Constructing Franklin Magic Squares

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Kubusdag Eindhoven – 14 oktober 2007

impossible number puzzles

Benjamin Franklin *discovered* squares with numbers $1, \ldots, 64$ arranged

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

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each ROW having sum $8 \cdot 32.5 = 260$.

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each ROW having sum $8 \cdot 32.5 = 260$,
each COLUMN having sum $8 \cdot 32.5 = 260$.

Benjamin Franklins square is NOT *magic*

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
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DIAGONALS have sums 262 and 258

Benjamin Franklins square is NOT *magic* but

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BENT DIAGONALS have sums 260.

Benjamin Franklins square is NOT *magic* but

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BENT DIAGONALS have sums 260 (in both orientations).

Benjamin Franklin squares are more than magic:

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

SHIFTED BENT DIAGONALS have magic sum 260 (all types $\vee, >, \wedge, <$)

Franklin Magic squares further feature

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

HALF ROWS having fixed sum $(4 \cdot 32.5 = 130)$

Franklin Magic squares further feature

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

HALF ROWS having fixed sum $(4 \cdot 32.5 = 130)$,
HALF COLUMNS having fixed sum $(4 \cdot 32.5 = 130)$

Franklin Magic squares finally have

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

2x2 SUBSQUARES all have sum $4 \cdot 32.5 = 130$
even if wrapped-around

a general n by n Franklin Magic square M has

1. entries $1, 2, \ldots, n^2$;
2. each row and each column has a fixed entry sum $n(1+n^2)/2$;
3. each two by two sub-square $\begin{bmatrix} M_{i,j} & M_{i,j+1} \\ M_{i+1,j} & M_{i+1,j+1} \end{bmatrix}$ has sum $2(1+n^2)$;
4. each half row starting in column 1 or $n/2+1$ has sum of entries equal to $n(1+n^2)/4$, and similar for half columns starting in row 1 or $n/2+1$;
5. each shifted bent diagonal has entry sum equal to $n(1+n^2)/2$.

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5. each shifted bent diagonal has entry sum equal to $n(1+n^2)/2$.

CHALLENGE:
find more Franklin Magic squares!

REQUIREMENT:
n is multiple of 4

a 4 by 4 Franklin Magic square would have entries $1, \ldots, 16$

X	?	?	?
?	?	?	?
?	?	?	?
?	?	?	?

a 4 by 4 Franklin Magic square would have entries $1, \ldots, 16$

X	$17 - X$	?	?
$17 - X$	?	?	?
?	?	?	?
?	?	?	?

half rows, half columns have sum 17

a 4 by 4 Franklin Magic square would have entries $1, \ldots, 16$

X	$17 - X$	?	?
$17 - X$	X	?	?
?	?	?	?
?	?	?	?

half rows, half columns have sum 17
2x2 subsquares have sum 34

a 4 by 4 Franklin Magic square would have entries $1, \ldots, 16$

X	$17 - X$	?	?
$17 - X$	X	?	?
?	?	?	?
?	?	?	?

half rows, half columns have sum 17
2x2 subsquares have sum 34
but entries should be distinct ! No such thing !

a 16 by 16 Franklin Magic square must have entries $1, \ldots, 256$.

1	192	209	112	225	96	49	144	241	80	33	160	17	176	193	128
252	69	44	149	28	165	204	117	12	181	220	101	236	85	60	133
14	179	222	99	238	83	62	131	254	67	46	147	30	163	206	115
247	74	39	154	23	170	199	122	7	186	215	106	231	90	55	138
15	178	223	98	239	82	63	130	255	66	47	146	31	162	207	114
246	75	38	155	22	171	198	123	6	187	214	107	230	91	54	139
4	189	212	109	228	93	52	141	244	77	36	157	20	173	196	125
249	72	41	152	25	168	201	120	9	184	217	104	233	88	57	136
16	177	224	97	240	81	64	129	256	65	48	145	32	161	208	113
245	76	37	156	21	172	197	124	5	188	213	108	229	92	53	140
3	190	211	110	227	94	51	142	243	78	35	158	19	174	195	126
250	71	42	151	26	167	202	119	10	183	218	103	234	87	58	135
2	191	210	111	226	95	50	143	242	79	34	159	18	175	194	127
251	70	43	150	27	166	203	118	11	182	219	102	235	86	59	134
13	180	221	100	237	84	61	132	253	68	45	148	29	164	205	116
248	73	40	153	24	169	200	121	8	185	216	105	232	89	56	137

