

# Bibliography on the Busy Beaver Problem

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## Introduction

In 1981 the editor of this bibliography was attracted to the Busy Beaver Problem by a reference in the Dutch translation of Ogilvy's *Tomorrow's Math* [Ogi72]. Tracing back in time one finds of course Rado's publication "On non-computable functions" [Rad62a, Rad62b]. The course of events since then is not always easy to trace. In the period 1962–1989 there were many successive high tides and low tides in the research efforts and in the results thereof. Moreover, researchers sometimes worked in isolation, and were unaware of the results of one another, though it seems most of the researchers involved live in the Federal Republic of Germany and the United States of America.

Literature search via e.g. Science Citation Index and via personal correspondence with some of the researchers in this field, gradually did result in this list of articles, books, reports and theses. Separately, reviews in Computing Reviews (CR), in the Journal of Symbolic Logic (JSL), in Mathematical Reviews (MR), and in the Zentralblatt für Mathematik und ihre Grenzgebiete (Zbl) are listed. A review between brackets contains nothing more than the author's summary.

Most of this bibliography was compiled before 1986. Only a few entries have been added haphazardly since then.

## References

### [A]       Articles, books, reports, theses

- [BaE93]   Jon Barwise and John Etchemendy. *Turing's World 3.0: An introduction to computability theory*. CLSI Lecture Notes 35, CSLI Publications, Stanford (California), 1993. ISBN 1881526100. [esp. par. 8.5, pages 103–104]
- [Bát09]   Norbert Bátfai. *Recombinations of Busy Beaver machines*. arXiv.org:0908.4013 [cs.CC], September 2009.  
URL at arXiv.org
- [Bec80]   Frank S. Beckman. *Mathematical Foundations of Programming*. Addison Wesley, 1980. ISBN 020114462X. [esp. pages 193–195 and 184]
- [Bel70]   A.G. Bell. Partitioning integers in N dimensions. *The Computer Journal*, 13(3), pages 278–283, August 1970. — (Zbl 199.317).

- [BJZ96] A.M. Ben-Amram, B.A. Julstrom and U. Zwick. A note on Busy Beavers and other creatures. *Mathematical Systems Theory*, 29(4), pages 375–386, 1996. — MR 97d:68049; (Zbl 848.68036). DOI: 10.1007/BF01192693
- [BeP02] A.M. Ben-Amram and H. Petersen. Improved bounds for functions related to busy beavers. *Theory of Computing Systems*, 35(1), pages 1–11, January-February 2002. — (MR 2002j:68026); (Zbl 0993.68036). DOI: 10.1007/s00224-001-1052-0
- [BöI69] K.H. Böhling and K. Indermark. *Endliche Automaten I*. B.I. Hochschulschriften, H 703, Bibliographisches Institut, Mannheim, 1969.
- [Boo74] G. Boolos and R. Jeffrey. *Computability and Logic*. Cambridge University Press, Cambridge, 1974. [esp. pages 27 & 34–42]
- [Bra64] A.H. Brady. *Solutions of restricted cases of the halting problem applied to the determination of particular values of a non-computable function*. PhD-Thesis, Oregon State University, Corvallis (Oregon), December 1964.
- [Bra65] A.H. Brady. Solutions of restricted cases of the halting problem used to determine particular values of a noncomputable function. *Notices of the American Mathematical Society*, 12, Abstract 65T-270, pages 476–477, June 1965.
- [Bra66] A.H. Brady. The conjectured highest scoring machines for Rado’s  $\Sigma$  for the value  $k=4$ . *IEEE Transactions on Electronic Computers*, EC-15, pages 802–803, October 1966. — JSL 40.617; (Zbl 156.19). DOI: 10.1109/PGEC.1966.264572
- [Bra75a] A.H. Brady. Solution of the non-computable “Busy Beaver” game for  $k=4$ . *Abstracts for: ACM Computer Science Conference (Washington, DC, February 18-20, 1975)*, page 27, ACM, 1975.
- [Bra75b] A.H. Brady. The solution to Rado’s “Busy Beaver Game” is now decided for  $k=4$  (Preliminary Report). *Notices of the American Mathematical Society*, 22(1), page A-25, 1975.
- [Bra83] A.H. Brady. The determination of the value of Rado’s noncomputable function  $\Sigma$  for four-state Turing machines. *Mathematics of Computation*, 40(162), pages 647–665, April 1983. — MR 84d:03049; Zbl 518.03013.
- [Bra88] A.H. Brady. The Busy Beaver game and the meaning of life. In Rolf Herken (ed), *The Universal Turing Machine: A half-century survey*, pages 259–277, Oxford University Press, 1988. — JSL 56.1091; (MR91b:03004); Zbl0685.68054.
- [BrI68] W. Brauer and K. Indermark. *Algorithmen, rekursive Funktionen und formale Sprachen*. B.I. Hochschulschriften, 817\*, Bibliographisches Institut, Mannheim, 1968.
- [BrK69] Ju.Ja. Breĭtbarť and V.A. Kozmidiadi. Two subclasses of Turing machines reducible to finite automata. *Doklady Akademii Nauk SSSR*, 187(1), pages 9–10,

