Technische Universiteit
Eindhoven
University of Technology

Examination cover sheet

(to be completed by the examiner)

Course name: **sequencing and scheduling** Course code: **2P450**

Date: **January 23, 2015**

Start time: **13:30** End time: **16:30**

Number of pages: **2**

Number of questions: **4** (10 sub-questions)

Maximum number of points/distribution of points over questions: **5 points per sub-question**

Method of determining final grade: **total divided by 5**

Answering style: **formulation**, order, **foundation of arguments**, multiple choice:

Exam inspection: **by appointment**

Other remarks:

Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- □ Notebook
- ✓ Calculator
- □ Graphic calculator
- □ Lecture notes/book
- □ One A4 sheet of annotations
- □ Dictionar(y)(ies). If yes, please specify:

Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will in any case be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

Exam Sequencing and scheduling, 2P450,
January 23, 2015, 13:30-16:30

This test contains four questions on two pages, with 10 sub-items, for which a total of 50 points can be scored. Each good answer scores 5 points. The final grade is obtained by dividing the amount of points by five. Questions may be answered in English and/or in Dutch. Please explain your answers. You are NOT allowed to use the handouts distributed in class/or on the web.

Question 1. Consider the problem of minimizing makespan on m parallel identical machines $\mathrm{P}m||C_{\max}$.

(a) Describe the LPT-rule and state its worst-case ratio α (in terms of m).

(b) Sketch the proof that for any instance I the LPT-schedule has a makespan $C_{\max}^{\mathrm{LPT}}(I)$ at most $\alpha C_{\max}^{\mathrm{OPT}}(I)$.

(c) Provide for each m an instance I_m for which $C_{\max}^{\mathrm{LPT}}(I_m)$ **equals** $\alpha C_{\max}^{\mathrm{OPT}}(I_m)$.

Question 2. Consider the problem $1|r_j|\hat{L}_{\max}$ of minimizing maximum *delivery time* of jobs scheduled on a single machine under release dates (also known as *head-body-tail*-problem). Here the delivery time is given als $\hat{L}_j := C_j + q_j$. In particular consider the instance given by

jobs j	1	2	3	4	5	6
p_j	8	3	4	3	4	4
r_j	0	6	16	19	12	10
q_j	1	2	3	4	5	6

(a) Formulate and apply an appropriate rule ρ for solving the PREEMPTIVE case. Compute the optimal *preemptive* schedule for the instance given above, and list the results in a table with the following entries:

jobs j	1	2	3	4	5	6
C_j^ρ						
$\hat{L}_j^\rho$						

(b) Verify optimality of the preemptive schedule by providing a *compact proof of optimality* in the form of a subset $S \subseteq \mathcal{J}$, for which $\min_{j\in S} r_j + p(S) + \min_{j\in S} q_j = \hat{L}_{\max}^\rho$. Explain how you found this set S.

Question 3. Consider the problem $Q3||\sum_j C_j$ of minimizing total completion time, on three parallel machines with distinct speeds $\sigma_1, \sigma_2, \sigma_3$.

(a) Describe for the general case, how to find the optimal solution.

(b) In particular we may consider the instance given by the following parameters

jobs j	1	2	3	4	5	6	7
p_j	6	8	5	7	9	4	10

machines i	1	2	3
σ_i	2	3	4

Apply the method described in (a) to find the optimal schedule for this instance.

(c) If machine i ($i = 1, 2, 3$) is **not able** to handle jobs j with $j = 0$ modulo $i + 1$ (that is, if $i + 1$ divides j), what would then be an appropriate rule or method for the general case?

Question 4. Consider the problem of scheduling n jobs on m machines, where each job consists of m subtasks, and where for each job j its i-th sub-task has to be processed on machine i. Here the processing time required is p_{ij}. We consider as objective function $\sum_j C_{mj}$, where C_{mj} denotes the completion time of the last subtask of job j (hence on machine m).

(a) Prove or disprove that there always exists an optimal schedule in which the jobs are processed on machine 2 in the same order as on machine 1.

(b) Prove or disprove that there always exists an optimal schedule in which the jobs are processed on machine m in the same order as on machine $m - 1$.