Some facts

- a 4 by 4 Franklin Magic square does not exist
- Franklin produced some 8 by 8 and some 16 by 16 Franklin squares (± 1750)
- web sites of magic squares display Franklin Magic squares of order 8 and 16, even 32
- constructing squares of order $8k$ is not so difficult
- March 22, 2007: HYPE in the Netherlands
  high school students Hoekstra, Schulte, Alkema construct an ALMOST-FRANKLIN-MAGIC square of order 12 (Dec 14, 2006)

Question

do there exist Franklin Magic squares of order 12, 20, 28, . . . ?

HSA Magic Square

1	142	11	136	8	138	5	139	12	135	2	141
120	27	110	33	113	31	116	30	109	34	119	28
121	22	131	16	128	18	125	19	132	15	122	21
48	99	38	105	41	103	44	102	37	106	47	100
73	70	83	64	80	66	77	67	84	63	74	69
60	87	50	93	53	91	56	90	49	94	59	88
85	58	95	52	92	54	89	55	96	51	86	57
72	75	62	81	65	79	68	78	61	82	71	76
97	46	107	40	104	42	101	43	108	39	98	45
24	123	14	129	17	127	20	126	13	130	23	124
25	118	35	112	32	114	29	115	36	111	26	117
144	3	134	9	137	7	140	6	133	10	143	4

HSA Magic Square

1	142	11	136	8	138	5	139	12	135	2	141
120	27	110	33	113	31	116	30	109	34	119	28
121	22	131	16	128	18	125	19	132	15	122	21
48	99	38	105	41	103	44	102	37	106	47	100
73	70	83	64	80	66	77	67	84	63	74	69
60	87	50	93	53	91	56	90	49	94	59	88
85	58	95	52	92	54	89	55	96	51	86	57
72	75	62	81	65	79	68	78	61	82	71	76
97	46	107	40	104	42	101	43	108	39	98	45
24	123	14	129	17	127	20	126	13	130	23	124
25	118	35	112	32	114	29	115	36	111	26	117
144	3	134	9	137	7	140	6	133	10	143	4

2 by 2 squares have sum $4 \cdot \frac{1}{2}(1 + 144) = 290$

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121	22	131	16	128	18	125	19	132	15	122	21
48	99	38	105	41	103	44	102	37	106	47	100
73	70	83	64	80	66	77	67	84	63	74	69
60	87	50	93	53	91	56	90	49	94	59	88
85	58	95	52	92	54	89	55	96	51	86	57
72	75	62	81	65	79	68	78	61	82	71	76
97	46	107	40	104	42	101	43	108	39	98	45
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2 by 2 squares have sum $4 \cdot \frac{1}{2}(1 + 144) = 290$
each **third** row or column has sum 290

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120	27	110	33	113	31	116	30	109	34	119	28
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60	87	50	93	53	91	56	90	49	94	59	88
85	58	95	52	92	54	89	55	96	51	86	57
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25	118	35	112	32	114	29	115	36	111	26	117
144	3	134	9	137	7	140	6	133	10	143	4

2 by 2 squares have sum $4 \cdot \frac{1}{2}(1 + 144) = 290$
each **third** row or column has sum 290
each bent-diagonal has sum $12 \cdot \frac{1}{2}(1 + 144) = 870$

Questions

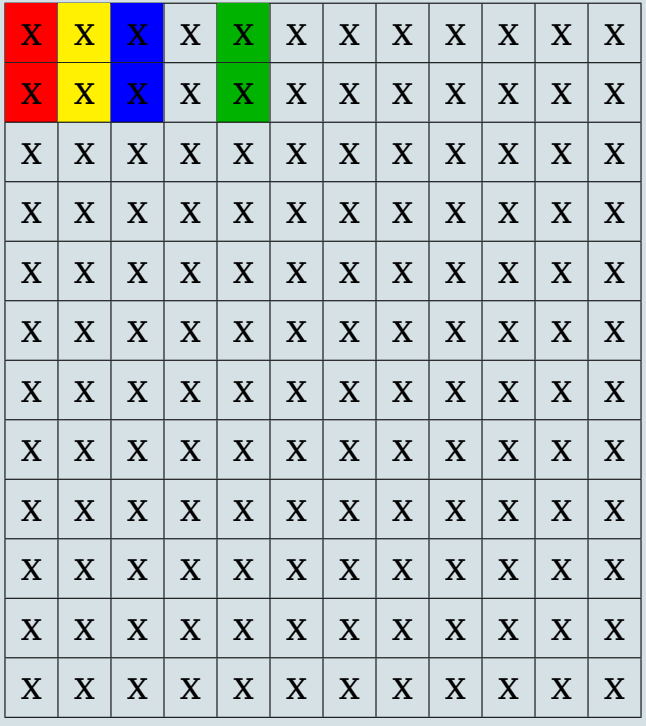
- do there exist Franklin Magic squares of order 12, 20, 28, . . . ?
- can a brute-force search by computer find such square?
- or is there some sophisticated way to construct these squares?