- 1969; in Russian. — Translation (E. Mendelson): *Soviet Mathematics Doklady*, 10(4), pages 763–764, 1969. — MR 42#7435; Zbl 193.331.
- [BKS06] Selmer Bringsjord, Owen Kellett, Andrew Shilliday, Joshua Taylor, Bram van Heuveln, Yingrui Yang, Jeffrey Baumes and Kyle Ross. A new Godelian argument for hypercomputing minds based on the Busy Beaver problem. *Applied Mathematics and Computation*, 176(2), 516–530.  
DOI: 10.1016/j.amc.2005.09.071
- [Bur90] M. Buro. *Ein Beitrag zur Bestimmung von Rados  $\Sigma(5)$ , oder Wie fängt man fleißige Biber?* Schriften zur Informatik und angewandten Mathematik, Bericht 146, Rheinisch-Westfälische Technische Hochschule, Aachen, November 1990.
- [Cha70] G.J. Chaitin. On the difficulty of computations. *IEEE Transactions on Information Theory*, IT-16, pages 5–9, 1970. — (Zbl 184.205).  
DOI: 10.1109/TIT.1970.1054390
- [Cha74a] G.J. Chaitin. Information-theoretic computational complexity. *IEEE Transactions on Information Theory*, IT-20, pages 10–15, January 1974. — MR 55#2529; Zbl 282.68022.  
DOI: 10.1109/TIT.1974.1055172
- [Cha74b] G.J. Chaitin. Information-theoretic limitations of formal systems. *Journal of the Association for Computing Machinery*, 21(3), pages 403–424, July 1974. — CR 15 Rev.27635; MR 56#13775; Zbl 287.68027.  
DOI: 10.1145/321832.321839
- [Cha75] G.J. Chaitin. Randomness and mathematical proof. *Scientific American*, 232(5), pages 47–52, May 1975.
- [Cha77] G.J. Chaitin. Algorithmic information theory. *IBM Journal of Research and Development*, 21(4), pages 350–359, July 1977. Errata, 21(5), page 496, September 1977. — CR 19 Rev.33065; (Zbl 362.94035).  
DOI: 10.1147/rd.214.0350 (Paper)  
DOI: 10.1147/JRD.1977.5391018 (Errata)
- [Cha82a] G.J. Chaitin. Gödel’s theorem and information. *International Journal of Theoretical Physics*, 22, pages 941–954, 1982.
- [Cha82b] G.J. Chaitin. Algorithmic information theory. In *Encyclopedia of Statistical Sciences*, 1, pages 38–41, John Wiley, New York, 1982.
- [Cha84] G.J. Chaitin. *Computing the busy beaver function*. IBM Research Report RC 10722, IBM Thomas J. Watson Research Center, Yorktown Heights (NY), 1984.
- [Cha87] G.J. Chaitin. Computing the busy beaver function. In T.M. Cover and B. Gopinath (eds), *Open Problems in Communication and Computation*, pages 108–112, Springer Verlag, New York, 1987. ISBN 3540966218.
- [CGS63] R.W. Coffin, H.E. Goheen and W.R. Stahl. Simulation of a Turing machine on a digital computer. In *Proceedings of the 1963 Fall Joint Computer Conference (Las*

- Vegas, Nevada, November 1963*), pages 35–43, AFIPS Conference Proceedings, 24, Spartan Press, Baltimore (Maryland), 1963. — CR 5 Rev.5742.
- [Dal74] R.P. Daley. Non-complex sequences: characterizations and examples. In *Proceedings 15th Annual Symposium on Switching and Automata Theory (Univ. of New Orleans, New Orleans, October 14–16, 1974)*, pages 165–169, IEEE Computer Society, Long Beach (California), 1974. — (MR 58#19342).
- [Dal76a] R.P. Daley. Non-complex sequences: characterizations and examples. *Journal of Symbolic Logic*, 41(3), pages 626–638, September 1976. — (MR 58#5117); Zbl 365.68054.
- [Dal76b] R.P. Daley. On the simplicity of Busy Beaver sets (Preliminary Report). *Notices of the American Mathematical Society*, 23(5), page A497, 1976.
- [Dal78] R.P. Daley. On the simplicity of Busy Beaver sets. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 24(3), pages 207–224, 1978. — MR 80h:03061; Zbl 421.03031.
- [Dal80a] R.P. Daley. The Busy Beaver method. In J. Barwise, H.J. Keisler, and K. Kunen (eds), *The Kleene Symposium (Proceedings of the Symposium held June 18–24, 1978 at Madison, Wisconsin)*, pages 333–345, Studies in Logic and the Foundations of Mathematics, 101, North-Holland, Amsterdam, 1980. — MR 82a:03040; Zbl 501.03024.  
DOI: 10.1016/S0049-237X(08)71266-7
- [Dal80b] R.P. Daley. Quantitative and qualitative information in computations. *Information and Control*, 45(3), pages 236–244, 1980. — (MR 81k:68029); (Zbl 457.68042).  
DOI: 10.1016/S0019-9958(80)90615-4
- [Dal81a] R.P. Daley. Retraceability, repleteness and Busy Beaver sets. In J. Gruska and M. Chytil (eds), *Mathematical Foundations of Computer Science 1981 (Proceedings 10th Symposium on MFCS, Štrbské Pleso, Czechoslovakia, August 31–September 4, 1981)*, pages 252–261, Lecture Notes in Computer Science, 118, Springer Verlag, Berlin, 1981. — MR 83m:03049; Zbl 486.03021.  
DOI: 10.1007/3-540-10856-4\_91
- [Dal81b] R.P. Daley. Busy Beaver sets and the degrees of unsolvability. *Journal of Symbolic Logic*, 46(3), pages 460–474, September 1981. — MR 83a:03035; Zbl 486.03022.
- [Dal82] Robert P. Daley. Busy Beaver sets: characterizations and applications. *Information and Control*, 52(1), pages 52–67, 1982. — MR 85h:03039; Zbl 503.03017.  
DOI: 10.1016/S0019-9958(82)80085-5
- [Dav78] M. Davis. What is computation? In L.A. Steen (ed), *Mathematics Today: Twelve Informal Essays*, pages 241–267, Springer Verlag, New York, 1978.
- [Dew84a] A.K. Dewdney. A computer trap for the Busy Beaver, the hardest-working Turing machine. *Scientific American*, 251(2), pages 10–12, 16, 17, August 1984. — Translation: Computer-Kurzweil, *Spektrum der Wissenschaft*, pages 8–9, 12, 14–16, November 1984.

