The 43rd International Mathematical Olympiad: A Reflective Report on IMO 2002

Tom Verhoeff
Faculty of Mathematics and Computing Science
Eindhoven University of Technology
T.Verhoeff@TUE.NL

August 2002

Abstract

This report describes my experiences as an observer at the 43rd International Mathematical Olympiad, accompanying the Dutch team leader. It gives a chronological overview of the IMO 2002 events, interspersed with personal musings. It is addressed at those interested in the IMO, but should also prove useful to members of other olympiad communities.

My main motivation for writing this report is that I have never encountered a publication that presents the many details that make up an IMO. There exist various published collections of IMO problems and solutions. But these do not explain how the problems are selected, how the work of the contestants is marked, and what else happens at an IMO. In this report, I have tried to capture the spirit of the IMO, especially as it is experienced from the point of view of a delegation leader.

The IMO 2002 problem set, selected solutions, some statistics, and a glossary of IMO terms are included at the end. The PDF version of this report contains hyperlinks, both internally and to the web.

Contents

1 Introduction **3**
1.1 Disclaimer 4

2 July 19: Arrival in ‘Brigadoon’ **4**
2.1 First Jury Meeting 4

3 July 20: Working Hard on the Shortlist **5**

4 July 21: Selecting Problems for the Paper **6**
4.1 The Elimination Process 8

5 July 22: Finalizing the Paper **10**

6 July 23: Elections and Opening Ceremony **11**
6.1 Coordination Briefing 12

7 July 24: First Examination **15**
7.1 Advisory Board Meets Jury 16
7.2 Marking Schemes 16
7.3 Mathematical Juggling 19

8 July 25: Second Examination **20**

9 July 26: First Day of Coordination **20**

10 July 27: Second Day of Coordination **21**

11 July 28: Final Jury Meeting and Excursion **21**

12 July 29: Closing Ceremony and Banquet **24**

13 July 30: Departure and Comparison to the IOI **24**

A Motions in Problem Selection **28**

B Exam Paper **29**

C Solutions **31**

D Statistics **36**

E Glossary **38**

1 Introduction

I attended, as an observer, the 43rd International Mathematical Olympiad (IMO) from July 19 to 30, 2002. IMO 2002 was hosted by the United Kingdom in Glasgow. I accompanied the team from the Netherlands, under the experienced leadership of Jan Donkers.

This trip report collects my personal reflections on the event. Therefore, it is good to know a little bit about my background. I will briefly digress on relevant details in this introduction.

The two main goals of my trip were to

- Compare the IMO and the International Olympiad in Informatics (IOI), and to find out how the IMO handles a number of issues that it has in common with the IOI;
- Find out what it takes to host an IMO.

The IOI is, like the IMO, an annual competition for talented high-school students from all over the world. The competition is complemented with a cultural program to foster mutual understanding and friendly relationships. The difference is, of course, that the IOI deals with computing science (informatics) rather than mathematics. The IOI was clearly modeled after the IMO.

Although the first IOI was held 30 years after the first IMO (that is, in 1989), it has grown to a comparable size: almost 80 countries are expected to participate in the IOI in August 2002. I have been involved in the IOI since 1994. At that time, I started a modest web site about the IOI, now available at

`www.scienceolympiads.org` .

This web site grew to include information about other science olympiads: Mathematics, Physics, Biology, and recently Astronomy.

At the time of this writing, I chair the IOI Scientific Committee (ISC). The ISC supervises the long-term technical development of the IOI competition, and monitors the preparation and execution of specific instances of the IOI competition. It is especially in this role that I attended IMO 2002, because I wanted to learn from the IMO.

I am also a mathematician, originally specialized in discrete mathematics, and later turned to computing science, especially various of its mathematical aspects. This explains my warm feelings for the IMO, as reflected by the extra attention for the IMO on my web site.

Understandably, the Dutch national olympiads interest me as well, especially in informatics (NIO) and mathematics (NWO). These two hold a special position on the web site (mostly in Dutch).

In 1995, the Netherlands hosted the IOI. I chaired the Host Scientific Committee in charge of the IOI'95 competition. At that time, the IOI was considerably smaller than it is now.

The Netherlands has participated in the IMO since 1969, but it never hosted the IMO. The IMO is bigger than the IOI, in number of invited countries, as well as in duration and size of the participating teams (more about that later). This adds considerably to the cost of organizing the event. Consequently, hosting the IMO is not easily within reach of smaller countries. Nevertheless, a spark of interest for hosting the IMO in the Netherlands has recently been ignited.

1.1 Disclaimer

The nature of my report is descriptive only. It does not prescribe how an IMO should be run. You can view it as an exercise in history or anthropology. It describes one instance in the IMO series, from one viewpoint only.

My report should also not be viewed as an official record of IMO 2002. It has not been prepared with that in mind, nor has it been reviewed or approved for that purpose. I have done my best to stick to the facts, but my lack of experience in IMO matters may have led to some (unintended) distortions.

Nevertheless, I hope that my report will be of use to those interested in the IMO. Also members of other olympiad communities, such as the IOI, are encouraged to take note.

Appendix E is a glossary of typical IMO terminology.

2 July 19: Arrival in 'Brigadoon'

As seems customary at the high-school olympiads, the attendants pay their trip up to one of the designated entry points. In my case, this is Glasgow Airport. There we are welcomed by happy IMO representatives, who see us to chartered buses. At this point, leaders are separated from early arriving deputies and contestants.

The bus trip takes longer than I had expected. Instead of taking us to nearby Glasgow, we drive on in northern direction to Dunblane. The Hilton Hydro in Dunblane turns out to be the place where the Jury conducts its secret business. The IMO 2002 organizers have nicknamed it 'Brigadoon'. Hilton Hydro used to be a recovery hospital (Kurort), but with its narrow corridors, many staircases, and clearly visible fire precautions, it perfectly fits my image of a British hotel.

Bill Richardson runs the registration desk, where we receive a bunch of goodies, including a badge in huge letters. Without deputies and contestants around, registration is simple and almost informal. Bill, together with his wife Krys, turn out to be the invisible movers of IMO 2002. They were responsible for many things (including the recovery of some lost digital cameras).

2.1 First Jury Meeting

At 21:00, after a remarkably good dinner, we already have our first Jury meeting. Everybody is there on time. Adam McBride, Head of the Department of Mathematics at the University of Strathclyde in Glasgow, sets the tone of the Jury meetings by his efficient (Scottish?) style of chairing. He clearly knows this game well.

Each of the 84 participating countries is represented in the Jury by one leader. The seating is pre-assigned and neatly laid out: leaders in the front (7 rows in 3 groups of 4), observers in the back (2 rows). There is plenty of desk space. Each leader has a blue wand with a three-letter country abbreviation for voting. Observers observe and are not allowed to speak.

