**Exam Sequencing and scheduling, 2P450,**
**April 13, 2012, 14:00-17:00**

This test contains three questions on two pages, with 10 sub-items, for which a total of 50 points can be scored. Each good answer scores 5 points. The final grade is obtained by dividing the amount of points by five. Questions may be answered in English and/or in Dutch. Please explain your answers. You are NOT allowed to use the handouts distributed in class/or on the web.

**Question 1.** Consider the problem ($P_m$): $Jm||C_{\max}$ of minimizing maximum completion time on a set of $m$ machines, in a job shop context. Further, we also consider problem ($Q_m$): $Fm||C_{\max}$ of minimizing maximum completion time on a set of $m$ machines, in a flow shop context.

(a) Show that one problem class is a special case of the other. Further show that for $Qm$ we have to distinguish between *permutation* schedules and *unrestricted* schedules by providing an instance for which the best *unrestricted* schedule is strictly better than the best *permutation* schedule. For which $m$ is there no distinction?

(b) Show how to solve (in general terms, not by example) problem $Q_2$.

(c) Consider an arbitrary instance $I$ of $P_2$. So we have say $n$ jobs, each consisting of one or two operations, to be processed on one or two different machines, in prefixed order. Describe how to compute an optimal schedule in a number of steps that is almost linear in $n$. Prove that your approach yields a minimum length schedule.

**Question 2.** Consider the problem of minimizing the number of late jobs in a setting with a single machine, and a series of $n$ jobs with due dates $d_j > 0$, for $j = 1, \ldots, n$.

(a) Show how to solve this problem. What is the running time of your algorithm?

(b) Consider the following instance of $1||\sum_j U_j$:

| job $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $p_j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $d_j$ | 23 | 5 | 20 | 8 | 19 | 11 | 21 | 14 | 22 | 17 |

Give an optimal schedule found by application of the algorithm..

(c) Why is in the three-field notation for this problem the due date parameter $d_j$ not mentioned in the second field?

**Question 3.** Consider the problem of minimizing the maximum lateness of jobs scheduled on a single machine. Job $j$ is available from $r_j$ onwards, and has due date $d_j$. Consider the following instance of $1|r_j|L_{\max}$:

| job $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p_j$ | 2 | 1 | 1 | 3 | 2 | 3 | 1 | 2 | 2 | 4 | 2 | 1 |
| $r_j$ | 0 | 1 | 12 | 7 | 3 | 9 | 0 | 7 | 12 | 9 | 6 | 17 |
| $d_j$ | 6 | 2 | 11 | 20 | 20 | 16 | 10 | 8 | 10 | 12 | 11 | 22 |

(a) Solve the above problem, *allowing for preemption.* Give the answer by providing a preemptive schedule, and completing the following table (please mark preempted jobs):

| job $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $d_j$ | 6 | 2 | 11 | 20 | 20 | 16 | 10 | 8 | 10 | 12 | 11 | 22 |
| $C_j$ | | | | | | | | | | | | |
| $L_j$ | | | | | | | | | | | | |

(b) Derive from your schedule the set $S \subseteq J$, for which the appropriate lower bound on optimal value $L_{\max}^{\text{OPT}}$ equals the realized maximum lateness. Describe the lower bound in its general form.

(c) In an *on-line* context, the jobs would have been revealed to us in order and at the time of their due dates. In order to make a reasonable non-preemptive schedule, without knowing the future, we might have chosen to form a so-called $\alpha$-schedule: jobs are scheduled tentatively in a preemptive way; once their $\alpha$-point has passed, they are really available for processing.

Give the definition of the alpha-point $T_j^{\alpha}$ in general, and compute for the given preemptive schedule for each job its $\alpha$-point, for $\alpha = 0.5$.

Describe the result by completing the following table

| job $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T_j^{\alpha}$ | | | | | | | | | | | | |

(d) Give the resulting non-preemptive schedule based on postponing the release of a job to its alpha-point, for $\alpha = 0.5$. Describe the answer by completing the following table:

| job $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $d_j$ | 6 | 2 | 11 | 20 | 20 | 16 | 10 | 8 | 10 | 12 | 11 | 22 |
| $C_j$ | | | | | | | | | | | | |
| $L_j$ | | | | | | | | | | | | |