- [Dew84b] A.K. Dewdney. Computer recreations. *Scientific American*, 251(5), pages 19–22, 27 esp. 27, November 1984. — Translation: Computer-Kurzweil, *Spektrum der Wissenschaft*, pages 8–13 (esp.13), January 1985.
- [Dew85a] A.K. Dewdney. Computer recreations. *Scientific American*, 252(3), pages 14–19 esp. 19, March 1985. — Translation: Computer-Kurzweil, *Spektrum der Wissenschaft*, pages 8–12 (esp. 12), May 1985.
- [Dew85b] A.K. Dewdney. Computer recreations. *Scientific American*, 252(4), pages 12–16 esp. 16, April 1985. — Translation: Computer-Kurzweil, *Spektrum der Wissenschaft*, pages 8–12 (esp.12), June 1985.
- [Dew89] A.K. Dewdney. The Busy Beaver problem. In *The Turing omnibus: 61 excursions in computer science*, pages 241–244, Computer Science Press, Rockville (MD), 1989. ISBN 0716781549.
- [Dre81] R.F. Drenick. Large-scale system theory in the 1980's. *Large Scale Systems: Theory and Applications*, 2(1), pages 29–43, February 1981.
- [Dun65] C. Dunham. A candidate for the simplest uncomputable function. *Communications of the Association for Computing Machinery*, (Letter to the Editor), 8(4), page 201, April 1965.
- [Dun68] C. Dunham. An uncompletable function. *American Mathematical Monthly*, 75, pages 1104–1105, December 1968. — (Zbl 186.10).
- [Dun86] Charles B. Dunham. A simpler approach to the Busy Beaver problem. *SIGACT News*, 17(3), page 29, 1986.  
DOI: 10.1145/382254.990522
- [End77] H.B. Enderton. Elements of recursion theory. In J. Barwise (ed), *Handbook of Mathematical Logic*, pages 527–566 [esp. pages 530–533], Studies in Logic and The Foundations of Mathematics, 90, North-Holland, Amsterdam, 1977.
- [Gács81] P. Gács. On the relation between descriptive complexity and algorithmic probability. In *Proceedings 22nd Annual Symposium on Foundations of Computer Science (Nashville, Tennessee, 1981)*, pages 296–303, IEEE, New York, 1981.  
DOI: 10.1109/SFCS.1981.31
- [Gács83] P. Gács. On the relation between descriptive complexity and algorithmic probability. *Theoretical Computer Science*, 22(1-2), pages 71–93, 1983. — MR 84h:60010.  
DOI: 10.1016/0304-3975(83)90139-1
- [GaY75] D. Gajski and H.M. Yamada. A Busy Beaver problem in cellular automata. In *Proceedings of the International Symposium on Uniformly Structured Automata and Logic (Internat. Inst. Advanced Study Soc. Informat. Sci., Tokyo, August 21-23, 1975)*, pages 171–183, IEEE Computer Society, New York, 1975. — (MR 58#4725).
- [Gar75] M. Gardner. Random numbers, Addendum. In *Mathematical Carnival*, pages 169–170, Penguin Books (Pelican), Harmondsworth, (1975)1982.