Answers: Yes and No

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

Searching for a Franklin magic square of order 12



RED equals BLUE equals GREEN.

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

RED equals BLUE equals GREEN.
RED plus YELLOW is magic, hence YELLOW plus GREEN is magic.

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

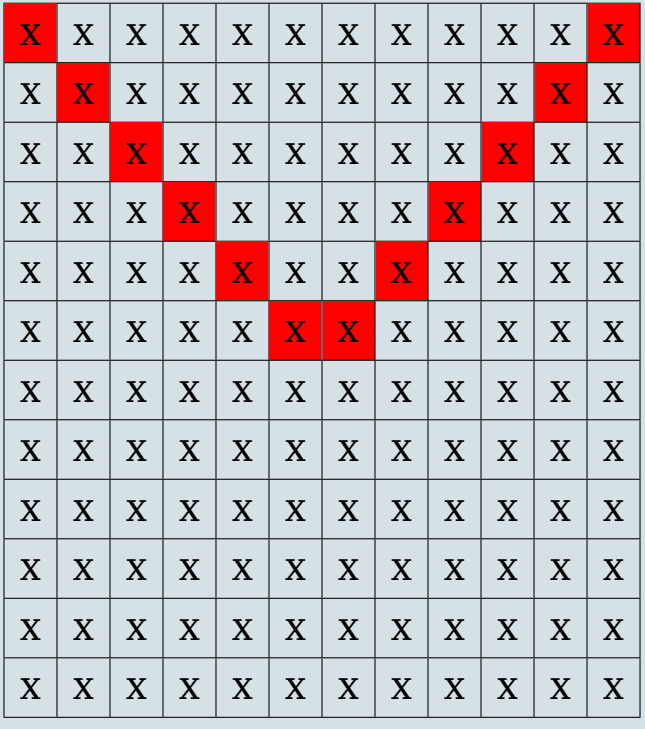
YELLOW plus GREEN is magic; GREEN plus BLUE is magic

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

YELLOW plus GREEN is magic; GREEN plus BLUE is magic
YELLOW equals BLUE !

Searching for a Franklin magic square of order 12



Bent-diagonal with magic sum

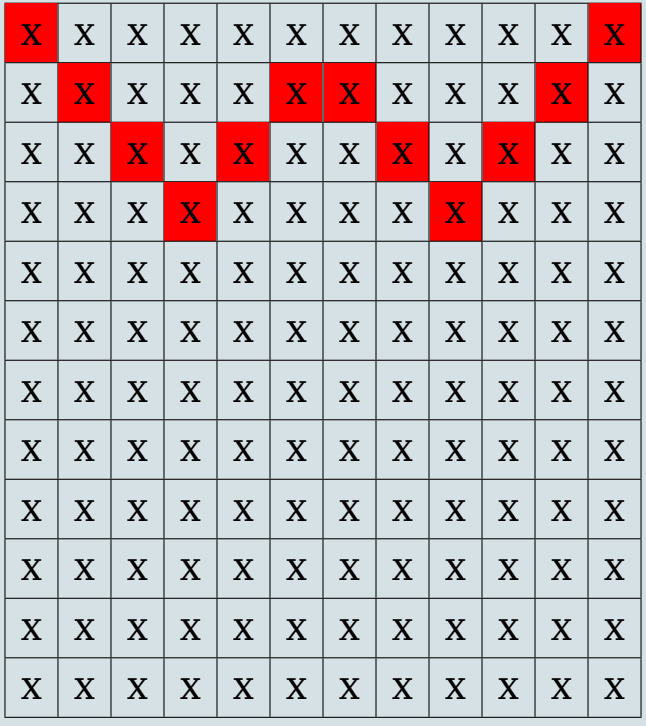
Searching for a Franklin magic square of order 12

