The IMO: About Talent, Fun, and Math Circles

That the letters IMO stand for *International Mathematical Olympiad*, is nowadays well known among mathematicians in the Netherlands, host of IMO 2011. That this olympiad concerns more than just a mathematics competition is not so well known. In this article, Tom Verhoeff, treasurer of the Dutch Mathematical Olympiad and organizer at IMO 2011, will highlight another face of the IMO.

Some eight years ago, I described in [33, 34] the International Mathematical Olympiad (usually abbreviated as IMO) from the perspective of the jury consisting of all team leaders, who select and translate the problems, grade the submitted work, and establish the final results. That NAW article closed with the words: "I think it would be an excellent challenge to bring the IMO to the Netherlands. ... Who will help?" Well, it got that far. This summer, the Netherlands hosts IMO 2011 [3, 5], a mathematics competition without equal, with participants from more than one hundred countries. However, it would be shortsighted to view the IMO as just a (large and difficult) mathematics competition. So, what else is there?

Objectives and history

In the regulations of IMO 2011 [11], we find that *the aims of the IMO 2011 are:*

- *to discover, encourage and challenge mathematically gifted young people in all countries;*
- *to foster friendly international relationships among mathematicians of all countries;*
- *to create an opportunity for the exchange of information on school syllabuses and practices throughout the world;*
- *to promote mathematics generally.*

Although each issue of the IMO has its own regulations, these objectives exist already for many, many years. A competition is not mentioned. The competition is 'only' a means chosen to achieve an end. To better understand this choice for a competition, we need to explore the past.

The genesis of the IMO is described fragmentarily [2, 7, 12, 15, 16, 17, 24, 31], though it has received more attention since its 50-th anniversary [8]. The first IMO was held in 1959 in Romania, with participants from the seven Eastern-bloc countries. However, that event did not arise from a vacuum, but was the consequence of the prolonged attention for mathematics in that region.

Figure 1: Logo of IMO 1959 (left); logo of IMO 2011 (right)

Before the IMO

While the French baron Pierre de Coubertin was busily occupied trying to revive the Olympic Games—and probably inspired by his efforts—the first national mathematics competition ever was organized in 1894. This happened in Hungary. Note that the modern Olympic Games 'only' started in 1896.

The Hungarian physicist baron Loránd Eötvös became Minister of Religion and Education in 1894. To celebrate this memorable event, the Hungarian society for mathematics and physics started a national mathematics competition for recently graduated students from secondary schools. The emphasis was put on testing mathematical creativity and not mathematical knowledge. The competition was named *Eötvös Competition*, in 1938 renamed to the *Kürschák Mathematical Competition* so as to honor the contributions of József Kürschák. The participants get four hours to solve three problems. The first problem of the Eötvös Competition in 1894 [19] gives a good impression of the lower bound on the difficulty level:

> Prove that the expressions $2x + 3y$ and $9x + 5y$ are divisible by 17 for precisely the same set of integer pairs x and y.

The status of this competition is underscored by the fact that that the top ten would be admitted to university without taking the notoriously tough admittance exam.

In the year 1893, the first issue had appeared of the Hungarian mathematical and physical journal for secondary schools, usually alluded to by its shortened title *KöMaL* [18]. This monthly periodical consists for a large part of problems in various categories, and solutions by students, who are always proud to see their names in print. Hungarian mathematicians unanimously agree on the overriding importance of the Eötvös–Kürschák competition and the journal KöMaL for their personal development [9, 23].

Figure 2: Loránd Eötvös (left), József Kürschák (middle), Spiru Haret (right)

In Romania something similar happened [2, 12]. At the end of the nineteenth century, Romania had a mathematician, Spiru Haret, as Minister of Education. Haret reformed the (mathematics) education after the French model in 1897. In 1895, the youth journal *Gazeta Matematică* was launched, again full of problems. Not much later, also an associated competition was started. After the Second World War, far-reaching educational reforms took place, now after the Russian model, with mathematics as a central topic. The competition obtained government support, was integrated into the curriculum, and since then it was referred to as *National Mathematical Olympiad*. At the important international mathematical congress of 1956 in Romania the idea for an international mathematical olympiad was first proposed.

Also in other countries, in particular the Soviet Union [14], but also the United States of America [31], China and other Asian countries [36], mathematics competitions were established early on.

The IMO

Due to the long experience with mathematics competitions in Romania, the first IMO in 1959 was a well-organized event. At the second IMO, again in Romania, deputy leaders were added to the team, and the team leaders arrived a few days in advance to select and translate the problems. After the second IMO, only the number of contestants per country has varied a few times, from the initial eight to the current six. Besides such small changes, the IMO is still run as in the very beginning.

In the school year 1961/62, the Netherlands started a mathematical olympiad and the mathematical youth journal *Pythagoras*, which both celebrate their 50-th anniversary in 2011.

