TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Exam (Theoretical Part) Proving with Computer Assistance (2IF44) on Tuesday May 11, 2004, 09.00 – 11.00 hrs.
Note: This is an 'open book examination', so you may use as information all written material that you bring with you.
The answers to the questions should be written down clearly formulated and in a well-organized manner.

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1. This question is about untyped λ-calculus.

(4) (a) Show that the λ-terms $(\lambda x\,.\,xx)y$ and $(\lambda xy\,.\,yx)xx$ are *not* β-convertible.

(6) (b) We define $Y := \lambda p\,.\,(\lambda x\,.\,p(xx))(\lambda x\,.\,p(xx))$. Show that, for arbitrary λ-term t:

$$t(Yt) =_\beta Yt\ .$$

(10) 2. This question is about $\lambda\!\rightarrow$–Church.

Find an inhabitant of the type $(\sigma \to \tau) \to (\varphi \to \sigma) \to \tau$ in the following context:

$$\Gamma \equiv x : (\varphi \to \tau) \to \tau\ ,$$

and give the corresponding derivation.

You may do this in flag notation, but give clear indications where the *axiom*-rule has been used.

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(10) 3. This question is about $\lambda 2$.

Show that the following term is typable in context $\Gamma \equiv \text{nat} : *$.

$$(\lambda\alpha : * \,.\, \lambda\beta : * \,.\, \lambda x : \alpha \,.\, \lambda y : \beta \,.\, \lambda f : \alpha \to (\beta \to \alpha) \,.\, fxy)\ \text{nat}\ \text{nat} \quad .$$

So give a derivation which calculates the type of this term. Mention the derivation rules that you use.

Note: You may employ the shortened flag notation, so it is allowed to

(1) denote the context with flags,

(2) omit applications of the ax-rule (also called start rule).

(10) 4. This question is about λC.

Give a derivation in flag notation that can act as a proof of the logical proposition

$\forall_{x \in S}(\neg P(x) \Rightarrow (P(x) \Rightarrow (Q(x) \wedge R(x))))$,

for all sets S and predicates P, Q and R on S.

Hint: Use the definition of $\bot$.

Note: (1) You may combine *start* and *weakening* if necessary.

(2) You may ignore the (s_1, s_2)-rule (also called: form rule), that is to say: you may assume that any Π-type that you encounter is already well-typed itself.

(3) You need not mention which derivation rules you use.

The numbers between brackets in front of the questions denote the maximal number of points you may obtain for that (part of) question. Your final mark is established by adding the points you scored, dividing this sum by four and rounding it off to a whole number.