Exam Sequencing and scheduling, 2P450,
January 12, 2009, 14:00-17:00

This test contains four questions on two pages, with 10 sub-items, for which a total of 50 points can be scored. Each good answer scores 5 points The final grade is obtained by dividing the amount of points by five. Questions may be answered in English and/or in Dutch. Please explain your answers. You are NOT allowed to use the handouts distributed in class.

Question 1. Consider the following flowshop-like problem: we have m machines, and n jobs that need to be processed. Job j consists of subtasks T_{ij}, for $i = 1, \ldots, m$, needing processing on machine M_i for a period of time p_{ij}. The flowshop characteristic entails that T_{ij} must be finished before $T_{(i+1)j}$ can be processed. A machine can handle one job at a time, and tasks cannot be preempted. The objective is to minimize the *average* job-completion $\frac{1}{n} \sum_{j=1}^{n} C_{mj}$.

(a) Prove that when looking for an optimal schedule, we may restrict ourselves to schedules in which the job order on the first machine is equal to the order on the second machine;

(b) Prove — or — disprove by an counterexample, that when looking for an optimal schedule, we may restrict ourselves to schedules in which the job order on the last machine is equal to the order on the one but last machine;

Question 2. We consider the job shop problem represented by the disjunctive graph representation, in which nodes represent tasks, conjunctive arcs (arrows) represent job-precedences, and disjunctive edges (lines without direction) represent common ressource use. With each node we associate its processing time. A schedule is then represented by giving a direction to the disjunctive edges in such a way that the resulting graph does not contain any directed cycle (loop). The objective is to minimize $C_{\max}$.

(a) Explain how to efficiently compute the earliest starting time for each task.

(b) Explain how to find the maximum length path (where the length of a path is the sum of the values associated with the nodes) is found, and how the lebgth of thei *critical path* relates to the value of the solution.

(c) When trying to find a better schedule, it may be a good thing to change the direction of a single disjunctive edge. On the other hand, just swapping the direction of such edge may result in a graph in which you have a directed cycle. Prove that swapping the direction of an edge that lies on the critical path cannot lead to a graph in which you have a directed cycle.

Question 3. Consider the following instance of the problem of minimizing the number of tardy jobs: We have a single machine, and 5 jobs, with processing times and due dates given in the following table

j	1	2	3	4	5
p_j	7	8	4	6	6
d_j	9	17	18	19	21

(a) Describe how to find, in general, for this type of problem a schedule in which the number of tardy jobs is minimized; what is the running time of your algorithm?; find using this method an optimal schedule.

(b) Now we add a little flavor to the problem by giving our jobs a *weight* w_j. Now our objective is to minimize the *weighted number of tardy jobs.* Describe an approach based on *dynamic programming* to solve this problem. Again, how fast is your algorithm? Solve the problem for the given instance if the above mentioned jobs have weights:

j	1	2	3	4	5
w_j	5	3	4	2	3

Question 4. We consider a scheduling problem in which jobs arrive over time and we want to minimize lateness. The data of our instance are as follows

j	1	2	3	4	5
r_j	0	2	5	7	8
p_j	5	5	4	3	3
q_j	3	5	9	10	11

(a) Find an optimal schedule in case we allow preemption. Explain your method in general and apply it to the instance.

(b) Next we consider this as an *on-line* problem where indeed jobs arrive over time. We will use the outcome of the preemptive schedule as a heuristic for the non-preemptive case: compute for each job its α-point in the preemptive schedule, for $\alpha = 0.5$. For the *real* solution, pretend that a job is released at its α point and is processed as early as possible. What is the outcome for this instance? [here the α point is the first point in time in which a job has been processed for a fraction α.

(c) Provide an unpperbound on the worst case ration for this heuristic in terms of α.