- [Gar79] M. Gardner. The random number  $\Omega$  bids fair to hold the mysteries of the universe. *Scientific American*, 241(5), pages 20–34, November 1979.
- [Gre64] M.W. Green. A lower bound on Rado’s sigma function for binary Turing machines. In *Proceedings IEEE 5th Annual Symposium on Switching Circuit Theory and Logical Design (Princeton University, Princeton, N.J., November 11-13, 1964)*, pages 91–94, Special Publication S-164, IEEE, New York, October 1964. — JSL 40.617.  
DOI: 10.1109/SWCT.1964.3
- [Har06] James Harland. The Busy Beaver, the placid platypus and other crazy creatures. In Joachim Gudmundsson and Barry Jay (eds), *CATS’06: Proceedings of the 12th Australasian Symposium on Theory of Computing (Hobart, Australia, January 16-19, 2006)*, pages 79–86, Conferences in Research and Practice in Information Technology, 51, Australian Computer Society, Darlinghurst, 2006.
- [Har07] James Harland. Analysis of Busy Beaver machines via induction proofs. In Joachim Gudmundsson and Barry Jay (eds), *CATS’07: Proceedings of the 13th Australasian Symposium on Theory of Computing (Ballarat, Victoria, Australia, January 30-February 2, 2007)*, pages 71–78, Conferences in Research and Practice in Information Technology, 65, Australian Computer Society, Darlinghurst, 2007.
- [Hen77] F. Hennie. *Introduction to Computability*. Addison-Wesley, Reading (Mass.), 1977. [esp. page 157]
- [Her88] Rolf Herken (ed). *The Universal Turing Machine: A half-century survey*. Oxford University Press, 1988. [esp. pages 259–277  $\equiv$  [Bra88]] — MR91b:03004.
- [Her09] Joachim Hertel. Computing the uncomputable Rado sigma function: An automated, symbolic induction prover for nonhalting Turing machines. *The Mathematics Journal*, 11(2), pages 270–283, 2009.  
Paper, Notebook
- [Hol04] Alex Holkner. Acceleration techniques for Busy Beaver candidates. In *Proceedings of the Second Australian Undergraduate Students’ Computing Conference (AUSCC’04, Melbourne, Australia, December 8-10, 2004)*, 2004.
- [Hop84] J.E. Hopcroft. Turing machines. *Scientific American*, 250(5), pages 70–80 & 124 (bibl.), May 1984.
- [HoU69] J.E. Hopcroft and J.D. Ullman. *Formal Languages and their relation to Automata*. Addison-Wesley, Reading (Mass.), 1969. — CR 11 Rev.20188.
- [Hot68] G. Hotz and H. Walter. *Automatentheorie und formale Sprachen (I. Turingmaschinen und rekursive Funktionen)*. B.I. Hochschulschriften, 821/821a, Bibliographisches Institut, Mannheim, 1968.
- [HoR63] R.W. House and T. Rado. An approach to artificial intelligence. In *Artificial Intelligence, Preprints Artificial Intelligence Sessions of the Winter General Meeting (New York, January 27 - February 1, 1963)*, pages 5–15, Special Publication S-142, IEEE, New York, January 1963.

- [Jjp74] J.P. Jones. Recursive undecidability - An exposition. *American Mathematical Monthly*, 81(7), pages 724–738, 1974. — MR 50#9568; Zbl 326.02034.  
Stable URL at JSTOR
- [Jnd73] N.D. Jones. *Computability Theory - an introduction*. ACM Monograph Series, Academic Press, New York, 1973. [esp. pages 58–67]
- [JoR93] T. Jones and G.J.E. Rawlins. Reverse hillclimbing, genetic algorithms and the busy beaver problem. In S. Forrest (ed), *Proceedings 5th International Conference on Genetic Algorithms (ICGA93) (Urbana Champaign, Illinois, July 1993)*, pages 70–75, Morgan Kaufmann. ISBN 1558602992.
- [Jul92] Bryant A. Julstrom. A bound on the shift function in terms of the Busy Beaver function. *ACM SIGACT News*, 23(3), pages 100–106, Summer 1992.  
DOI: 10.1145/141914.141918
- [Jul93] B.A. Julstrom. Noncomputability and the Busy Beaver problem. UMAP Unit 728. *Journal of Undergraduate Mathematics and its Applications*, 14(1), pages 39–74, 1993.
- [KaS77] H.P. Katseff and M. Sipser. Several results in program size complexity. In *Proceedings Eighteenth Annual Symposium on Foundations of Computer Science (Providence, Rhode Island, October 31, 1977)*, pages 82–89, 1977.
- [KaS81] Howard P. Katseff and Michael Sipser. Several results in program size complexity. *Theoretical Computer Science*, 15(3), pages 291–309, September 1981. — CR 22 Rev.38850; (MR 82i:68028); (Zbl 459.68014).  
DOI: 10.1016/0304-3975(81)90083-9
- [Kel05] Owen Kellett. *A multi-feceted attack on the Busy Beaver problem*. Master’s thesis, Rensselaer Polytechnic Institute, New York, July 2005.
- [Kle52] S.C. Kleene. *Introduction to Metamathematics*. North-Holland, Amsterdam, 1952.
- [Kop81] Rona Jane Kopp. *The Busy Beaver Problem*. MA-thesis, State University of New York, Binghamton (New York), 1981.
- [Kor66] R.R. Korfhage. *Logic and Algorithms*. John Wiley, New York, 1966. — CR 8 Rev.11339.
- [Laf09] Grégory Lafitte. Busy Beavers gone wild. In T. Nearly, D. Woods, T. Seda and N. Murphy (eds), *Proceedings International Workshop on The Complexity of Simple Programs (Cork, Ireland, December 6–7, 2008)*, Electronic Proceedings in Theoretical Computer Science, 1, June 2009.  
DOI: 10.4204/EPTCS.1.12
- [Lee60] C.Y. Lee. Automata and finite automata. *The Bell System Technical Journal*, 39(5), pages 1267–1295, September 1960. — JSL 36.534; MR 22#3633.  
URL at Alcatel-Lucent