This first meeting is brief. Adam explains the drill, summarized well by his statement "we are going to work you hard" (you'll have to imagine the Scottish accent with r's as drum rolls).

Apparently in contrast to last year's IMO, the leaders received the IMO 2002 shortlist without solutions.[1] Each booklet is labeled with a name and a sequence number. This is obviously confidential material. Observers do not get a copy (even when begging :-).

The IMO 2002 shortlist has 27 problems categorized in four fields (at one page per field):

- 6 in Number Theory (N1 through N6)
- 8 in Geometry (G1 through G8)
- 6 in Algebra (A1 through A6)
- 7 in Combinatorics (C1 through C7)

These problems were selected and polished by the Problem Selection Committee chaired by Imre Leader from the University of Cambridge. By the end of April, they had received 130 problems from 41 of the participating countries.

The leaders are requested to rate the difficulty level of each problem they try as easy, medium, or hard. Furthermore, they can register their gut reaction in the 'beauty contest', that is, rate how much they would like to see each problem in the exam paper as 'not at all', 'perhaps', or 'very much so'. If any problem is known to be in the public domain, this must be reported, preferably with a traceable reference.

It is emphasized that by tradition the shortlist must remain confidential until the next IMO. Many leaders wish to use these problems in training.

Later that evening I have a chat with Imre in the bar. He invites me to the Coordination Briefing where the problem coordinators will receive final instructions on their duty of determining scores.

3 July 20: Working Hard on the Shortlist

The first whole day for leaders at the IMO is easy to describe. They all work hard on the shortlist, either alone or in groups. Well, almost all, because some cannot

[1] Later, I was told that last year was an exception rather than the rule.

resist the temptation to join an excursion arranged for observers. I try my hand at some of the combinatorics problems.

It is important to understand why the host decided to have the leaders work on the problems without providing solutions. It is not because solutions must remain even more secret than the problems themselves. The real reason is that you cannot assess a problem, unless you work on it as a contestant. Once you see a solution, it all looks so simple. And from my own experience I can tell that this is absolutely true.

What —in my mind— makes the problems especially challenging for the contestants is the short amount of time available to solve them. On each of the two examinations, they get four and a half hours for three totally unrelated problems. By tradition there is one easier, one medium, and one harder problem, giving them lots of food for thought. I must admit that I am a relatively slow and untrained problem solver. More experienced problem solvers tell me that the $4\frac{1}{2}$ hours give plenty of time for preliminary exploration (false starts, incorrect ideas, blind alleys, etc.), and they remind me that writing down a solution is not the time-consuming part. Furthermore, I guess that most of the contestants would not do much better if they were given $4\frac{1}{2}$ days.

Late in the afternoon, I go for a 50-minute run north toward Ashfield. For my way back, I choose to follow a sign proclaiming 'public path', blissfully ignorant of its semantics. After some 100 meters, the path is no longer discernible in the soggy field with sheep. Later, I rediscover it as a narrow, overgrown 'public bath' (:-). But the view is wonderful and rewarding. This is the kind of problem solving that Paul Zeitz contrasts to 'doing exercises at the hotel's gym' in his beautiful book *The Art and Craft of Problem Solving*.

At 20:15 in the evening, a 40-page booklet with solutions and comments is handed out to the leaders. Again, these copies are labeled for individual leaders, and not available to observers.

4 July 21: Selecting Problems for the Paper

Yesterday was straightforward. The coming days call for a miracle, of which Adam will repeatedly and proudly remind us. The Jury must select the 6 problems for the exam paper from the 27 shortlisted problems. Adam explains that there will be five sessions each day: two in the morning, two in the afternoon, and one in the evening.

The first session today, starting at 09:00, is still relatively easy, because it deals with meta-issues only. First thing, the poll forms are collected for processing.

Imre, as chair of the Problem Selection Committee, presents some mathematical background information on each of the shortlisted problems. He does this with great flair and passion. It is clear that the Problem Selection Committee has worked hard on the shortlist. They checked it, for instance, against the extensive collections of mathematical problems. Imre emphasizes that they love all the problems on the shortlist. Along the way, I learn that certain topics are taboo, if no solutions are known without them, such as[2]:

- Pell's equation
- Complex numbers
- Fermat points

Of course, contestants may use knowledge about these topics, and it could give them some advantage, but it must not be required knowledge for solving a problem.

Next, Adam asks for very general, that is, not problem-specific, questions about the shortlist. There are none. He also asks whether any problems may be known, that is, in the public domain. Again, none.

Imre comments on the difficulty level of the problems as assessed by the Problem Selection Committee. Some 8 to 10 problem are classified as easy or easy-ish, and another 8 or so as hard. One or two problems must be selected from each of the easy(-ish) and hard classes.

Once the poll results have been photocopied and distributed, Adam runs through the whole list. There are a few discrepancies with the views of the Problem Selection Committee, but no big surprises. However, I do have the impression that some leaders have guessed at the difficulty level of problems they did not try.

Now, Adam asks for problem-specific comments, problem by problem. Twelve problems receive attention. Here are some typical comments:

- Knowledge of Fermat points gives an unfair advantage.
- A similar problem may have appeared in a national olympiad (no reference given).
- This problem is not really that easy.
- The bound to be proved is far from sharp.
- The solution is similar to that in a Russian pre-selection contest some years ago.
- For this problem, all-or-nothing scores can be expected.

Alternate solutions are requested, so that they can be photocopied and distributed. You might think that this is unlikely to happen after the thorough preparations of the Problem Selection Committee, but in the next couple of days, 16 new solutions are offered.

Just before the first break, John Webb, secretary of the IMO Advisory Board (AB, see glossary in Appendix E) explains the AB election procedure:

[2] An official taboo list does not exist; and this is on purpose. It was not completely clear to me whether infinite combinatorics belongs here as well.

1. Each candidate makes a brief statement.
2. Each candidate selects a representative to observe the counting of the votes.
3. Candidates withdraw.
4. Jury debates and votes.

The AB election will take place two days from now, after the exam paper has been set and all translations are approved.

4.1 The Elimination Process

After the break, the hard part starts: eliminating 21 problems. There appears to be no standardized, regulated process for the elimination. IMO tradition has it that this is approached afresh at every IMO. The only common thread appears to be the following format:

1. Someone modestly makes a suggestion (mathematicians are notably shy in social situations).
2. The Jury debates the suggestion.
3. If the debate converges, then someone carefully formulates a motion (almost as if it were a mathematical proposition).
4. If the motion is seconded, it is numbered and written on a slide.
5. Further comments specific to the motion are accepted.
6. The motion is orally translated into the official languages (French, German, Russian, and Spanish).
7. Votes are cast (in favor or against) and counted.

However, this format is written down nowhere, nor is it explained to newcomers.

