This is a closed book exam. Answers may be given in English or Dutch. For each question the number of points is indicated in the margin. These points add up to 100. Your grade is the sum of the points you scored on all questions divided by 10 (not rounded off).

1. Give a DFA $D$ with alphabet \{a, b\} and no more than 4 states for the language $L = \{w \in \{a, b\}^* \mid w$ has substring $aa$ and ends in $b\}$. Prove that $\mathcal{L}(D) = L$ by providing the pathsets of the states of $D$.

2. The transition diagram of NFA $N$ is given by

(10) (a) Using the NFA-to-DFA algorithm transform $N$ into a DFA $D$ that accepts $\mathcal{L}(N)$. Provide both the transition table and the transition diagram of $D$. [hint: the resulting DFA has 4 states]

(10) (b) Using the NFA-to-RE algorithm based on GNFA’s construct a regular expression for $\mathcal{L}(N)$. Provide all intermediate GNFA’s and the constructed regular expression.

3. Given are an alphabet $\Sigma$, a symbol $c \in \Sigma$, and a regular language $L$ over $\Sigma$. Language $L'$ is defined by

\[
L' = \{ucv \mid uv \in L\}
\]

(strings in $L'$ are obtained by taking strings from $L$ and inserting the symbol $c$ in them at an arbitrary position). Let $(Q, \Sigma, \rightarrow, q_0, F)$ be an NFA with $\mathcal{L}(N) = L$. Show how to construct an NFA $N'$ such that $\mathcal{L}(N') = L'$ [hint: use 2 different copies of $N$]. Give some informal arguments why your construction is correct (no formal proof required).
4. Given is language \( L = \{a^{3n}b^{2n} \mid n \geq 0\} \).

(a) Construct a push-down automaton with no more than 5 states accepting \( L \) and give an invariant table for it.

(b) Give a context-free grammar \( G \) for \( L \) with only 1 variable and prove \( L(G) \subseteq L \).

5. Function \( f : \{a, b, c\}^* \to \{A, B\}^* \) is defined by

\[
\begin{align*}
f(\varepsilon) &= \varepsilon \\
f(aw) &= Af(w) \quad \text{for all } w \in \{a, b, c\}^* \\
f(bw) &= Bf(w) \quad \text{for all } w \in \{a, b, c\}^* \\
f(cw) &= f(w) \quad \text{for all } w \in \{a, b, c\}^*
\end{align*}
\]

(string \( f(w) \) is obtained from \( w \) by deleting all \( c \)'s, and changing all \( a \)'s into \( A \)'s and all \( b \)'s into \( B \)'s). Give a classical Turing machine with at most 6 states that computes \( f \) and show the computation of \( f(bcac) \) by this Turing machine.

6. Given are labeled transition systems \( S \) and \( T \):

\[
\begin{align*}
S : & \quad \begin{array}{c}
p_0 \\
p_1 \\
p_2 \\
p_3 \\
\end{array} \\
& \quad \begin{array}{c}
\tau \\
a \\
\tau \\
b \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
T : & \quad \begin{array}{c}
q_0 \\
q_1 \\
q_2 \\
q_3 \\
\end{array} \\
& \quad \begin{array}{c}
\tau \\
a \\
\tau \\
b \\
\end{array}
\end{align*}
\]

Compute a coloring scheme for the combination of these LTSs (your computation should be presented in the form of a table) and indicate whether and why \( S \) and \( T \) are branching bisimilar or not.