- [Lee63] C.Y. Lee. Some recent results in Turing machine theory - an outline. In *Automata Theory (Advanced Concepts in Information Processing Systems) (June 10-21, 1963)*, pages 1–21, Engineering Summer Conferences, 6305, University of Michigan, Ann Arbor (Michigan), 1963.
- [LeP81] H.R. Lewis and C.H. Papadimitriou. *Elements of the Theory of Computation*. Prentice-Hall, Englewood Cliffs NJ, 1981. [esp. problem 6.1.2, pages 300–301]
- [Lin63] S. Lin. *Computer Studies of Turing Machine Problems*. PhD-thesis, The Ohio State University, Columbus (Ohio), 1963.
- [LiR65] S. Lin and T. Rado. Computer studies of Turing machine problems. *Journal of the Association for Computing Machinery*, 12(2), pages 196–212, April 1965. — CR 6 Rev.8651; JSL 40.617; MR 33#3847; Zbl 137.10. DOI: 10.1145/321264.321270
- [LSW83] J. Ludewig, U. Schult, and F. Wankmüller. *Chasing the Busy Beaver - Notes and Observations on a Competition to Find the 5-state Busy Beaver*. Forschungsberichte des Fachbereichs Informatik, 159, Universität Dortmund, Dortmund, 1983.
- [Lyn72] D.S. Lynn. New results for Rado’s sigma function for binary Turing machines. *IEEE Transactions on Computers*, C-21(8), pages 894–896, August 1972. — Zbl 248.02038. DOI: 10.1109/TC.1972.5009047
- [MPC99] Penousal Machado, Francesco B. Pereira, Amílcar Cardoso and Ernesto Costa. Busy Beaver : The influence of representation. In R. Poli et al. (eds), *Genetic Programming*, pages 29–38, Lecture Notes in Computer Science, 1598, Springer Verlag, Berlin, 1999. DOI: 10.1007/3-540-48885-5\_3
- [MaS90] Rona Machlin and Quentin F. Stout. The complex behavior of simple machines. *Physica D*, 42(1–3), pages 85–98, 1990. DOI: 10.1016/0167-2789(90)90068-Z
- [Mar91] J.C. Martin. *Introduction to Languages and the Theory of Computation*. McGraw-Hill, New York, 1991. ISBN 0071008519. [esp. example 21.1, page 373]
- [MaB90] Heiner Marxen and Jürgen Buntrock. Attacking the Busy Beaver 5. *Bulletin of the EATCS*, 40, pages 247–251, February 1990. — (Zbl 0744.68043).
- [Mic93] Pascal Michel. Busy beaver competition and Collatz-like problems. *Archive for Mathematical Logic*, 32(5), pages 351–367, 1993. — MR 94f:03048; (Zbl 779.03009). DOI: 10.1007/BF01409968
- [Mic04] Pascal Michel. Small Turing machines and generalized busy beaver competition. *Theoretical Computer Science*, 326(1-3), pages 45–56, 2004. DOI: 10.1016/j.tcs.2004.05.008

- [Mic09] Pascal Michel. *The Busy Beaver Competition: a historical survey*. arXiv.org:0906.3749 [math.LO], September 2009.  
URL at arXiv.org
- [Min67] M.L. Minsky. *Computation: finite and infinite machines*. Prentice Hall, Englewood Cliffs (N.J.), 1967. — MR 50#9050; (Zbl 195.024).
- [NOI02] Etsushi Nameda, Toru Ohira and Takashi Ikegami. Dynamical systems approach to the busy beaver problem. *Computer Physics Communications*, 147(1-2), pages 637–640, August 2002. — (Zbl 0992.68503).  
DOI: 10.1016/S0010-4655(02)00356-9
- [Obe58] W. Oberschelp. Varianten von Turingmaschinen. *Archiv für Mathematische Logik und Grundlagenforschung*, 4, pages 53–62, 1958. — JSL 36.534; MR 20#4494; Zbl 84.10.  
URL at Universität Göttingen
- [OST88] A. Oberschelp, K. Schmidt-Göttsch, and G. Todt. Castor Quadruplorum. *Archive for Mathematical Logic*, 27(1), pages 35–44, 1988. — MR 89k:03045; Zbl 646.03034.  
DOI: 10.1007/BF01625831
- [Ogi72] C.S. Ogilvy. *Tomorrow's math*. Oxford University Press, New York, 2nd ed, (1962)1972. [esp. pages 44–47 & 169] — Translation (C. van der Linden): *De wiskunde van morgen*, Het Spectrum, Utrecht/Antwerpen, 1965. [esp. pages 39–42 & 117]
- [Pag69] D. Pager. On the problem of finding minimal programs for tables. *Information and Control*, 14, pages 550–554, 1969. — (Zbl 182.333).  
Also *Kibernet. Sbornik*, 7, pages 94–98, 1970. — (Zbl 214.19).
- [Pag71] D. Pager. On two-way, two-tape automata. *Mathematical Systems Theory*, 5(2), pages 164–167, 1971. — MR 46#1513; (Zbl 218.603).  
DOI: 10.1007/BF01702873
- [Pag74] D. Pager. Further results on the problem of finding minimal length programs for decision tables. *Journal of the Association for Computing Machinery*, 21(2), pages 207–212, April 1974. — CR 15 Rev.26957; MR 50#1557; Zbl 303.68031.  
DOI: 10.1145/321812.321816
- [PeC01] Francisco B. Pereira and Ernesto Costa. The influence of learning in the evolution of busy beavers. In E.J.W. Boers (ed.), *Applications of Evolutionary Computing*, Proceedings Evo Workshops 2001 (Como, Italy, April 18–20, 2001), LNCS 2037, pages 421–430, Springer-Verlag, Berlin, 2001. — (Zbl 0978.68706).  
DOI: 10.1007/3-540-45365-2\_44
- [PeF02] Alessandro Perrone and Gianluigi Ferraris. ABBA, agent based beaver applications : busy beaver in swarm. In H. Yin (ed.), *Intelligent data engineering and automated learning*, Proceedings IDEAL2002 (Manchester, UK, August 12–14, 2002), LNCS 2412, pages 273–278, Springer-Verlag, Berlin, 2002. — (Zbl