Sum remains magic

Searching for a Franklin magic square of order 12

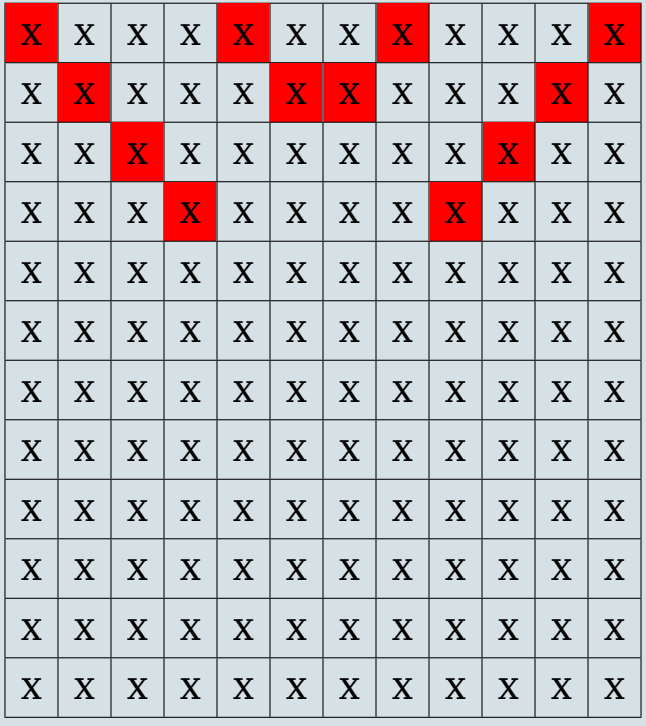
Sum remains magic

Searching for a Franklin magic square of order 12



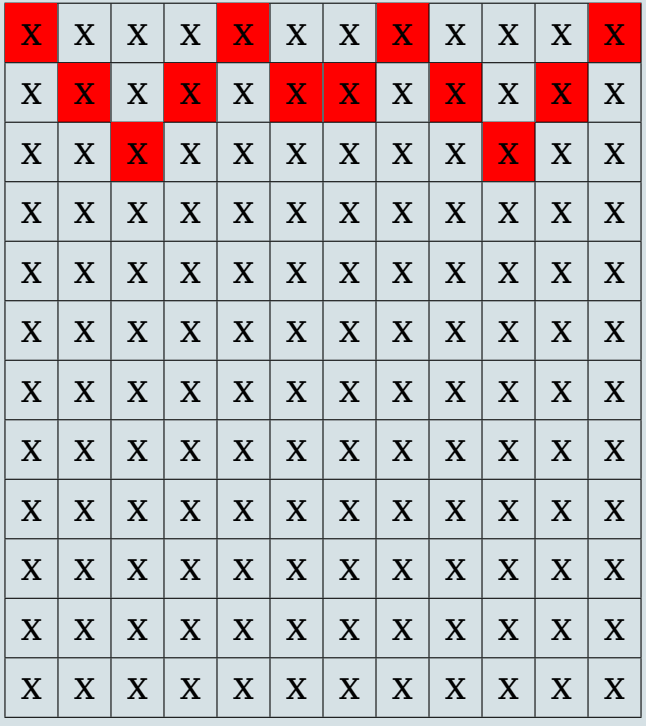
Sum remains magic

Searching for a Franklin magic square of order 12



Sum remains magic

Searching for a Franklin magic square of order 12



Sum remains magic

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

Sum remains magic

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

magic RED sum equals sum of THREE magic 2x2 squares

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

RED equals YELLOW (only used 2x2 magic subsquare property)

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

RED equals YELLOW (only used 2x2 magic subsquare property)
RED plus YELLOW is magic ROW (870)

