TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Examination (Theoretical Part) Bewijzen met Computerondersteuning (2R844) on Monday May 12, 2003, 14.00 – 16.00 hrs.
Note: This is an 'open book examination', so you may use as information all written material that you bring with you.
The answers to the questions should be written down clearly formulated and in a well-organized manner.

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1. This question is about untyped λ-calculus.

   We define λ-terms *zero*, *one*, *two* and *plus* by:

   $zero := \lambda xy \,.\, x$,

   $one := \lambda xy \,.\, xy$,

   $two := \lambda xy \,.\, x(xy)$,

   $plus := \lambda mnuv \,.\, mu(nuv)$.

(5) (a) Show that $plus\ one\ one =_\beta two$.

(5) (b) Prove that it does *not* hold that: $plus\ one\ one =_\beta plus\ zero\ zero$.

(10) 2. This question is about $\lambda\!\rightarrow$–Church.

   Find an inhabitant of the type $\sigma \rightarrow ((\sigma \rightarrow \tau) \rightarrow \varphi)$ in the context $\Gamma \equiv x : \sigma \rightarrow (\tau \rightarrow (\sigma \rightarrow \varphi))$ and give the corresponding derivation (you may do this in flag notation, but give clear indications where the *axiom*-rule has been used).

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(10) 3. This question is about $\lambda 2$.

Show that the following term is typable in context $\Gamma \equiv \text{nat} : *$.

$$(\lambda\alpha : * . \lambda\beta : * . \lambda f : \alpha \to \alpha . \lambda g : \alpha \to \beta . \lambda x : \alpha . g(f(fx))) \ \text{nat} \quad .$$

So give a derivation which calculates the type of this term. Mention the derivation rules that you use.

Note: You may employ the shortened flag notation, so it is allowed to:

(1) denote the context with flags,

(2) omit all applications of the ax-rule (also called: start rule).

(10) 4. This question is about λC.

Give a derivation in flag notation that can act as a proof of the logical proposition $\forall_{x\in S}(P(x)) \Rightarrow \forall_{y\in S}(P(y) \vee Q(y))$.

(Use the second-order definition of $\vee$.)

Note: (1) You may combine *start* and *weakening* if necessary.

(2) You may ignore the (s_1, s_2)-rule (also called: form rule), that is to say: you may assume that any Π-type that you encounter is already well-typed itself.

The numbers between brackets in front of the questions denote the maximal number of points you may obtain for that (part of) question. Your final mark is established by adding the points you scored, dividing this sum by four and rounding it off to a nearby whole number.