The start of the elimination process betrays hesitation, as if the players still need to get used to each other. This is not so surprising, if one realizes that the Jury meets only once a year and furthermore its composition changes from year to year. Adam asks for guidance rather than taking the initiative. The opening moves in the game touch upon suggestions such as:

- First decide how many problems are selected from each category (N=Number Theory, G=Geometry, A=Algebra, and C=Combinatorics).
- Choose hard problems first.
- Eliminate clear losers first.
- First select one problem from each category (N, G, A, C).

Each of these suggestions is accompanied by motivations, sometimes referring to past IMOs. Informally, it is confirmed that by tradition we will have at least one problem from each of the four categories. Finally, the first motion is put forward:

> First, select two hard problems, as numbers 3 and 6 in the paper.

You must know that each of the two IMO examinations involves three problems: numbers 1 to 3 on the first day, and numbers 4 to 6 on the second day. On each day, the problems are numbered in order of increasing difficulty (according to the Jury's perception). The first problem on a day is clearly easier and the third one clearly harder; that is, there is an intentional gradient in difficulty level. Typically, problem 1 is the easiest and problem 6 the hardest, but problem 2 could be harder than problem 5. If there is a significant difference in overall difficulty of the two days, then the second day is preferred to be the harder one.

The motion is seconded, as required. After translations and voting, it is easily carried by 73 in favor and 1 against. When observing the handling of this first motion, I fear slow progress. Fortunately, the process runs like a smooth train after a few motions.

By lunch, only five motions have been handled, and the two hard problems have not yet been selected. During the breaks, many leaders hand out little gifts, often in the form of problem sets from their national contests.

I will spare you the details of all the discussions, motions, and votings. For a summary of the motions concerning problem selection, see Appendix A. Here are some of the points raised:

- G6 is an unusual problem, combining geometry and combinatorics, without requiring special knowledge.
- C2 and C7 are "newspaper" problems; these are important in advertising the IMO to the general public.
- C2 is more a "puzzle" than mathematics.
- N6 is "really too difficult".
- N6 could be classified as an algebra problem.
- If you know about generating functions, then N5 is too easy; otherwise, it is too hard.
- A4 is a "natural" 4-variable functional equation; this is rare, because multi-variable functional equations are usually either trivial or very unnatural.

Before dinner, I run for almost 45 minutes in a southern direction toward Bridge of Allan. The return via another public path reinforces my earlier perception. The final stretch takes me along the edge of the local golf course, where I feel an intruder.

By the end of the fifth session, the following has been accomplished:

- N2, C1 are candidates for problems 1 and 4,
- A2, A4, G3, G4, C2, C3 are candidates for problems 2 and 5,
- N6, G6 are candidates for problems 3 and 6.

5 July 22: Finalizing the Paper

Today, we will have another five sessions. Halfway through the second session, the final selection is determined in Motion 26:

1: C1, 2: G3, 3: N6; 4: N2, 5: A4, 6: G6

Imre immediately discloses the origin of these problems:

COL, KOR, ROM; ROM, IND, UKR

It is considered a big honor to have a problem on the IMO paper. The "English section" will now finalize the English formulation of the selected problems for the paper. This takes longer than expected. The main principles for the revision are

- Short sentences,
- Natural order of concepts,
- Uniform presentation.

After a late lunch, the Jury scrutinizes the English version of each selected problem for unfamiliar notation, misleading phrasing, or ambiguities. I am somewhat surprised at how long this takes, especially in view of the fact that there has not been a single remark about the formulation of the problems so far. For example, there is a lot of discussion about 'integer' and 'natural number', especially in the context of $n > 1$. Also the double summation in problem 6 attracts attention:

$$\sum_{1 \le i < j \le n} \frac{1}{O_i O_j}$$

Can you expect a high-school student to understand such notation? It is proposed to include a general clarification sheet for this (see tomorrow). Furthermore, some complain about the notation $O_i O_j$ for the length of a line segment. It is agreed that this can also be 'translated' as $|O_i O_j|$ or $\overline{O_i O_j}$, whatever is more appropriate for the team. Mathematical notation is apparently not that universal.

For comparison, I have included the original and final formulations of the problems in Appendix B. After approving the English formulations, the official French, German, Russian, and Spanish versions are prepared. Problem by problem, each of these is projected for approval, though fewer people pay attention now.

Jan and I translate the selected problems into Dutch on my employer's Apple PowerBook, while others work on their translations. The organizers have provided 12 laptop computers and 2 printers for that purpose. Only a handful of the leaders has laptops. By the way, we are completely without Internet here. And even the telephones in our hotel rooms have been disconnected to reinforce our (sense of) isolation.

Altogether 55 translations are prepared and posted on the wall of the Jury meeting room for inspection. Many translations are shared between countries. Our Dutch translation will also be used for the Flemish contestants on the Belgian team. It is interesting to study the translations. Quite a number of translations turn out to be handwritten, which explains the remarkably small number of computers. Most translations use the exact same mathematical notation as in the English version, written left to right, even if the text is right to left. I encounter only one that writes the formulas right to left. Even the $\sum$ for summation is written backwards, and also the variable names have been 'translated'.

6 July 23: Elections and Opening Ceremony

The first morning session starts with approval of all translations. Adam points out that the translations have been available to all leaders for scrutiny. He asks if any errors or other deviations have been spotted. The motion for approval is carried by such a vast majority that the 'against' votes are not even cast.

The next item on the agenda is the 'language-free' explanation for the double summation:

$$\sum_{1 \le i < j \le n} \frac{1}{O_i O_j} = \begin{array}{cccccc} \frac{1}{O_1 O_2} & + \frac{1}{O_1 O_3} & + & \cdots & + & \frac{1}{O_1 O_n} \\ & + \frac{1}{O_2 O_3} & + & \cdots & + & \frac{1}{O_2 O_n} \\ & & & \ddots & & \vdots \\ & & & & + & \frac{1}{O_{n-1} O_n} \end{array}$$

which easily receives the approval of everyone. I now hear that similar explanations have been handed out to contestants in other IMOs, though I have never seen them mentioned in connection with past IMO problem sets. It seems to me that either the explanatory note should not accompany the exam paper at all, or it should be included in the published problem collections. In case of this double summation, it could just have been used as answer to specific clarification requests.

The elections for the IMO Advisory Board are carried out according to the plan announced two days ago. There are 6 candidates for the position of AB chair, and almost 15 for the general position(s). If the new chair is already a member of the AB, then two persons rather than one must be elected for the general positions.

I did not have a good idea what the discussion might be about, once the candidates have withdrawn. It turns out to be brief. Some leaders explain why they made their nomination. Others have a question or suggestion about the election procedure.

József Pelikán from Hungary is elected new AB chair by a convincing margin. In his speech he emphasized his intention to uphold the IMO traditions. Instead of sitting through the speeches and elections for the two general AB positions, I run for 50 minutes. A gently sloping road takes me east toward Sheriffmuir. At the MacCrae monument starts a public path back to Dunblane, but the drizzle makes me decide to backtrack my footsteps.