As an example, I mention a medium problem from the first IMO [10]:

> **Problem 2.** For which real values of x is
>
> $$\sqrt{(x+\sqrt{2x+1})} + \sqrt{(x-\sqrt{2x+1})} = A,$$
>
> given (a) $A = \sqrt{2}$, (b) $A = 1$, (c) $A = 2$, where only non-negative real numbers are admitted for square roots?

Compare this to a medium problem from IMO 2010, submitted by Hans Zantema from the Netherlands and discussed in the previous issue of NAW [37], which touches on modern research:

> **Problem 5.** In each of six boxes $B_1, B_2, B_3, B_4, B_5, B_6$ there is initially one coin. There are two types of operation allowed:
>
> **Type 1:** Choose a nonempty box B_j with $1 \le j \le 5$. Remove one coin from B_j and add two coins to B_{j+1}.
>
> **Type 2:** Choose a nonempty box B_k with $1 \le k \le 4$. Remove one coin from B_k and exchange the contents of (possibly empty) boxes B_{k+1} and B_{k+2}.
>
> Determine whether there is a finite sequence of such operations that results in boxes B_1, B_2, B_3, B_4, B_5 being empty and box B_6 containing exactly $2010^{2010^{2010}}$ coins. (Note that $a^{b^c} = a^{(b^c)}$.)

It is clear that the mathematical olympiad is not just a recreational event, but a systematic activity, carried by a broad vision, to stimulate and develop the interest in mathematics. This makes it hard to determine a proper position for the mathematical olympiad, because it has strong ties with education, research, *and* public relations. Kenderov, former president of the World Federation of National Mathematics Competitions (WFNMC [38]), explicitly touches on this problem in [16]. It also poses a challenge for the recently established Platform for Mathematics in the Netherlands [22].

Talent and fun

Back to the objectives of the IMO. How do you discover mathematical talent? And how can you stimulate and develop that talent? Fortunately, there is more attention for these concerns nowadays, as witnessed by publications like [1, 4]. (The previous issue of NAW carries a review of the first edition of [1]; the second edition appeared shortly after the review was written.) One of the misunderstandings that is addressed there is that talent will flourish anyway, even without extra attention. This is reinforced by another misunderstanding, namely that talent is purely innate. You've got it, or not. And if you've got it, then everything will turn out fine by itself. Research [27] has revealed that talent is a much broader phenomenon, and that for a significant part it is also influenced by upbringing, from very young on. Having talent for X not necessarily means that someone is very skilled at X from the very start, but it can also mean that someone is highly interested in X and motivated to spend time on X.

The regular school curriculum is not specifically aimed at discovering talent. Good pupils without talent can obtain high grades (by diligent memorization and practicing), and talented but bored pupils sometimes get low grades. Furthermore, talent for mathematics does not necessarily coincide with exceptional performance in other subjects. Children with talent for mathematics may not stand out in everyday school life, and hence they often do not qualify for enrichment classes or similar activities. Finally, general enrichment classes do not develop (mathematical) talent; they just keep pupils busy.

In music and sports, and also for languages, it has long been recognized that you need to start young, very young indeed, and usually this does not involve school. In these areas, there are many possibilities for extra attention. Talent in X: one is not (only) born with it, but to a significant extent it is also acquired and imbued. That process includes developing an interest in X, which in turn requires that one can have fun doing X. Fun and talent pose a chicken-egg paradox. To become skilled in something, it helps if you enjoy doing it, and to enjoy doing something, it helps if you already have some ability. Fun and talent best evolve together, in small steps.

Figure 3: Logo of the Dutch Kangaroo contest (left), Junior Math Olympiad (middle), Dutch Mathematical Olympiad (right)

To discover mathematical talent one needs to look at other things than the grade for the school subject mathematics. The Kangaroo mathematics contest and the Dutch Mathematical Olympiad (including the Junior Math Olympiad) try to help discover talent for mathematics at an early age. They do so by offering attractive and challenging mathematical problems that require limited prior knowledge but for which you do need to think creatively. It is inexcusable that there still exist schools that deny their pupils the opportunity to participate. It appears to be notoriously hard to predict who has (mathematical) talent. Children themselves, their parents, and their teachers, often do not know, but participation in such a contest can provide a—sometimes surprising—indication.

Once talent for mathematics has been discovered (which could also mean that great interest or motivation has been aroused, without an ability to perform well), it is important to develop that talent. Someone with talent learns more quickly, and to develop talent further, material needs to be offered more rapidly than in the regular school pace. The Dutch

Mathematical Olympiad provides material and training, on a larger scale than in the past, both centrally and via ten regional support centers at universities [21]. But also the summer camps of *Vierkant voor Wiskunde* [35] (Square for Mathematics) and math clubs, within or outside school, can contribute.

