

# Algorithmic Adventures

From Knowledge to Magic



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Perfection is based upon small things,  
but perfection itself is no small thing at all.

Michelangelo Buonarroti

## Algorithmic Cooking

*An algorithm provides simple and unambiguous advice on how to proceed step by step in order to reach a specified goal.*

To what extent may one view a cooking recipe as an algorithm?

### Ingredients for apricot flan:

- 3 egg whites
- 1 pinch of salt
- 6 tablespoons of hot water
- 100 g cane sugar
- 3 egg yolks
- 1 teaspoon of lemon peel
- 150 g flour
- 1/2 teaspoon of baking powder
- 400 g peeled apricots

### Recipe for apricot flan:

1. Put greaseproof paper into a springform pan!
2. Heat the oven up to 180°C!
3. Heat up 6 tablespoons of water!
4. Mix three egg whites with the hot water and a pinch of salt, beat them until you get whipped egg white!

... ..

## Cooking Recipe

4.6 Mix the content of G for 2 minutes.

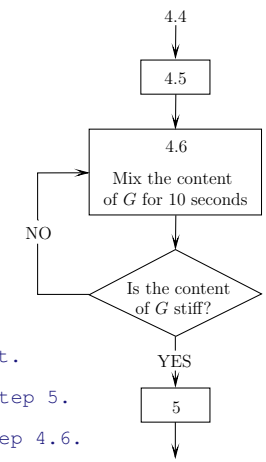
Refine this as:

4.6 Mix the content of G for 10 seconds.

4.7 Test whether the content of G is stiff or not.

If the answer is "YES", then continue with step 5.

If the answer is "NO", then continue with step 4.6.



Flowchart

## Computer Algorithms

- Fix a list of fundamental instructions (operations) that a computer can execute without any doubt.
- Possible (abstract) inputs: (infinitely many) **problem instances**
- Required output solving the problem

*An algorithm for solving a problem (a task) has to ensure that it works correctly for each possible problem instance.*

*To work correctly means that, for any input, it finishes its work in a finite time and produces the correct result.*

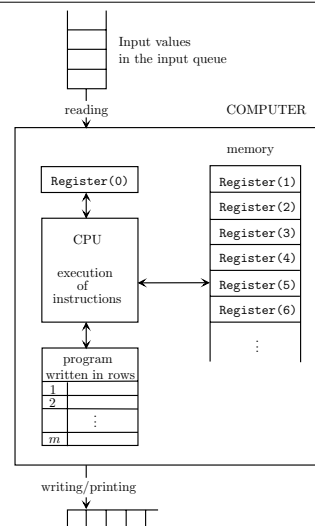
## Computer Programs and Programming

A **program** is a *sequence of computer instructions* that is represented in a form understandable by a computer.

1. A program does not need to be a representation of an algorithm. A program may be a meaningless sequence of computer instructions.
2. An algorithm need not be written in the form of a program. An algorithm can also be described in a natural language or in the language of mathematics. A program must be expressed in a special formalism of the given **programming language**.

**Programming** is the *activity of rewriting algorithms* into programs.

## Computer Model



## Computer Model (Terminology)

- A **memory** that consists of a large number of memory cells, called **registers**, numbered by positive integers, called **addresses** of the registers. Each register can save an arbitrarily large number.
- A special memory in which the whole program is saved. Each line of the program consists of exactly one instruction of the program. The lines are numbered starting at 1.
- There is a special register `Register(0)` that contains the number of the just executed instruction (line) of the program.
- A CPU (central processing unit) that is connected to all other parts of the computer, doing all the work.

## Programming Language 'TRANSPARENT', Part 1

- (1) Read into Register( $n$ ).
- (2) Register( $n$ )  $\leftarrow k$
- (3) Register( $n$ )  $\leftarrow$  Register( $j$ ) + Register( $i$ )
- (4) Register( $n$ )  $\leftarrow$  Register( $j$ ) - Register( $i$ )
- (5) Register( $n$ )  $\leftarrow$  Register( $j$ ) \* Register( $i$ )
- (6) Register( $n$ )  $\leftarrow$  Register( $j$ ) / Register( $i$ )
- (7) Register( $n$ )  $\leftarrow \sqrt{\text{Register}(m)}$

## Programming Language 'TRANSPARENT', Part 2

- (8) If Register( $n$ ) = 0, then go to row  $j$
- (9) If Register( $n$ )  $\leq$  Register( $m$ ), then go to row  $j$
- (10) Go to row  $j$
- (11) Output  $\leftarrow$  Register( $j$ )
- (12) Output  $\leftarrow$  "Text"
- (13) End.
- (14) Register(Register( $i$ ))  $\leftarrow$  Register( $j$ )

## Never-Ending Execution

One of our most important demands on the definition of an algorithm for a computing task is that the algorithm finishes its work for any input and provides a result.

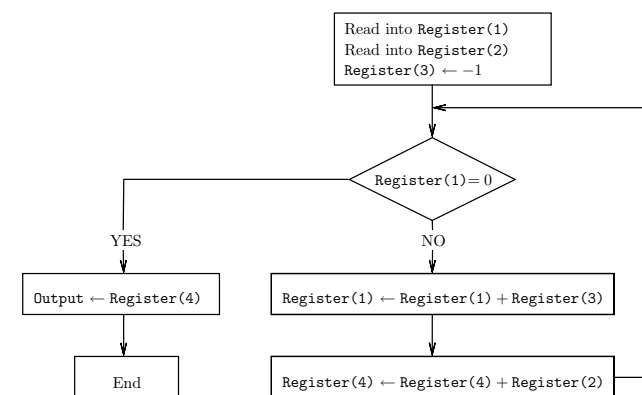
In the formal language of computer science, we speak about **halting**.

If an algorithm  $A$  finishes its work on an input (a problem instance) in a finite time, then we say that **algorithm  $A$  halts on  $x$** .

In this terminology, we force a halt of the algorithm on every possible input and in such a case we say that  $A$  **always halts**.

A program can engage in never-ending execution.

## Example



What does this program compute? What if Register(1) starts  $< 0$ ?

## Summary

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- One has to be able to *apply* an algorithm even if one is not an expert in solving the considered problem. One does not need to understand *why* the algorithm provides the solution of the problem. It suffices to be able to execute the simple activities the algorithm consists of.
- Defining the notion of an algorithm, one has to list all such *simple activities* and everybody has to agree that all these activities are executable by a machine.
- An algorithm is designed not only to solve a problem instance, but it must be applicable to solving *all* possible instances of a given problem.
- We require a guarantee that an algorithm for a problem successfully finds a solution for *each* problem instance.