Searching for a Franklin magic square of order 12

x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x
x	x	x	x	x	x	x	x	x	x	x	x

RED has magic sum (435)
used 2x2 magic subsquare property, plus magic row sum

Let $P = \{1, 3, 5\}, Q = \{2, 4, 6\}, R = \{7, 9, 11\}, S = \{8, 10, 12\}$,

a Franklin Magic square M of order 12 is a square with

1. entries $1, 2, \ldots, 144$;
2. each two by two sub-square $\begin{bmatrix} M_{i,j} & M_{i,j+1} \\ M_{i+1,j} & M_{i+1,j+1} \end{bmatrix}$ has sum 290;
3. in row (column) 1, entries in first half ($P + Q$) add up to 435;
4. in row (column) 1, entries in second half ($R + S$) add up to 435;
5. in row (column) 1, entries in first half odd positions plus entries in second half of even positions ($P + S$) add up to 435;

Note:
in row 1, entries in P and entries in R have the same sum

Let $P = \{1, 3, 5\}, Q = \{2, 4, 6\}, R = \{7, 9, 11\}, S = \{8, 10, 12\}$,

a Franklin Magic square M of order 12 remains Franklin Magic even

1. reflected along diagonal
2. reflected along horizontal axis of symmetry
3. reflected along vertical axis of symmetry
4. interchanging columns $1, 3, 5$ (similar for Q, R, S);
5. interchanging columns in P with those in R (similar for Q and S);
6. applying above tricks to rows

Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	1	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	1	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

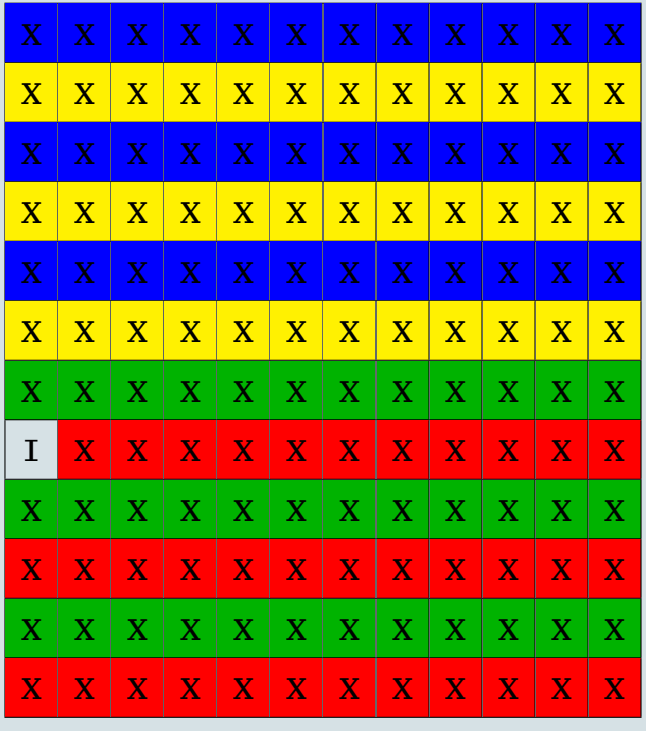
Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	1	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

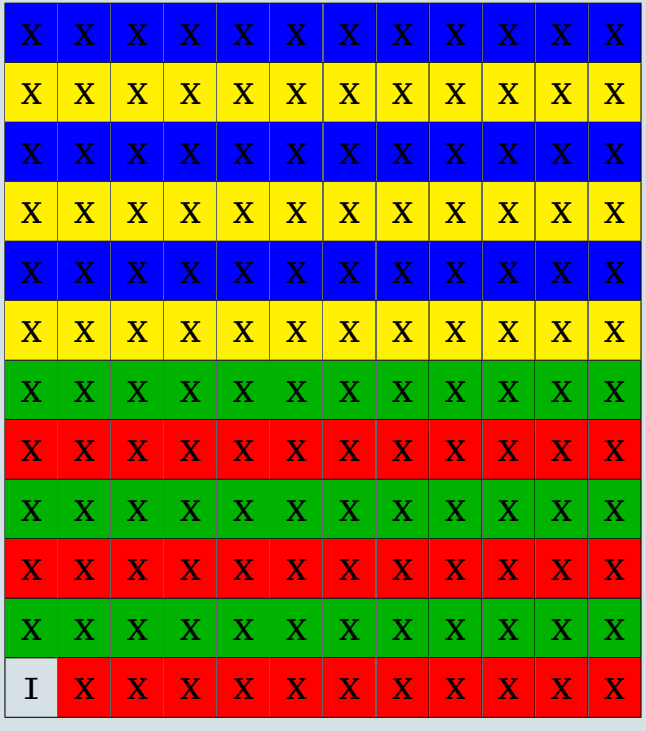
Position of 1 in a Franklin magic square of order 12

