Exam Sequencing and scheduling, 2P450,
April 8, 2011, 14:00-17:00

This test contains three questions on two pages, with 10 sub-items, for which a total of 50 points can be scored. Each good answer scores 5 points. The final grade is obtained by dividing the amount of points by five. Questions may be answered in English and/or in Dutch. Please explain your answers. You are NOT allowed to use the handouts distributed in class/or on the web.

Question 1. Consider the problem (P): $O3||C_{\max}$ of minimizing maximum completion time on a set of three identical machines. In particular consider the instance I given by

jobs j	1	2	3	4
p_{1j}	6	8	9	2
p_{2j}	2	5	2	7
p_{3j}	8	4	4	7

We further consider the problem (Q): $O3|\text{pmtn}|C_{\max}$ of minimizing maximum completion time on a set of three identical machines, where we allow jobs to be preempted.

Let $P(I')$ and $Q(I')$ denote the optimal solution values of the two problems, defined on the same instance I'.

(a) How do the values $P(I')$ and $Q(I')$ relate, for general instances I'? Prove your claim.

(b) For the particular instance I defined above, give an non-trivial lower bound on the solution value $Q(I)$.

(c) Does there exist a solution of Q with a matching value?

(d) Give a general non-trivial lower bound $LB(I')$ for $Q(I')$, that is, for arbitrary instances I'. Does there always exist a solution to P with $P(I') = LB(I')$? If yes, prove; if no, give a counterexample.

Question 2. Consider the problem of minimizing in a system of a single machine, and a series of jobs with due dates $d_j > 0$, the number of late jobs.

(a) Describe the general strategy to find the maximum amount of jobs one can process on time.

(b) Consider the following instance of $1||\sum_j U_j$:

job j	1	2	3	4	5	6	7	8
p_j	6	6	5	4	4	8	7	5
d_j	26	30	32	34	19	13	25	24

(c) Consider the same instance of jobs as in (b), but with the additional property that job j has a *release date* $r_j = d_j - p_j$. Now we are interested in scheduling ALL jobs, with a schedule minimizing maximum lateness. Provide a good schedule and argue why it is (as good as) optimal.

Question 3. Consider a problem with m uniform machines, with speeds $s_1 \geq s_2 \geq \ldots \geq s_m > 0$. We have n tasks that may be preempted. The jobs have processing requirements p_j with $p_1 \geq p_2 \geq \ldots \geq p_n > 0$, for $j = 1, \ldots, n$. That means that if a machine i is to process the whole job j, it will take $\frac{p_j}{s_i}$ time units. We are interested in minimizing the makespan.

(a) Argue that if a schedule of length ℓ exists, then, for any integer k, $0 < k < \min\{m, n\}$, there is also a schedule of length ℓ or less for the instance I_k, defined by considering only the k first machines, and the k first tasks. Use the result on I_k to express a lower bound on $C_{\max}$.

(b) Argue that if a schedule of length ℓ exists, then

$$\ell \geq \frac{\sum_{j=1}^{n} p_j}{\sum_{i=1}^{m} s_i}$$

(c) If instead we would like to minimize sum of completion times, and we do not allow preemption, how do we find the optimal schedule?