1020.68841).  
10.1007/3-540-45675-9\_45

- [Pet06] H. Petersen. Computable lower bounds for Busy Beaver Turing machines. In Z. Esik, C. Martin-Vide and V. Mitrană, *Recent Advances in Formal Languages and Applications*, pages 305–319, Studies in Computational Intelligence, 25, Springer Verlag, Berlin, 2006.  
DOI: 10.1007/978-3-540-33461-3\_12
- [RaW63] M.O. Rabin and H. Wang. Words in the history of a Turing machine with a fixed input. *Journal of the Association for Computing Machinery*, 10(4), pages 526–527, October 1963. — (CR 5 Rev.6392); Zbl 192.67.  
DOI: 10.1145/321186.321195
- [Rad62a] T. Rado. On non-computable functions. *The Bell System Technical Journal*, 41(3), pages 877–884, May 1962. — CR 3 Rev.3264; JSL 32.524; MR 24#A3063.  
URL at Alcatel-Lucent
- [Rad62b] T. Rado. *On Non-computable Functions*. Bell Telephone System Technical Publications, Monograph 4199, 1962.
- [Rad63] T. Rado. On a simple source for non-computable functions. In *Proceedings of Symposium on Mathematical Theory of Automata (Polytechnic Institute of Brooklyn, April 24-26, 1962)*, pages 75–81, MRI Symposium Series XII, Polytechnic Press, Brooklyn (N.Y.), 1963. — JSL 32.524; (MR 30#1043); Zbl 132.248.
- [Ros03] Kyle Ross. *Use of optimisation techniques in determining values for the quadruplorum variants of Rado’s Busy Beaver function*. Masters thesis, Rensselaer Polytechnic Institute, New York, 2003.
- [Sal85] A. Salomaa. *Computation and Automata*. Cambridge University Press, Cambridge, 1985. [esp. example 4.1, pages 81–82, and exercise 4.1, page 114.]
- [San78] H.P. Sankappanavar. Decision problems: history and methods. In A.I. Arruda, N.C.A. da Costa, and R. Chaqui (eds), *Mathematical Logic (Proceedings of the First Brazilian Conference on Mathematical Logic, Universidad Estadual de Campinas, July 4-6, 1977)*, pages 241–291, Lecture Notes in Pure and Applied Mathematics, 39, Marcel Dekker, New York, 1978. — Zbl 385.03011.
- [Smo80] C. Smoryński. Some rapidly growing functions. *The Mathematical Intelligencer*, 2(3), pages 149–154, 1980. — MR 82c:03064; Zbl 453.03049.  
DOI: 10.1007/BF03023057
- [Sta65] W.R. Stahl. Genetic adaptation as a solvability problem for a Turing machine. In W.A. Kalenich (ed), *Proceedings IFIP Congress (New York, May 24-29, 1965)*, vol. 2, pages 570-571, Spartan Books, New York, 1965.
- [SCG64] W.R. Stahl, R.W. Coffin, and H.E. Goheen. Simulation of biological cells by systems composed of string-processing finite automata. In *Proceedings AFIPS 1964 Spring Joint Computer Conference (Washington, D.C., April 1964)*, pages 89–102, Spartan Books, Baltimore (Maryland), 1964. — CR 5 Rev.6523.

- [Tha70] J.W. Thatcher. Self-describing Turing machines and self-reproducing cellular automata. In A.W. Burks (ed), *Essays on Cellular Automata*, pages 103–131 & 354–368 (bibl.), University of Illinois Press, Urbana (Ill.), 1970. [esp. pages 111–113]
- [Tur36] A.M. Turing. On computable numbers, with an application to the Entscheidungsproblem. *Proceedings of the London Mathematical Society, Second Series*, 42(3), pages 230–265, November 1936. — Zbl 16.97.  
DOI: 10.1112/plms/s2-42.1.230
- [Tur37] A.M. Turing. On computable numbers, with an application to the Entscheidungsproblem: A correction. *Proceedings of the London Mathematical Society, Second Series*, 43(7), pages 544–546, May 1937. — Zbl 18.193.  
DOI: 10.1112/plms/s2-43.6.544
- [Vit76] P.M.B. Vitányi. On a problem in collective behavior of automata. *Discrete Mathematics*, 14(1), pages 99–101, 1976. — MR 52#16133; (Zbl 322.68030).  
DOI: 10.1016/0012-365X(76)90011-X
- [Wal82] T.R.S. Walsh. The Busy Beaver on a one-way infinite tape. *ACM Sigact News*, 14(1), pages 38–43, Winter 1982. — (Zbl 0565.68048).  
DOI: 10.1145/1008892.1008896
- [Wan57] H. Wang. A variant to Turing’s theory of computing machines. *Journal of the Association for Computing Machinery*, 4(1), pages 63–92, 1957. — MR 20#4492.  
DOI: 10.1145/320856.320867
- [Wan60] H. Wang. Towards mechanical mathematics. *IBM Journal of Research and Development*, 4, pages 2–22, 1960. — Zbl 97.4.  
Also *Kibernet. Sbornik*, 5, pages 114–165, 1962. — Zbl 106.8.  
Also in K.M. Sayre and F.J. Crosson (eds), *The Modelling of the Mind*, pages 91–120, University of Notre Dame Press, South Bend (Indiana), 1963.  
DOI: 10.1147/rd.41.0002
- [Wei73] B. Weimann. *Untersuchungen über Rado’s Sigma-Funktion und eingeschränkte Halteprobleme bei Turingmaschinen*. Dissertation, Mathematisch-Naturwissenschaftlichen Fakultät, Rheinische Friedrich-Wilhelms-Universität, Bonn, November 1973.
- [Wei85] B. Weimann. Leserbrief. *Spektrum der Wissenschaft*, page 4, August 1985.
- [WCF73] B. Weimann, K. Casper, and W. Fenzl. Untersuchungen über haltende Programme für Turingmaschinen mit 2 Zeichen und bis zu 5 Befehlen. In *GI Gesellschaft für Informatik eV: 2. Jahrestagung (Karlsruhe, October 2-4, 1972)*, pages 72–81, Lecture Notes in Economics and Mathematical Systems, 78, Springer Verlag, Berlin, 1973. — Zbl 259.02028.
- [Woo87] D. Wood. *Theory of Computation*. Harper & Row, New York, 1987. [esp. pages 331–332]

