} + \mathbf{v}t,\ 0 \le t \le 1.$

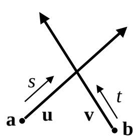
At intersection:

$\mathbf{p}(s) = \mathbf{q}(t)$

Solve for s and t (next sheet);

Check if $0 \le s \le 1$ and $0 \le t \le 1$;

If so, intersection is $\mathbf{p}(s)$.



65

Solving for s and t

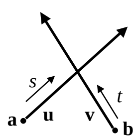
$\mathbf{p}(s) = \mathbf{q}(t)$, or

$\mathbf{a} + \mathbf{u}s = \mathbf{b} + \mathbf{v}t$, or

$(\mathbf{u} \quad \mathbf{v})\begin{pmatrix} s \\ t \end{pmatrix} = \mathbf{b} - \mathbf{a}$, or

$\begin{pmatrix} s \\ t \end{pmatrix} = (\mathbf{u} \quad \mathbf{v})^{-1}(\mathbf{b} - \mathbf{a})$, or

$$\begin{pmatrix} s \\ t \end{pmatrix} = \frac{1}{u_x v_y - u_y v_x}\begin{pmatrix} v_y & -v_x \\ -u_y & u_x \end{pmatrix}\begin{pmatrix} b_x - a_x \\ b_y - a_y \end{pmatrix}$$

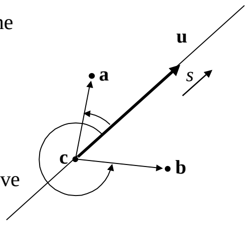


66

Position points/line

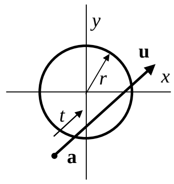
Check if points **a** and **b** are on the same side of line $\mathbf{p}(s) = \mathbf{c} + \mathbf{u}s$

Use $Det(\mathbf{u}, \mathbf{v}) = |\mathbf{u}||\mathbf{v}|\sin\theta$:
Points are on the same side if $Det(\mathbf{u}, \mathbf{a}-\mathbf{c})$ and $Det(\mathbf{u}, \mathbf{b}-\mathbf{c})$ have the same sign.



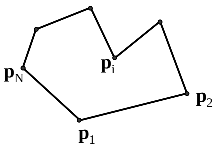
Line/circle intersection

Find intersections of :
line : $\mathbf{p}(t) = \mathbf{a} + \mathbf{u}t,\ 0 \le t \le 1$ and
circle : $\mathbf{x} \cdot \mathbf{x} = r^2$.
At intersection :
$\mathbf{p}(t) \cdot \mathbf{p}(t) = r^2$, or
$(\mathbf{a} + \mathbf{u}t) \cdot (\mathbf{a} + \mathbf{u}t) = r^2$, or
$\mathbf{u} \cdot \mathbf{u}t^2 + \mathbf{a} \cdot \mathbf{u}t + \mathbf{a} \cdot \mathbf{a} - r^2 = 0$.
Solve quadratic equation for t :
0, 1, or 2 solutions.



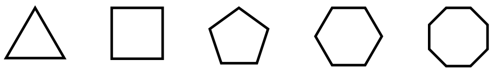
Polygons

- Sequence of points $\mathbf{p}_i$, $i = 1,\ldots, N$, connected by straight lines
- Index arithmetic: modulo N
  $\mathbf{p}_0 = \mathbf{p}_N$, $\mathbf{p}_{N+1} = \mathbf{p}_1$, etc.



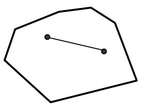
Regular N-gon

$$\mathbf{p}_i = (r\cos\alpha_i, r\sin\alpha_i)$$
$$\alpha_i = 2\pi(i + 1/2)/N - \pi/2$$

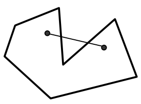


Convex and concave

- Convex:
  - each line between two arbitrary points inside the polygon does not cross its boundary

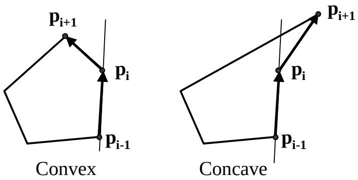


- Concave:
  - not convex



Convexity test

Assume polygon is oriented counterclockwise.
Polygon is concave, if
$\mathrm{Det}(\mathbf{p}_i - \mathbf{p}_{i-1}, \mathbf{p}_{i+1} - \mathbf{p}_i) > 0$ for all i



Convex   Concave

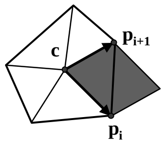
Polygon area and orientation

$a = \sum_{i}^{N} Det(\mathbf{p}_i - \mathbf{c}, \mathbf{p}_{i+1} - \mathbf{c})/2$, $\mathbf{c}$ is *arbitrary* point

$area = |a|$

$a > 0$: counterclockwise orientation

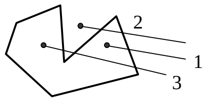
$a < 0$: clockwise orientation



73

Point/polygon test

Given a polygon. Test if a point $\mathbf{c}$ is inside or outside.



Solution :

Define a line $L = \mathbf{c} + \mathbf{v}t, t \geq 0$.

$\mathbf{v}$ can be chosen arbitrarily, f.i. (1, 0).

Let n be the number of crossings of L with the polygon. If n is odd : point is inside, else it is outside.

74

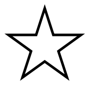
Point/polygon test (cntd.)

- Beware of special cases:
  - Point at boundary
  - $\mathbf{v}$ parallel to edge
  - $\mathbf{c} + \mathbf{v}t$ through vertex

75

Puzzles

- Define a procedure to clip a line segment against a rectangle.
- Define a procedure to calculate the intersection of two polygons.
- Define a procedure to draw a star.
- Same, with the constraint that the edges $\mathbf{p}_{i-1}\,\mathbf{p}_i$ and $\mathbf{p}_{i+2}\,\mathbf{p}_{i+3}$ are parallel.



76