Number Guessing with Lies

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The Game

1. Alice picks a number N in the range 0 through 15.
2. Bob asks a series of Yes/No questions.
3. Alice answers each question, and may lie once.
4. Bob then tells the number N and which answer was a lie (if any).

How can Bob do this?

Question Q_1

Is your number one of these?

	1		3	4		6		8		10			13		15

Question Q_2

Is your number one of these?

	1	2			5	6		8			11	12			15

Question Q_3

Is your number one of these?

								8	9	10	11	12	13	14	15

Question Q_4

Is your number one of these?

	1	2		4			7		9	10		12			15

Question Q_5

Is your number one of these?

				4	5	6	7					12	13	14	15

Question Q_6

Is your number one of these?

		2	3			6	7			10	11			14	15

Question Q_7

Is your number one of these?

	1		3		5		7		9		11		13		15

Figuring it out

- Let the answers be a_i ($0 =$ No; $1 =$ Yes) for $i = 1, \ldots, 7$
- Compute

$$\begin{aligned} p_1 &= a_1 + a_3 + a_5 + a_7 \pmod 2 \\ p_2 &= a_2 + a_3 + a_6 + a_7 \pmod 2 \\ p_3 &= a_4 + a_5 + a_6 + a_7 \pmod 2 \end{aligned}$$

- Compute $q = p_1 + 2p_2 + 4p_3$ (each p_i is 0 or 1)
- If $q = 0$, then there was no lie
- If $q \neq 0$, then answer a_q was a lie: flip a_q (replace it by $1 - a_q$)
- Alice' secret number was $N = 8a_3 + 4a_5 + 2a_6 + a_7$

How It Works

Q_3									8	9	10	11	12	13	14	15
Q_5					4	5	6	7					12	13	14	15
Q_6			2	3			6	7			10	11			14	15
Q_7		1		3		5		7		9		11		13		15
Q_1		1		3	4		6		8		10			13		15
Q_2		1	2			5	6		8			11	12			15
Q_4		1	2		4			7		9	10		12			15

Questions Q_3, Q_5, Q_6, and Q_7 do a *Binary Search*; works *without* lie.

The three other questions help detect a single lie:

$$\begin{array}{cccccccc}
\cdot & \boxed{Q_1} & \cdot & Q_3 & \cdot & Q_5 & \cdot & Q_7 \\
\cdot & \cdot & \boxed{Q_2} & Q_3 & \cdot & \cdot & Q_6 & Q_7 \\
\cdot & \cdot & \cdot & \cdot & \boxed{Q_4} & Q_5 & Q_6 & Q_7
\end{array}$$

There are $8 = 2^3$ possibilities: no lie, or 7 possible lies.

Error-Correcting Hamming(7,4) Code

Less efficient solution repeats questions Q_3, Q_5, Q_6, Q_7 three times.

We used a Hamming(7,4) code.

It has 4 data bits, 3 parity/check bits, and can correct one bit error.

The 4 data bits encode a value from 0 through 15.

Each question corresponds to the transmission of a bit.

A lie corresponds to a bit error.

Can be generalized to $2^k - k - 1$ data bits and k parity/check bits.

Variation (for kids): lie *every* time except once.