Opening Ceremony

Due to our isolated location, it had escaped me that yesterday was the arrival day for deputies and contestants. Today, we will see them, but no more than that. At the IMO, leaders are strictly separated from their contestants most of the time.

At 14:00, we board the buses to Glasgow, where the opening ceremony will take place. Barony Hall, a restored Victorian Gothic church now owned by the University of Strathclyde, is already filled with deputies, contestants, and guides when we arrive. Narrow spiral stairs lead us to a balcony, which is just too small to hold all of us comfortably. Leaders wave at their contestants, almost in compassion.

The opening ceremony itself is simple. An introductory video that refuses to start on command, a few short speeches, some Scottish folk dancing on live bagpipe tunes (2 ladies, 1 bagpipe), and an alphabetic parade of all teams (of course, minus the leaders, who now jump up to take pictures). At the end, Colin Wright entertains us with marvelous mathematical juggling. And then it is back to Brigadoon.

6.1 Coordination Briefing

Today, the remaining problem coordinators have arrived in Dunblane. They will be involved in determining the scores once the examinations are over. Imre, who is also Chief Coordinator, prepared them over the last couple of months, including a weekend of training. Tonight is their final briefing. The thoroughness of this briefing taught me more about the IMO than anything else. I feel privileged to attend.

We start with a group of about 30 persons who have missed part of the training or who just want to refresh their memory. Later the group grows to almost 60. As it turns out, I am the only outsider. Imre whips out some handwritten slides and dashes off to summarize the main points.

- Coordinators mark the work of the contestants, using photocopies of the scripts.
- Coordination Consultants primarily act as translators, either before or during coordination sessions.

- Leaders mark the original scripts of their own contestants.
- Each leader meets 2 coordinators for each problem at a scheduled 45-minute time slot.

For the coordination sessions there will be 8 coordinators per problem, distributed over 4 tables. Each problem group is led by a Senior Coordinator, also known as problem captain.

Imre explains that the scores will be used to allocate medals, and that the following performance is, typically, required to win a medal:

Gold solve 4 to 5 problems

Silver solve 3 problems

Bronze solve 1 problem and some bits

He reminds the coordinators that, of the three cutoffs (gold-silver, silver-bronze, and bronze-none), the cutoff between bronze and no medal is where marks will be most clustered. So this is where each mark gained or lost is most important. Therefore, it is terribly important to get the scores for the 'bits' right.

Tomorrow, the coordinators must produce the so-called marking schemes for the problems to guide the allocation of points. They are encouraged to ...

- develop each marking scheme as an 8-person team;
- start working on the problem without solution, so as to get a feel for where the hard steps are;
- generate lots of solutions.

Imre reminds them that during coordination the marking schemes evolve and that it is the Senior Coordinators' responsibility to keep their entire problem group up to date.

The most critical aspect of a marking scheme is the distinction between the 0^+ region (failure) versus the 7^- region (success):

Region	Score	Description
7^-	7 *	Complete solution
	6	Tiny slip (and contestant could immediately repair it)
	5	Small gap or mistake (but non-central)
0^+	2	Lots of genuine progress
	1 *	Non-trivial progress
	0 *	"Busy work", lots of writing, special cases only

In the table, default scores are marked by an asterisk ('*'). Note that scores 3 and 4 are missing; they are used in special situations only. Marking is not about breaking the solution into bits and awarding marks for each bit. It is often not additive, but rather based on taking the maximum, to avoid building up undue credit for lots of small observations:

$$1 \oplus 1 \oplus 1 \quad = \quad 1$$

Most discussion will occur at the bottom end. Of course, there are different kinds of 0's, but even a 'very good 0' is still 0. Do not give points out of sympathy with the leaders or contestants. Score inflation is not going to help anyone.

Imre shows some marking schemes and points out good and bad features:

- Good: indicating how known solutions and errors fit in the scheme
- Bad: being additive, or not dealing with alternate approaches

In a coordination session, the two coordinators will usually meet a leader accompanied by a deputy or an observer. There are various types of leaders to expect:

- *honest*, but on the side of their contestants (the usual case); might mark high: "not sure it is 5 or 6, so ask for 6"
- *super honest*; might mark low
- *bully*; intimidates, uses loud voice
- *forceful but fair*; apparently bullying, but in fact has a point
- *non-expert*; might not understand contestants' scripts

The work of the contestants, in IMO terminology known as scripts, also comes in various flavors:

- many are *very good*, but long, complicated, unusual
- many are *very bad* (and also long)
- most are *not in English*
- most have a *neat* part, but also a *rough* part

It is the leader's duty to explain complicated solutions, not that of the coordinators. Never having seen an IMO script, I still have to experience this reality.

The advice to coordinators hardly needs mentioning:

- be consistent and fair; that is, firm but willing to admit you are wrong;
- be polite at all times;
- can always ask for help from the problem captain;
- let the leaders decide the order of handling their contestants;

- keep records of tough decisions;
- at the end, leader and coordinator sign the score form; this is final;
- if no consensus, ask to come back later.

Coordinators are also reminded of very bad things that can happen, such as cheating, and very good things:

> *Special prizes* can be awarded for particularly elegant solutions or generalizations.

7 July 24: First Examination

Today is the first examination day. At 08:50 the leaders gather in the meeting room to answer any questions that come in. The Jury has regrouped according to common languages. We have no idea about the situation of the contestants in Glasgow, where the four-and-a-half-hour exam will start at 09:00. During the first half hour of the examination, the contestants can ask questions concerning obscurities in the exam paper.

Somewhat to my surprise, questions arrive very soon after the examination has started. We are told that 6 fax machines have been set up to receive questions. Each fax is given a tracking number and is logged on a flipover. The relevant leader gets the fax and a copy on a transparency to formulate a response. Once the response is ready, the leader queues at the overhead projector to present question and response to the Jury, translating where necessary. Sometimes there is a little discussion, but more often there is immediate approval. The approved response is written on the fax, checked by an organizer, and faxed back to Glasgow.

In the first half hour, about 20 questions have been handled. Most of them concern problem 1, for example

- whether 'distinct' means 'pairwise distinct';
- whether X-sets are counted over one given coloring, or over all possible colorings.

Several questions about problem 2 ask for the definition of 'incentre'. The first Dutch question arrives after 45 minutes. It appears that some faxes have accumulated a delay of up to 40 minutes. Altogether, 67 questions are handled, but not one of them reveals a serious flaw in the paper.

Once the examination is over, the scripts written by the contestants will be

- collected,
- photocopied,
- put into color-coded folders, each holding the script from one contestant for one problem,

- transferred to Dunblane, and
- distributed to the problem coordinators and leaders.

It is agreed that the scripts will not be handed over to the leaders until after discussion of the marking schemes later this evening.

