

TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Examination (Theoretical Part) Bewijzen met Computerondersteuning (2R844)
on Thursday March 20, 2003, 09.00 – 11.00 hrs.

Note: This is an 'open book examination', so you may use as information all written material that you bring with you.

The answers to the questions should be written down clearly formulated and in a well-organized manner.

1. This question is about untyped λ -calculus.

We define λ -terms K and S by:

$K := \lambda xy . x$, and

$S := \lambda xyz . xz(yz)$.

- (5) (a) Show that $SKKx =_{\beta} Kxy$.
- (5) (b) Prove that no term H can exist such that, for every P and Q :
 $HPQ \rightarrow_{\beta} K$ if $P =_{\beta} Q$ and
 $HPQ \rightarrow_{\beta} S$ if $\neg(P =_{\beta} Q)$.
(Hint: consider the term $(\lambda u . Huv)v$.)

(10) 2. This question is about $\lambda \rightarrow$ -Church.

Show that the following term is legal in the empty context:

$\lambda x : (\sigma \rightarrow \tau) \rightarrow \sigma . \lambda y : \sigma \rightarrow \tau . \lambda z : \varphi . y(xy)$

and give the corresponding derivation (you may do this in flag notation, but give clear indications where the *axiom*-rule has been used).

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- (10) 3. This question is about $\lambda 2$.

Find an inhabitant of the following type in the context $\Gamma \equiv nat : * .$

$$\Pi \alpha : * . \Pi \beta : * . (nat \rightarrow \alpha) \rightarrow ((\alpha \rightarrow (nat \rightarrow \beta)) \rightarrow (nat \rightarrow \beta)) .$$

So give a derivation which calculates an inhabitant of this type. Mention the derivation rules that you use.

Note: You may employ the shortened flag notation, so it is allowed to:

- (1) denote the context with flags,
- (2) omit all applications of the ax-rule (also called: start rule).

- (10) 4. This question is about λC .

Give a derivation that can act as a proof of the universal validity of the logical proposition $(\neg p \Rightarrow p) \Rightarrow p$.

Here we need *classical* logic, so start with the declaration:

$$c : (\Pi \gamma : * . ((\gamma \rightarrow \perp) \rightarrow \perp) \rightarrow \gamma)$$

which expresses the axiom that for all propositions q it holds that: $\neg \neg q \rightarrow q$.

Note: (1) You may combine *start* and *weakening* if necessary.

(2) You may ignore the (s_1, s_2) -rule (also called: form rule), that is to say: you may assume that any Π -type that you encounter is already well-typed itself.

The numbers between brackets in front of the questions denote the maximal number of points you may obtain for that (part of) question. Your final mark is established by adding the points you scored, dividing this sum by four and rounding it off to a nearby whole number.