In this context, it is worthwhile to note that the Dutch participants at the IMOs before 1977 were not trained and mostly finished at or near the bottom. Therefore, in 1976, Jan van de Craats began systematic training activities. Not only did the results at the IMO improve drastically, the students clearly showed that they had much fun while training [31, p.807]. Complete statistics can be found on [10].

The IMO itself also pays attention to the element of fun. Besides enjoyable math, there are social-cultural excursions and there is a recreational program. This, in turn, contributes to the objective of fostering friendly international relationships.

Mathematical circles

One of my math professors described the mathematical enterprise as a pyramid, whose top is populated with proven productive talent, and whose base consists of hard workers with interest and developing talent. As in other fields, such as music and sports, the base needs to be fed constantly to push the top higher and higher. The metaphor of an ancient Egyptian pyramid is misleading, because there every layer has a volume that is proportional to the square of the distance to the top. In social pyramids, on the other hand, the size of the layers grows exponentially with the distance from the top. Or, to turn it around, the height of a

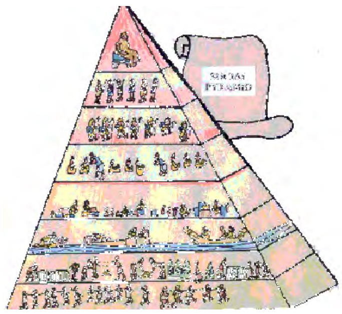

Figure 4: A misleading picture of a social pyramid

social pyramid is the logarithm of the size of the base. You need a considerable base to reach some height. In that respect, mathematics can still learn from music and sports, where the base is very large and consequently the top higher. Not everyone can reach the top of the pyramid, but also at all other levels you can (learn to) have fun.

The IMO is not the solution for everything, but in many countries it does set a good example and shows how high the top can be. Recently, this has been explained again in [26]. The IMO has brought about many national and regional competitions, including for instance since 2009 the Benelux Mathematical Olympiad among Belgium, the Netherlands, and Luxembourg. Also, the IMO puts the topic of math talent firmly on the agenda. In that way, the IMO is indeed much more than a mathematics competition, namely a vehicle "to discover, encourage, and challenge mathematically gifted young people in all countries" and "to promote mathematics generally".

A little warning is in place. Not everyone with a talent for mathematics likes to compete and is encouraged by a contest [25, 29]. If the level of the competition is not properly adjusted to the participants, then it can be counterproductive. The IMO attempts to mitigate this by awarding bronze, silver, and gold medals to roughly half of the contestants. That protocol also emphasizes that it is not in the first place a competition against other persons, but rather against the problem set. In a similar vein, the IMO is not set up as a competition between countries, although unofficially a country ranking is usually calculated.

Besides competitions and journals for secondary schools, a third pillar evolved to care for talent in Central and Eastern Europe, especially in the 1960s: math clubs, also known as mathematical circles. Such a mathematical circle can operate in diverse ways, but in general it revolves around regular, non-competitive sessions to have fun with mathematics. Over the years, a lot has been written about them [6, 13, 28, 30, 32]. In the US, the many mathematical circles are nowadays organized in the *National Association of Math Circles* (NAMC [20]).

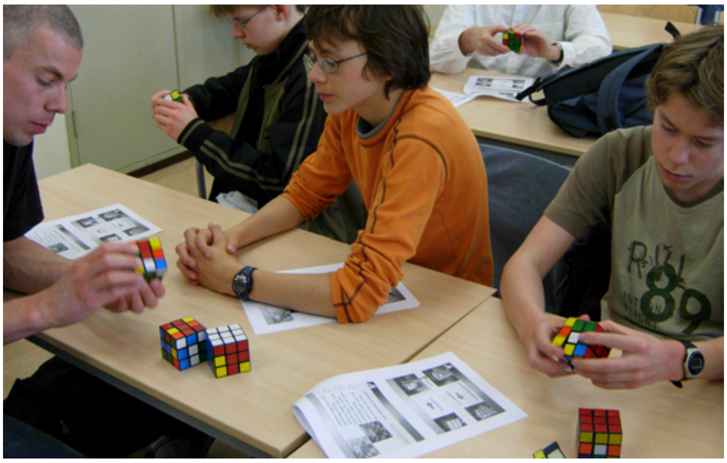

Figure 5: Mathematical circle session on algebra at the Free University in Amsterdam

Soon, IMO 2011 in the Netherlands will be over, but the need to discover and stimulate math talent and to promote joy in mathematics remains. It would be wonderful when, in the wake of IMO 2011, also here mathematical circles would come to bloom at schools and universities, to develop math talent in a friendly atmosphere. At this moment, only the Free University of Amsterdam has a math circle [39]. The regional training activities for the Dutch Mathematical Olympiad are organized and attended with enthusiasm, and they provide a solid starting point. Will you help?

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