7.1 Advisory Board Meets Jury

At 11:00, the IMO Advisory Board meets with the Jury. The first items on the agenda concern future IMO hosts. IMO 2004 will be hosted by Greece, and a detailed schedule is now available. For IMO 2005 and IMO 2006, the AB proposes as hosts Mexico and Slovenia respectively. This is accepted by a majority. The Mexican and Slovenian leaders make a brief statement. For the years 2007, 2008, and 2009 there are candidates (Vietnam, Brazil, Germany), but no decisions will be made now.

Then there is a financial report, followed by an inconclusive discussion about the election procedure. It is suggested that the Problem Selection Committee adds a fifth category titled ‘Miscellaneous’. Another suggestion is to have a central address and possibly some kind of general (host-independent) IMO newsletter. Someone else proposes to have a central web portal hosted by UNESCO in Paris.

After the meeting, most of the leaders go on an excursion to Edinburgh, which Jan and I join. A smaller group visits a distillery and Loch Lomond. Along the way, we are treated to bits of Scottish history. The biggest sight is Edinburgh Castle, but also the steel bridge across the Firth of Forth is very impressive when viewed from below.

7.2 Marking Schemes

At 20:30 after dinner, the Jury meets to hear about the marking schemes, which were designed today. Imre introduces the 6 problem captains who will explain the rough marking schemes and listen to comments. In contrast to previous years, the organizers have decided to obtain formal approval from the Jury for the marking schemes.

Because I have never seen anything in writing about marking schemes, I will include summaries here. The final versions take up about 2 pages per problem and are further refined during coordination as new insights evolve. Complete solutions are provided in Appendix C.

For Problem 1 Béla Bollobás begins by explaining that there are many possible approaches. In general, treatment of special cases receives 0 marks. Observing that it suffices to show that the blue column lengths a_i are a permutation of the blue row lengths b_i receives 1 mark total, even without any proof or a restatement of the goal in terms of the product formula

$$a_0 a_1 \dots a_{n-1} \quad = \quad b_0 b_1 \dots b_{n-1} \,.$$

For each of the following approaches, typical errors and their deductions are summarized: induction proof, bijection proof, formula proof, calculating the number of columns (rows) with exactly k blue points, weighting proof, and integral proof. For instance, in an induction proof, missing the base case costs 1 mark if it is trivial, and 2 marks if it is non-symmetric, non-trivial. In case of a bijection proof, 2 marks are deducted for not stating (in any way) that the transformation is invertible. No marks are deducted for a statement of invertibility without proof.

There is no debate at all about this marking scheme.

For Problem 2 Tony Gardiner emphasizes that, again, many approaches are possible. This is actually an easy problem, in spite of being number 2, and it is hard to define the difference between non-trivial progress (1 mark) and lots of real progress (2 marks). Once you make real progress, you basically have a complete solution. Therefore, the marking scheme provides for 0, 1, 6, and 7 marks only, though Tony also added that he expected some cases of 2 and even 3 marks to emerge during the coordination process.

If $\angle AOB < 120°$ is not clearly used somewhere, then 1 mark is deducted. There are only a few remarks from the Jury.

For Problem 3 Tim Gowers points out that only real work that can actually appear in a complete solution will be rewarded. There are various seemingly interesting but useless results possible. No marks will be given for showing that $(m, n) = (3, 5)$ is a solution, and 1 mark for proving that

$$q(a) \mid p(a) \text{ for } \infty\text{-ly many } a \quad \Rightarrow \quad q \mid p$$

For Problem 4 Vin de Silva notes that there seems to be really only one solution. Concerning the (a)-part, 1 mark will be given for showing

$$\frac{1}{d_1 d_2} + \frac{1}{d_2 d_3} + \cdots + \frac{1}{d_{k-1} d_k} < 1 \quad \Leftrightarrow \quad D < n^2$$

At most 4 marks are given for doing the (a)-part only, and at most 3 marks for doing only the (b)-part assuming the (a)-part.

Some find the 4 marks too generous. Someone else asks whether a clear ‘typo’ by the contestant will not be treated as a ‘tiny slip’ (6 marks), but could still get the full 7 marks. This is indeed the case.

For Problem 5 Dan Crisan starts on a positive note: weaker students might be able to get some useful results, such as 1 mark for any of

(A) f is multiplicative, f is positive on $\mathbb{R}_+$, $f(x) = x^2$ on $\mathbb{N}$ (or any infinite subset of $\mathbb{R}$)

or even 2 marks for any of

(B) $f(x) = x^2$ on $\mathbb{Q}$ ($\mathbb{Q}_+$) or any other dense subset of $\mathbb{R}$ ($\mathbb{R}_+$)

(C) f is monotone, or f is continuous

It is emphasized that this is not additive, though 3 marks are given for (B) and (C) together, without making the connection to finish the problem. If the connection is indeed made, but without any justification, then 5 marks may be awarded.[3] No marks are given for mentioning the two constant solutions or $f(x) = 2^2$, because they can be obtained by blind manipulation. However, 1 mark is deducting for missing a constant solution, since it is impossible to finish the problem without excluding these. And 2 marks are deducted for missing both, because this suggests general sloppiness, and hence a 'small gap' rather than a 'tiny slip'.

This marking scheme draws the most fire:

- One objection is that this scheme rewards knowledge (on standard types of functional equations) that can be obtained by (hard) training, rather than mathematical ingenuity. However, this is in the nature of the problem rather than in the marking scheme.
- Another objection is that the 2 marks deduction for missing both constant solutions is too severe.
- Then there is the issue of 'closing off the proof'. Some contestants may find it so obvious, that they will not write more after doing (B) and (C), or they simply write 'Now, I am done.' Will they be able to get full marks?
- Finally, some consider it bad practice that no marks are rewarded for actually finding all the solutions that the problem asks for (without proving completeness). For this problem, finding the constant solutions is even considered "necessary progress" in view of the deductions when missing them, but they are not rewarded as progress.

It is agreed that the problem coordinators will revise the marking scheme in the light of this discussion. However, they need not present the revised marking scheme for final approval.

For Problem 6 Gerry Leversha says that they have $2+\varepsilon$ solutions, where ε is possibly $= 0$. Solving particular configurations, such as $n = 3$, yields 0 marks. Showing that, for any pair of the circles,

$$\frac{1}{O_i O_j} \leq \frac{\alpha_{ij}}{2}$$

[3] One needs to be quite careful here in quoting a general theorem from analysis to finish the proof. It may seem, but actually is *not* true in general that two increasing functions agreeing on a dense set are equal. Consider an increasing function f with $f(x) = x$ for $x < \sqrt{2}$, and $f(x) = x + 1$ for $x > \sqrt{2}$. Then $f(\sqrt{2})$ can still be chosen anywhere between $\sqrt{2}$ and $1 + \sqrt{2}$.

receives 1 mark, because this is the key step in making the problem more accessible (see Figure 1, called the bow-tie).

