1. Write down the letter(s) of the regular expression(s) of which the language equals $\{a, b\}^*$
A. $(a + b)^*$
B. $(a \cdot b)^*$
C. $(a \cdot (b + 1))^*$
D. $((a + 1) \cdot (b + 1))^*$
E. $(a \cdot b^*)^*$
F. $(a^* \cdot b^*)^*$

2. Let $L = \{(ab)^k \mid k \geq 0\} \cup \{(ba)^\ell \mid \ell \geq 0\}$. To which language is $L^C$ equal
A. $\{(ab)^k a \mid k \geq 0\} \cup \{(ba)^\ell b \mid \ell \geq 0\}$
B. $\{w \in \{a, b\}^* \mid \#_a(w) \neq \#_b(w)\}$
C. $\{w \in \{a, b\}^* \mid (|w| \text{is odd}) \lor (w \text{ contains substring } aa \text{ or } bb)\}$
D. $\{w \in \{a, b\}^* \mid (|w| \text{is odd}) \lor \#_a(w) \neq \#_b(w)\}$

3. Construct a DFA $D$ with alphabet $\{a, b\}$ and no more than 4 states for the language $L = \{w \in \{a, b\}^* \mid w \text{ has substring } bba\}$. Prove that $L(D) = L$ by providing the pathsets of the states of $D$. 

(please turn over)
4. The transition diagram of NFA $N$ is given by

(a) Using the NFA-to-DFA algorithm transform $N$ into a DFA $D$ that accepts $\mathcal{L}(N)$. Provide both the transition table and the transition diagram of $D$. [hint: the resulting DFA has 3 states]

(b) Using the NFA-to-RE algorithm based on GNFA construct a regular expression for $\mathcal{L}(N)$. Provide all intermediate GNFAs.

5. Given are an alphabet $\Sigma$, a symbol $a \in \Sigma$, and a regular language $L$ over $\Sigma$. Language $L'$ is defined by

$$L' = \{ u \in \Sigma^* \mid ua \in L \}$$

Give an answer to one of the following two questions.

- Let $(Q, \Sigma, \delta, q_0, F)$ be a DFA with $\mathcal{L}(D) = L$. Show how to construct a DFA $D'$ using $D$ such that $\mathcal{L}(D') = L'$. (no proof required)

- Let $(Q, \Sigma, \rightarrow, q_0, F)$ be an NFA with $\mathcal{L}(N) = L$. Show how to construct an NFA $N'$ using $N$ such that $\mathcal{L}(N') = L'$. (no proof required)