X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
1	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X	X	X

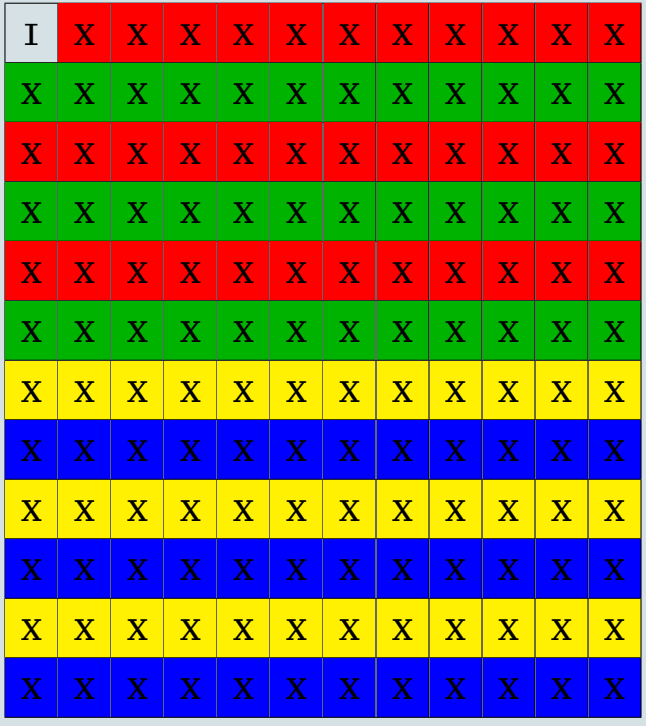
Position of 1 in a Franklin magic square of order 12



Position of 1 in a Franklin magic square of order 12



Position of 1 in a Franklin magic square of order 12



if there is a 12 by 12 FMS, there is one starting with 1

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	x	x	x	x	x	x	x	x	x	x	x
y_2	x	x	x	x	x	x	x	x	x	x	x
y_3	x	x	x	x	x	x	x	x	x	x	x
y_4	x	x	x	x	x	x	x	x	x	x	x
y_5	x	x	x	x	x	x	x	x	x	x	x
y_6	x	x	x	x	x	x	x	x	x	x	x
y_7	x	x	x	x	x	x	x	x	x	x	x
y_8	x	x	x	x	x	x	x	x	x	x	x
y_9	x	x	x	x	x	x	x	x	x	x	x
y_{10}	x	x	x	x	x	x	x	x	x	x	x
y_{11}	x	x	x	x	x	x	x	x	x	x	x

Fill in first row and column, rest follows from 2x2 squares

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	x	x	x	x	x	x	x	x	x	x	x
y_2	x	x	x	x	x	x	x	x	x	x	x
y_3	x	x	x	x	x	x	x	x	x	x	x
y_4	x	x	x	x	x	x	x	x	x	x	x
y_5	x	x	x	x	x	x	x	x	x	x	x
y_6	x	x	x	x	x	x	x	x	x	x	x
y_7	x	x	x	x	x	x	x	x	x	x	x
y_8	x	x	x	x	x	x	x	x	x	x	x
y_9	x	x	x	x	x	x	x	x	x	x	x
y_{10}	x	x	x	x	x	x	x	x	x	x	x
y_{11}	x	x	x	x	x	x	x	x	x	x	x

Fill in first row and column, rest follows from 2x2 squares
$M_{22} = 290 - 1 - x_1 - y_1$; $M_{23} = 290 - x_1 - x_2 - M_{22}$

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	x	x	x	x	x	x	x	x	x	x	x
y_2	x	x	x	x	x	x	x	x	x	x	x
y_3	x	x	x	x	x	x	x	x	x	x	x
y_4	x	x	x	x	x	x	x	x	x	x	x
y_5	x	x	x	x	x	x	x	x	x	x	x
y_6	x	x	x	x	x	x	x	x	x	x	x
y_7	x	x	x	x	x	x	x	x	x	x	x
y_8	x	x	x	x	x	x	x	x	x	x	x
y_9	x	x	x	x	x	x	x	x	x	x	x
y_{10}	x	x	x	x	x	x	x	x	x	x	x
y_{11}	x	x	x	x	x	x	x	x	x	x	x

