TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Exam (Theoretical Part) Proving with Computer Assistance (2IF44) on Thursday April 28, 2005, 14.00 – 17.00 hrs.
*Note:* This is an 'open book examination', so you may use as information all written material that you bring with you.
The answers to the questions should be written down clearly formulated and in a well-organized manner.

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1. This question is about untyped $\lambda$-calculus.

(5) (a) We use the following abbreviations for untyped $\lambda$-terms:
$\texttt{true} := \lambda xy \,.\, x$,
$\texttt{false} := \lambda xy \,.\, y$, and
$F := \lambda xy \,.\, x\, y\, \texttt{false}$ .
Prove that the $\lambda$-terms $F\ \texttt{true}\ \texttt{true}$ and $F\ \texttt{true}\ \texttt{false}$ are *not* $\beta$-convertible.

(3) (b) Let $L$ be a $\lambda$-term in untyped $\lambda$-calculus which has a $\beta$-normal form. Prove that all $M$ such that $L \twoheadrightarrow_\beta M$ have a $\beta$-normal form, as well. (You may make use of results stated in the handout, but if so, give a clear reference to the theorem, lemma etc. which is applicable.)

(2) (c) Show that, when $L$ is a $\lambda$-term which has a $\beta$-normal form, it may nevertheless occur that there is an infinite reduction sequence starting from $L$.

(10) 2. This question is about $\lambda\!\rightarrow$–Church.

Find an inhabitant of the type $(\sigma \to \varphi) \to (\sigma \to \tau)$ in the context $\Gamma \equiv x : (\tau \to \varphi) \to \tau$.

Give the corresponding derivation. You may use the flag notation, but give clear indications where the *axiom*-rule has been used.

See page 2.

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3. This question is about $\lambda 2$. We define

   $t \;:=\; \lambda\alpha : * \,.\, \lambda\beta : * \,.\, \lambda f : \alpha \rightarrow (\alpha \rightarrow \beta) \,.\, \lambda x : \alpha \,.\, f x x \;.$

(6) (a) Show that $t$ is typable in the empty context.
So give a derivation which calculates the type of this term. Mention the derivation rules that you use.
*Note:* You may employ the shortened flag notation, so it is allowed to denote the context with flags, and to omit applications of the ax-rule (also called start rule).

(2) (b) Assume that $nat : *$, $int : *$, $3 : nat$. We define

$$F \;:=\; \lambda m : nat \,.\, \lambda n : nat \,.\, m - n^2 \;.$$

Give types for $t\ nat$, $t\ nat\ int$, $t\ nat\ int\ F$ and $t\ nat\ int\ F\ 3$. *(Only the answers.)*

(2) (c) Describe $t\ nat\ int\ F\ 3$ as simple as possible.

(10) 4. This question is about $\lambda C$.

Give a derivation in flag notation that can act as a proof of the logical proposition $\neg(p \wedge \neg p)$, for all propositions $p$.

*Note:* (1) Use the second order definition of $\wedge$.

(2) You may combine *start* and *weakening* if necessary.

(3) You may ignore the $(s_1, s_2)$-rule (also called: form rule), that is to say: you may assume that any $\Pi$-type that you encounter is already well-typed itself.

(4) You need not mention which derivation rules you use.

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The numbers between brackets in front of the questions denote the maximal number of points you may obtain for that (part of the) question. Your final mark is established by adding the points you scored, dividing this sum by four and rounding it off to a whole number.