TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Exam (Theoretical Part) Proving with Computer Assistance (2IF44) on Wednesday March 16, 2005, 9.00 – 12.00 hrs.
Note: This is an 'open book examination', so you may use as information all written material that you bring with you.
The answers to the questions should be written down clearly formulated and in a well-organized manner.

---

1. This question is about untyped λ-calculus.

(5) (a) We define the λ-terms K and S by:
$K \;:=\; \lambda xy\,.\,x$, and
$S \;:=\; \lambda xyz\,.\,(xz)(yz)$.
Find a λ-term N in which neither K nor S occurs, and such that (1) N is in β-normal form and (2) $S(KK)Kxy \twoheadrightarrow_\beta N$.
Hint: When eliminating occurrences of K or S by using their definition, only do that for one at a time.

(5) (b) Let L and M be terms in untyped λ-calculus. Assume $L =_\beta M$ and assume that $L \twoheadrightarrow_\beta N$, with N in β-normal form. Prove that also $M \twoheadrightarrow_\beta N$. (You may make use of results stated in the handout, but if so, give a clear reference to the theorem, lemma etc. which is applicable.)

(10) 2. This question is about $\lambda{\rightarrow}$–Church.

Show that the following term is legal:
$\lambda x : (\sigma \to \tau) \to \tau \;\;.\;\; x(yz)$.

Give an appropriate context and the corresponding derivation. You may use the flag notation, but give clear indications where the *axiom*-rule has been used.

See page 2.

Exam (Theoretical Part) Proving with Computer Assistance (2IF44) on Wednesday March 16, 2005, 9.00 – 12.00 hrs.

---

(10) 3. This question is about $\lambda 2$.

Find, in the context $\Gamma \equiv \alpha : *,\ \beta : *,\ \gamma : *$, an inhabitant of the following type :

$$\Pi \delta : * \ .\ ((\alpha \to \gamma) \to \delta) \to ((\alpha \to \beta) \to ((\beta \to \gamma) \to \delta))\ .$$

So give a derivation which calculates an inhabitant of this type. Mention the derivation rules that you use.

Note: You may employ the shortened flag notation, so it is allowed to

(1) denote the context with flags,

(2) omit applications of the ax-rule (also called start rule).

(10) 4. This question is about λC.

Give a derivation in flag notation that can act as a proof of the logical proposition

$\exists_{x \in S}(P(x)) \ \Rightarrow\ \neg(\forall_{y \in S}(\neg P(y)))$,

for all sets S and predicates P on S.

Note: (1) Use the second order definition of $\exists$.

(2) You may combine *start* and *weakening* if necessary.

(3) You may ignore the (s_1, s_2)-rule (also called: form rule), that is to say: you may assume that any Π-type that you encounter is already well-typed itself.

(4) You need not mention which derivation rules you use.

The numbers between brackets in front of the questions denote the maximal number of points you may obtain for that (part of) question. Your final mark is established by adding the points you scored, dividing this sum by four and rounding it off to a whole number.