First row: x_5 (in Q), x_{10} (in R), x_{11} (in S) follow from rules
Similar: y_5, y_{10}, and y_{11} are fixed; 16 degrees of freedom.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	x	x	x	x	x	x	x	x	x	x	x
y_2	x	x	x	x	x	x	x	x	x	x	x
y_3	x	x	x	x	x	x	x	x	x	x	x
y_4	x	x	x	x	x	x	x	x	x	x	x
y_5	x	x	x	x	x	x	x	x	x	x	x
y_6	x	x	x	x	x	x	x	x	x	x	x
y_7	x	x	x	x	x	x	x	x	x	x	x
y_8	x	x	x	x	x	x	x	x	x	x	x
y_9	x	x	x	x	x	x	x	x	x	x	x
y_{10}	x	x	x	x	x	x	x	x	x	x	x
y_{11}	x	x	x	x	x	x	x	x	x	x	x

rough number of possibilities: $144^{16} \approx 3.4\,10^{34}$.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	x	x	x	x	x	x	x	x	x	x	x
y_2	x	x	x	x	x	x	x	x	x	x	x
y_3	x	x	x	x	x	x	x	x	x	x	x
y_4	x	x	x	x	x	x	x	x	x	x	x
y_5	x	x	x	x	x	x	x	x	x	x	x
y_6	x	x	x	x	x	x	x	x	x	x	x
y_7	x	x	x	x	x	x	x	x	x	x	x
y_8	x	x	x	x	x	x	x	x	x	x	x
y_9	x	x	x	x	x	x	x	x	x	x	x
y_{10}	x	x	x	x	x	x	x	x	x	x	x
y_{11}	x	x	x	x	x	x	x	x	x	x	x

number of possibilities: $1\,473\,501\,105$.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	x	x	x	x	x	x	x	x	x	x	x
y_2	x	x	x	x	x	x	x	x	x	x	x
y_3	x	x	x	x	x	x	x	x	x	x	x
y_4	x	x	x	x	x	x	x	x	x	x	x
y_5	x	x	x	x	x	x	x	x	x	x	x
y_6	x	x	x	x	x	x	x	x	x	x	x
y_7	x	x	x	x	x	x	x	x	x	x	x
y_8	x	x	x	x	x	x	x	x	x	x	x
y_9	x	x	x	x	x	x	x	x	x	x	x
y_{10}	x	x	x	x	x	x	x	x	x	x	x
y_{11}	x	x	x	x	x	x	x	x	x	x	x

number of possibilities: $24\,473\,864\,360$.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	x	x	x	x	x	x	x	x	x	x	x
y_2	x	x	x	x	x	x	x	x	x	x	x
y_3	x	x	x	x	x	x	x	x	x	x	x
y_4	x	x	x	x	x	x	x	x	x	x	x
y_5	x	x	x	x	x	x	x	x	x	x	x
y_6	x	x	x	x	x	x	x	x	x	x	x
y_7	x	x	x	x	x	x	x	x	x	x	x
y_8	x	x	x	x	x	x	x	x	x	x	x
y_9	x	x	x	x	x	x	x	x	x	x	x
y_{10}	x	x	x	x	x	x	x	x	x	x	x
y_{11}	x	x	x	x	x	x	x	x	x	x	x

number of possibilities: 121 404 978.

Searching for a Franklin magic square of order 12

1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
y_1	x	x	x	x	x	x	x	x	x	x	x
y_2	x	x	x	x	x	x	x	x	x	x	x
y_3	x	x	x	x	x	x	x	x	x	x	x
y_4	x	x	x	x	x	x	x	x	x	x	x
y_5	x	x	x	x	x	x	x	x	x	x	x
y_6	x	x	x	x	x	x	x	x	x	x	x
y_7	x	x	x	x	x	x	x	x	x	x	x
y_8	x	x	x	x	x	x	x	x	x	x	x
y_9	x	x	x	x	x	x	x	x	x	x	x
y_{10}	x	x	x	x	x	x	x	x	x	x	x
y_{11}	x	x	x	x	x	x	x	x	x	x	x

number of possibilities: 121 404 978, NONE of which extend to 12 by 12!
165 hours of computing time (3.5 hours using 50 computers in parallel).