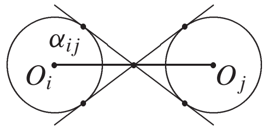

Figure 1: Bow-tie (α_{ij} is angle between $O_i O_j$ and inner tangent)

There is some debate over whether proving $n\pi/4$ as bound should receive 2 or 3 marks. The response is that it will only be rewarded 3 marks if the contestant's approach holds reasonable promise of reducing this bound to the required bound of $(n-1)\pi/4$.

We are done by 22:30. Final versions of the marking schemes will be made available tomorrow. The scripts of the first examination are now available for pick-up. Jan collects his 18 folders in three colors.

7.3 Mathematical Juggling

After a short break, we have an extra item on the program. Colin Wright, who performed in the Opening Ceremony, has been invited back to juggle more and present the mathematical theory of 3-ball juggling that he and some friends invented. He already did this extended juggling talk for the contestants. The organizers were so impressed that they decided we should have an opportunity to enjoy it as well. Colin explains the SiteSwap notation for juggling patterns as a sequence of integer cycle times. It is based on some symmetry premises, such as

- only one ball is thrown or caught at a time,
- the hands alternate,
- the hands are full for half the time.

He also mentions the key invariants that characterize the notation, such as

- the average cycle time equals the number of balls.

This notation helped them invent new juggling patterns. He finishes his fast-paced and funny performance by letting us construct a pattern, which he then juggles.

Later that night, Jan and I look over some of the Dutch scripts. Imre's description of what to expect from a script is borne out. Some are long, with a neat part and a rough part, following an unnecessarily complicated, but ingenious path. It strikes me that the Dutch contestants nicely number their accomplishments. This turns out to be very helpful in coordination.

8 July 25: Second Examination

The second examination day starts as the first, with half an hour of question time. The process has been polished. Altogether we see 43 questions, none of them disturbing. Afterwards, I run west and back for half an hour. Dunblane is very small indeed, because I soon got out of town in each of the four main wind directions.

Later in the morning, the leaders check out of the Hilton Hydro and move to Glasgow. There, we are put up in the Moat House Hotel on the river Clyde. The hotel connects to the Scottish Exhibition & Conference Centre (SECC), location of the examinations. The contestants stay in apartments of the University of Strathclyde at half an hour's walking distance. The deputies stayed with them, but they now also move to the Moat House Hotel, where coordination will take place.

At 13:30, when the second examination ends, the leaders and deputies meet the contestants at the gate of the huge exhibition hall. The atmosphere is hectic and happy as teams struggle to reunite. At last, the contestants can relax.

I wonder what they think of the schedule with two examination days in a row. At the IOI, the two competition days are separated by a compulsory day of rest, which the organizers use to complete the grading of the first day and prepare for the second. However, some former IMO contestants convince me that they would not be able to enjoy an excursion between the exams. In fact, they prefer to forget the first exam when starting on the second. There really is no benefit in looking back at the first exam. They find it better to continue while their concentration is still focused on exams.

All teams have lunch at the hotel; it is their first joint meal. In the afternoon, everyone can visit the Glasgow Science Centre just across the river. It has an IMAX theater and various hands-on science exhibits. Afterwards there is a reception for the leaders, with several speeches that hardly anybody will have understood.

9 July 26: First Day of Coordination

While the contestants and guides are off on their first excursion, the leaders and deputies start their first day of coordination. The schedule squeezes $84 * 6 = 504$ coordination sessions into two days. Some say it is madness, and mumble that three days might have been better, though all recent IMOs have used two days.

Imre finds time to explain his design of the coordination schedule to me. For each problem there are four pairs of coordinators, occupying tables numbered 1 to 4. Each team has been assigned to one table number for all 6 problems. To spread the load, Coordination Table 1 sees the top 14 teams and the bottom 7. It is important for consistency that teams close in ability are handled by the same coordinators. Ranking is informally based on past performance. Similarly, Coordination Table 2 gets the 14 teams just below the top 14 and the 7 teams just above the bottom 7. The other teams are randomly distributed over Coordination Tables 3 and 4, taking into account some further constraints. For instance, languages are spread over time, to minimize conflicting requests for Coordination Consultants; and no team coordinates two second-day problems on the morning of the first day of coordination.

Each coordination session is scheduled to last for 45 minutes, like last year. This is more generous than the 30 minutes of earlier years, but it does require 4 tables rather than 3 per problem. Coordination sessions typically occur in sequential pairs, with half-hour breaks between pairs. Sessions for which extra preparation is expected, are scheduled as first in a pair. Sessions for which much talking is expected are scheduled second in a pair, so that they can be extended into the break if necessary, without upsetting the schedule.

I discreetly observe some coordination sessions from a distance. All results are entered into Bill Richardson's computer and posted on a collapsible wall in the corridor. There is no secrecy about this, in contrast to the IOI.

10 July 27: Second Day of Coordination

Today is more of the same. I take the opportunity to run for some 50 minutes along the bank of the Clyde to Glasgow Green, a community park with various paved paths and lavish lawns. On the way back along the other side of the Clyde, underneath one of the many bridges, I notice the 'undesirables', as the local population calls them.

Coordination is making good progress. Some leaders and observers keep track of every detail, jotting all numbers on their little notepads. The results of the last problem coordinated for each team are not posted. This seems to be a new feature at IMO 2002. It is done to prevent leaders from taking advantage of such knowledge to influence the outcome of their remaining coordination sessions.

At the end of the day, not all disagreement has been resolved. The next morning, I hear that Imre and some coordinators worked late into the night to seek convergence and to finalize all results.

11 July 28: Final Jury Meeting and Excursion

At 09:00, the Final Jury meeting is held in a room that is too small for the size of the interested audience, in spite of the lack of tables. The leaders sit in the front with their wands.

The Chief Invigilator, Terry Heard, reports on the conduct of the examinations. A hall of 2300 m^2 was used, with 486 tables in 6 zones. No 2 competitors of the same team were in the same zone. The climate was well controlled, and there was no external noise or visual distraction. The examinations had started at 09:03 and 09:00 respectively, watched over by 33 invigilators. One candidate had fallen ill (nothing serious), another had slept for half an hour. The 15 000 pages of scripts were photocopied on 6 photocopiers and put into over 5 000 folders.

Imre, as Chief Coordinator, reports on the coordination. The schedule had mostly been adequate. The detailed marking schemes had always been applied when applicable. In the end all disagreement had been settled amicably.

Next, the Jury is asked to approve all scores of the contestants. This passes without any discussion.

Now the Jury must determine the cutoff scores for the medals. An anonymized table and histogram is shown (see Table 1 in Appendix D). The IMO 2002 Regulations prescribe:

- The total number of prizes will not be more than half the total number of contestants.
- The numbers of first (gold), second (silver), and third (bronze) prizes will be approximately in the ratio 1 : 2 : 3.