- [YDX97] Ruiguang Yang, Longyun Ding and Shurun Xu. Some better results estimating the shift function in terms of Busy Beaver function. *SIGACT News*, 28(1), pages 44–48, 1997.  
DOI: 10.1145/250037.250038
- [B] **Reviews**
- [Ádá67] A. Ádám. Zbl 132.248, (T. Rado, Proc. Sympos. Math. Automata, 1963), 1967.
- [Ani89] A.V. Anisimov. Zbl 646.03034, (A. Oberschelp et al., Arch. Math. Logic, 1988), February 1989.
- [Ast89] A. Astromoff. MR 89k:03045, (A. Oberschelp et al., Arch. Math. Logic, 1988), November 1989.
- [Bae84] R.M. Baer. MR 84d:03049, (A.H. Brady, Math. Comp., 1983), 1984.
- [Bae94] R.M. Baer. MR 94f:03048, (P. Michel, Arch. Math. Logic, 1993), June 1994.
- [Bae97] R.M. Baer. MR 97d:68049, (A.M. Ben-Amram et al., Math. Syst. Theory, 1996), 1997.
- [Ber84] J. Berman. Zbl 518.03013, (A.H. Brady, Math. Comp., 1983), June 1984.
- [Blu67] E.K. Blum. MR 33#3847, (S. Lin and T. Rado, JACM, 1965), March 1967.
- [Bor80] E. Börger. Zbl 421.03031, (R.P. Daley, Z. Math. Logik Grundlagen Math., 1978), September 1980.
- [Buc67] J.R. Büchi. Zbl 137.10, (S. Lin and T. Rado, JACM, 1965), 1967.
- [Buz83] S. Buzeteanu. Zbl 501.03024, (R.P. Daley, Stud. Logic Found. Math., 1980), September 1983.
- [Cog62] E.J. Cogan. MR 24#A3063, (T. Rado, Bell System Tech. J., 1962), December 1962.
- [Cal81] C. Calude. Zbl 453.03049, (C. Smoryński, Math. Intell., 1980), December 1981.
- [Cal82] C. Calude. MR 82a:03040, (R.P. Daley, Studies in Logic and Found. of Math., 1980), 1982.
- [Cal83a] C. Calude. Zbl 486.03021, (R.P. Daley, LNCS 118, 1981), February 1983.
- [Cal83b] C. Calude. Zbl 486.03022, (R.P. Daley, J. Symb. Logic, 1981), February 1983.
- [Cal83c] C. Calude. Zbl 503.03017, (R.P. Daley, Inf. Control, 1982), October 1983.
- [Can67] F.B. Cannonito. JSL 32.524, (T. Rado, Bell System Tech. J., 1962) & (T. Rado, Proc. Sympos. Math. Automata, 1963), 1967.
- [Can70] F.B. Cannonito. Zbl 192.67, (M.O. Rabin and H. Wang, JACM, 1963), 1970.
- [Cur37] H.B. Curry. Zbl 16.97, (A.M. Turing, Proc. London Math. Soc., 1936), July 1937.

