Errata and Addenda for "On Abstraction and Informatics" [2]

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p.7, below (5) The definition of morphism composition $\circ$ should read

$$(f \circ g)(x) \quad = \quad f(g(x))$$

Note that the definition as was given (before I corrected it), is indeed used by some mathematicians. Also see [1]. Traditional notation for functions is inherently confusing, because the implicit "directions" in the signature descriptor $f : A \to B$ (left to right) and function application $f(x)$ (right to left) are in conflict.

For signature composition, it makes sense to write the composition of $f : A \to B$ with $g : B \to C$ as $f \circ g : A \to C$, preserving the order ($A \xrightarrow{f} B \xrightarrow{g} C$; and taking for granted that then $(f \circ g)(x) = g(f(x))$, where the order switches).

For function application, it makes sense to write the composition of these same f and g as $(g \circ f)(x) = g(f(x))$, maintaining the order of the functions on the left and right, but their signatures do "match" in the opposite order ($A \xrightarrow{f} B \xrightarrow{g} C$).

All this can easily be resolved by writing function application in the reverse order: $x.f = f(x)$. In that case, we could define $x.(f \circ g) = (x.f).g$, and everything would be "in order". It seems less "natural" to reverse the order in the signature, and write $f : B \leftarrow A$

References

[1] Functional Composition. in *Wikipedia*. en.wikipedia.org/wiki/Function_composition (accessed 03-Oct-2011).

[2] T. Verhoeff. “On Abstraction and Informatics”, Proceedings of ISSEP 2011, Bratislava, Slovakia (to appear).