Given that there are 479 contestants, the lower bound for gold is easily determined at 29 points. There ensues some discussion about which boundary to determine next: between silver and bronze, or bronze and none. The rule about the total number of prizes is more strictly formulated than the one involving the ratios. Therefore, it is decided to determine the boundary between bronze and none. The cutoff at 14 points seems better than at 13, yielding 232 medalists instead of 248. The boundary between silver and bronze is now obvious at 23 points.

This year no recommendations for special prizes are made. Many contestants have done excellent work, but no outstanding new solutions or generalizations were encountered. After the meeting, three documents with various presentations of all results will be made available.

Now that the results of IMO 2002 have been finalized, there is time for matters concerning future IMOs:

- Some leaders are not happy about the way the marking scheme for problem 5 had been handled and wish to see a better procedure next year. Imre counters that the mood of the Jury had always been followed. In particular, the three open issues were handled as follows:
  - *What if a well-known theorem was quoted?* The coordinators had already indicated to accept this, *provided* the theorem is indeed well-known and stated clearly and correctly. This is an (unwritten) IMO standard: a contestant can quote a known theorem, as long as it is indeed known and it is quoted accurately.
  - *What if both constant solutions were missing?* The coordinators decided to deduct only 1 mark rather than 2. No such case occurred.
  - *What if only the constant solutions were found?* The coordinators decided to stick with the 0 marks. This situation is comparable to an induction proof. Missing the base case costs 1 mark, but you do not get 1 mark for just doing the base.

- If marking schemes are to be formally approved, then it is found essential to receive a printed version of the marking schemes at least one hour in advance. This has never been done before.
- It is noted that this is the first IMO where the marking schemes were formally approved by the Jury. We should be thankful to the organizers. The final decision should be with the coordinators, to avoid a conflict of interest. The coordinators are not an executive body of the Jury. The coordinators and the Problem Selection Committee know the problems much better than the Jury.
- Someone tries to explain that the binary marking schemes (0, 1, 6, 7) are a disadvantage to small countries, and asks the AB to reconsider.

The issue of possible new topics to allow (such as complex numbers, for example) is not mentioned at all.

The key figures in IMO 2002 are thanked abundantly, and rightfully so.

The organizers of IMO 2003 in Japan point out the main differences with IMO 2002. In particular,

- IMO 2003 will be one day longer;
- it will start 12 days earlier on July 7;
- the problem submission deadline is much earlier at February 15;
- the leaders will move around in a different pattern.

This concludes the final Jury meeting.

The Final Excursion is in the afternoon. The plan is that most participants board the steamship *Waverly* and have lunch and dinner there as well. A defect sustained yesterday causes considerable delay. Because of the limited capacity of the *Waverly*, there is a separate bus excursion for the coordinators. I end up taking the very enjoyable, though rainy, bus trip with the coordinators to Loch Lomond. During a short boat tour on the loch, one of the coordinators challenges us with this nice problem:

> *N* prisoners share a toilet with one light bulb operated by an ordinary switch. Only one prisoner can be in the toilet at any one moment. The inside of the toilet cannot be observed from outside. In every infinite sequence of toilet visits, every prisoner occurs infinitely often.
>
> The prisoners must initially agree upon an algorithm such that, after some finite amount of time, some prisoner will correctly announce that each prisoner has been to the toilet at least once. The initial state of the light is not known. What algorithm can they use?

Before the boat trip is over, we come up with a nice solution[4].

[4] Hint: Count the number of times the light is switched on.

In the evening, I collect various files with IMO 2002 data to prepare statistics (see Appendix D). I have learned that if you do not grab such information immediately, then it may take a long time to get it, or you may not even get it at all.

12 July 29: Closing Ceremony and Banquet

In the morning, we have our first spell of true free time. I go for another 50-minute run to Glasgow Green. So far, the weather has been remarkably fine for running.

The Closing Ceremony is scheduled for 15:30 in the Clyde Auditorium ('Armadillo') of the SECC next to the hotel. When entering, we are welcomed by the loud sound of a lonely bag-piper. Inside the huge theater, there is plenty of space. The Scottish Fiddle Orchestra provides the musical introduction. The awards ceremony itself is very simple and efficient. Up to 10 contestants are awarded their bronze or silver medals in parallel by 10 officials. In the intermezzo, four little girls do a sword dance on a bagpipe tune. Princess Anne presents the gold medals without much ado. Her speech betrays good preparation.

Immediately afterwards, buses transport the leaders and deputies through the rain to Barony Hall, now for a reception (which lasts too long with too little to drink) and one final speech. The Closing Banquet for all participants will be at *The Arches*, a club under the central station. Because of the heavy rain, we go by bus. It takes an awfully long time to get in. Inside, the arched ceilings explain the name. The buffet dinner is served at several locations. Unfortunately, the music is too loud for any decent form of communication. Some contestants seem to enjoy the games, such as a mechanical rodeo and jumping upside-down in a velcro suit against an air cushion. Some of us decide to leave early and take a train back to the hotel.

13 July 30: Departure and Comparison to the IOI

The Dutch team leaves on the 09:00 bus to Glasgow Airport. The trip back home is uneventful and provides me with some time to look back on IMO 2002. One obvious thing for me to do, is to compare the IMO with the International Olympiad in Informatics (IOI).

The IMO examination does not include all of mathematics, but it does aim to cover all the pure mathematics common to the high-school curricula of most countries. For the IOI, this would be unrealistic, because informatics has no strong tradition in the high-school curriculum of most countries. The IOI competition focuses on informatics problems of an *algorithmic nature*. Here are two (exceptionally) short examples. The first one is adapted from task PALIN at IOI 2000:

> Write an efficient program that, when given a sequence of characters, determines the minimum number of characters to be inserted into the input sequence to make it a palindrome.

The second is adapted from task MEDIAN at IOI 2000:

> Given is an odd number of objects, all of distinct weight. The only way to compare weights is through the function *Med3*(a, b, c) that returns the object of median (middle) weight among three distinct objects:
>
> $$\min\{a, b, c\} < \mathit{Med3}(a, b, c) < \max\{a, b, c\}$$
>
> Write an efficient program to determine the object of median weight among all given objects, using only function *Med3*.

Note that these problem descriptions are not complete and would not be usable as such at an IOI competition. They only illustrate what the nature of IOI problems is like.

Let me start my IMO-IOI comparison by considering the IMO process of selecting the problems from the shortlist, as described in §4.1. On one hand, the IMO process comes across as very orderly, careful, and open. More than 40 IMOs have contributed to this process and refined it into what I have witnessed. A strong tradition is carried in the memories of many experienced members of the Jury. On the other hand, an outsider still notices a gap between intentions and results. It is difficult to have a group choose rationally among so many good options in so little time. The contorted path of motions makes it impossible (even for mathematicians) to blame anyone for the result. That way —the reasoning might be— everyone must be happy (though this is not true). Or, maybe, I am too optimistic, and the Jury members have very diverse opinions on the desired outcome.