- [Cur38] H.B. Curry. Zbl 18.193, (A.M. Turing, Proc. London Math. Soc., 1937), 1938.
- [DewXX] A.K. Dewdney. Zbl. 0685.68054, (A.H. Brady, 1988), 19XX.
- [End75] H.B. Enderton. JSL 40.617, (S. Lin and T. Rado, JACM, 1965) & (A.H. Brady, IEEE Trans. Electr. Comp., 1966) & (M.W. Green, Proc. Switching Circuit Th., 1964), 1975.
- [Gil75] J.T. Gill. MR 50#1557, (D. Pager, JACM, 1974), July 1975.
- [Gil78] J.T. Gill. MR 56#13775, (G.J. Chaitin, JACM, 1974), November 1978.
- [Hel85] J. Helm. MR 85h:03039, (R.P. Daley, Inform. and Control, 1982), 1985.
- [Her70] G.T. Herman. Zbl 193.331, (Ju.Ja. Breĭtbart and V.A. Kozmidiadi, Doklady Akad. Nauk SSSR, 1969), 1970.
- [Hom75] H.H. Homuth. Zbl 282.68022, (G.J. Chaitin, IEEE Trans. Inform. Theory, 1974), February 1975.
- [Iss75] W. Issel. Zbl 287.68027, (G.J. Chaitin, JACM, 1974), April 1975.
- [Jjp83] J.P. Jones. MR unpublished, (R.P. Daley, Inform. and Control, 1982), September 1983.
- [Kas67] M. Kassler. CR 8 Rev.11339, (R.R. Korfhage, Logic and algorithms, 1966), March-April 1967.
- [Kin64] G.W. King. CR 5 Rev.6392, (M.O. Rabin and H. Wang, JACM, 1963), 1964.
- [Kog79] S.R. Kogalovskii. Zbl 385.03011, (H.P. Sankappanavar, Lect. Notes Pure Appl. Math., 1978), May 1979.
- [Kri81] M.S. Krishnamoorthy. CR 22 Rev.38850, (H.P. Katseff and M. Sipser, Theor. Comput. Sci., 1981), December 1981.
- [Lew73] F.D. Lewis. MR 46#1513, (D. Pager, Math. Systems Theory, 1971), July 1973.
- [Lud76] E. Lüdde. Zbl 303.68031, (D. Pager, JACM, 1974), January 1976.
- [Mar78] G.B. Marandzjan. Zbl 365.68054, (R.P. Daley, J. Symbolic Logic, 1976), August 1978.
- [McL80] T. McLaughlin. MR 80h:03061, (R.P. Daley, Z. Math. Logik Grundlag. Math., 1978), 1980.
- [Men59] E. Mendelson. MR 20#4494, (W. Oberschelp, Arch. Math. Logik Grundlagenforsch., 1958), July-August 1959.
- [Moo61] E.F. Moore. MR 22#3633, (C.Y. Lee, Bell System Tech. J., 1960), April 1961.
- [Moy74] J.A. Moyne. CR 15 Rev.27635, (G.J. Chaitin, JACM, 1974), December 1974.
- [Mul71] A.A. Mullin. MR 42#7435, (Ju.Ja. Breĭtbart and V.A. Kozmidiadi, Dokl. Akad. Nauk SSSR, 1969), November 1971.

- [Mul82] A.A. Mullin. MR 82c:03064, (C. Smoryński, Math. Intell., 1980), March 1982.
- [Obe91] A. Oberschelp. JSL 56.1091, (A.H. Brady, in: R. Herken (ed.), 1988), 1991.
- [Owi83] J.C. Owings Jr. MR 83m:03049, (R.P. Daley, LNCS 118, 1981), 1983.
- [Pao83] R.A. Di Paola. MR 83a:03035, (R.P. Daley, J. Symbolic Logic, 1981), 1983.
- [Par64] R.A. Parker. CR 5 Rev.6523, (W.R. Stahl et al., Proc. AFIPS 1964 Spring Joint Comput. Conf., 1964), 1964.
- [Poo74] U.W. Pooch. CR 15 Rev.26957, (D. Pager, JACM, 1974), July 1974.
- [Röd75] D. Rödding. MR 50#9568, (J.P. Jones, Amer. Math. Monthly, 1974), November 1975.
- [Ros59] A. Rose. MR 20#4492, (H. Wang, JACM, 1957), July-August 1959.
- [Ros62] A. Rose. Zbl 97.4, (H. Wang, IBM J. Res. Develop., 1960), 1962.
- [Sch78] C.-P. Schnorr. MR 55#2529, (G.J. Chaitin, IEEE Trans. Information Theory, 1974), February 1978.
- [Sch84] C.-P. Schnorr. MR 84h:60010, (P. Gács, Theoret. Comput. Sci., 1983), 1984.
- [Šep73] K. Šeper. Zbl 248.02038, (D.S. Lynn, IEEE Trans. Comput., 1972), July 1973.
- [Sha91] Stewart Shapiro. MR 91b:03004, (Rolf Herken (ed), The Universal Turing Machine: A half-century survey, 1988), February 1991.
- [Shp71] J.C. Shepherdson. JSL 36.534, (W. Oberschelp, Arch. Math. Logik Grundlagenforsch., 1958) & (C.Y. Lee, Bell System Tech. J., 1960), 1971.
- [Shr64] P. Sheridan. CR 5 Rev.5742, (R.W. Coffin et al., Proc. AFIPS 1963 Fall Joint Comput. Conf., 1963), 1964.
- [Sie77] D. Siefkes. Zbl 326.02034, (J.P. Jones, Amer. Math. Monthly, 1974), January 1977.
- [Sou78] N. Soundararajan. CR 19 Rev.33065, (G.J. Chaitin, IBM J. Res. Dev., 1977), May 1978.
- [Tam60] D. Tamari. Zbl 84.10, (W. Oberschelp, Arch. Math. Logik Grundlagenforsch., 1958), July 1960.
- [Tou76] G. Tourlakis. MR 52#16133, (P.M.B. Vitányi, Discrete Math., 1976), December 1976.
- [Wal70] R.E. Wall. CR 11 Rev.20188, (J.E. Hopcroft and J.D. Ullman, Formal languages and their relation to automata, 1969), November 1970.
- [Wee65] G.P. Weeg. CR 6 Rev.8651, (S. Lin and T. Rado, JACM, 1965), November-December 1965.
- [Yoe62] M. Yoeli. CR 3 Rev.3264, (T. Rado, Bell System Tech. J., 1962), November-December 1962.

## Updates

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