The IOI has a similar duty of determining the problems to be solved in the competition. Typically, there are also six competition problems in the IOI. However, the boundary conditions differ in a number of important ways:

- Each IOI is run in 8 days from arrival to departure of *all* participants; leaders do not arrive 4 or 5 days earlier as at the IMO.
- The IOI meetings, where problems are discussed, involve *leaders, deputy leaders, and qualified observers*; at the IMO, the problems are discussed by the leaders only.
- The description of a single IOI competition problem typically takes *2 pages*, including diagrams and examples of input and output formats; short problems fit on 1 page, long problems take 3 pages. Compare this to IMO problems[5], where a shortlist of 30 problems fits on 4 pages, rather than 60.
- The IOI contestants are required to express their algorithms in one of the allowed programming languages[6] and they must engineer their programs to run flawlessly, because marking is based on automated execution.

[5] IMO problems tend to be very compact. It is quite remarkable that still so many neat and diverse competition problems are invented, year after year. There is a slight tendency toward more complex and less general situations, but not much.

[6] Currently, Pascal, C, and C++ are allowed at the IOI.

- At the IOI, the details needed to mark a problem take considerably more effort to prepare than at the IMO. At the IMO, the marking schemes are developed by a team of coordinators in one day before the examination. Still, the IMO marking schemes receive more attention than is the case for most other maths exams.

I find it surprising that the Problem Selection Committee has played no role of significance beyond preparing the shortlist. The Jury has asked them a few clarifications only. The IOI established the IOI Scientific Committee (ISC) to monitor and assist in the preparation of the IOI competition. The entire IOI problem set is finalized well in advance of the event, though some last-minute polishing is unavoidable.

The ISC is also involved in supervising the longer-term development of the IOI competition. More general issues involving continuity of the IOI are handled by its International Committee (IC), the IOI counterpart of the IMO Advisory Board.

In my opinion, the strong points of the IMO, in comparison to the IOI, are:

- More *diversity in difficulty levels* of the exam problems. Also see Tables 2 and 3 in Appendix D.

- Leaders apply their *professional skills* more, especially because they are more involved in problem selection and in marking.

- The marking considers *all* information in the scripts that the contestants submit after the examination, and this information is examined by real mathematicians.

  Note that at the IOI, submitted programs are evaluated as *black boxes*. IOI contestants do not even get an opportunity to submit their design ideas for consideration in the evaluation process. Furthermore, they may submit only *one* program (otherwise, they might start gambling). This often places contestants in a dilemma. If they have a correct but inefficient program, they know it will not get them full marks (maybe 60%). So they work on a more efficient program, which they may not get completely right. In the end, they must decide which one to submit. In contrast, at the IMO, if a contestant proves a helpful lemma, say for even n, which constitutes real progress but does not solve the problem, then its contribution to the final mark will not be erased by messing about with odd n.

Some further differences are:

- Each of the two IOI competition days lasts for *five* hours, and there is a compulsory *non-competition day* between them.

- The IOI has only *four* contestants per delegation[7].

[7] What in the IMO appears to be called a team, is referred to as delegation in the IOI.

- At the IOI, leaders, deputies, and contestants are *together* most of the time.

Let me hasten to say that the IMO and IOI are complex events, which evolved over many years. It will not be easy for the IMO or the IOI to adopt practices from each other. But that should not stop us from thinking about it.

I enjoyed IMO 2002 immensely and I learned a lot by participating as an observer. I would like to express my sincere thanks to the persons who were willing to share their experiences with me.

A Motions in Problem Selection

Here is a list of all motions concerning problem selection at IMO 2002. The formulation is not necessarily the official version.

Nr.	Motion	Result
1	First, select two hard problems as problems 3 and 6	Carried
2	Eliminate G5, because some say it has been used	Carried
3	Eliminate A5 as candidate for problems 3 and 6	Carried
4	Eliminate A6 as candidate for problems 3 and 6	Carried
5	Eliminate N5, because no elementary solution found so far	Carried
6	Eliminate G7, G8 as candidates for problems 3 and 6	Carried
7	Eliminate C6 as candidate for problems 3 and 6	Carried
8	Vote for each of the remaining 5 pairs; in each round, eliminate pair with least number of votes; can vote for as many pairs as wanted; repeat until one pair left	Carried
9	Eliminate A3	Carried
10	Select problems 1 and 4	Carried
11	Eliminate C3, G2, A2 as candidates for problems 1 and 4	Carried
12	Eliminate A1 as candidate for problems 1 and 4	Carried
13	Select N1, C1 as problems 1 and 4 (order still open)	Failed
14	Eliminate N1 completely, because it is way too easy	Carried
15	Eliminate C1 completely	Failed
16	Select C1 as problem 1 or 4	Carried
17	Select N2 as problem 1 or 4	Carried
18	Revert to situation immediately after Motion 14	Failed
19	Create a shorter list for problems 2 and 5	Carried
20	Close the shorter list at A1, A3, A4, G3, G4, C2, C3	Carried
21	Eliminate A1 from the shorter list	Carried
22	Adjourn until tomorrow morning	Carried
23	Eliminate C2	Carried
24	Include one more problem on the shorter list	Failed
25	Vote for each of the remaining 8 pairs, eliminating pair with least number of votes in each round	Carried
25a	Amend Motion 25 to allow multiple votes per leader per round	Carried
26	Select order C1, G3, N6, N2, A4, G6	Carried

B Exam Paper

Here are the 6 problems in the IMO 2002 exam paper. For comparison, I have included the original formulation from the shortlist, and the final formulation in the exam paper.

Problem 1 (original) Let n be a positive integer. Each point (x, y) in the plane, where x and y are non-negative integers with $x + y < n$, is coloured red or blue, subject to the following condition: if a point (x, y) is red, then so are all points (x', y') with $x' \leq x$ and $y' \leq y$. Let A be the number of ways to choose n blue points with distinct x-coordinates, and let B be the number of ways to choose n blue points with distinct y-coordinates. Prove that $A = B$.

Problem 1 (final) Let n be a positive integer. Let T be the set of points (x, y) in the plane where x and y are non-negative integers and $x + y < n$. Each point of T is coloured red or blue. If a point (x, y) is red, then so are all points (x', y') of T with both $x' \leq x$ and $y' \leq y$. Define an X-set to be a set of n blue points having distinct x-coordinates, and a Y-set to be a set of n blue points having distinct y-coordinates. Prove that the number of X-sets is equal to the number of Y-sets.

Problem 2 (original) The circle S has centre O, and BC is a diameter of S. Let A be a point of S such that $\angle AOB < 120°$. Let D be the midpoint of the arc AB which does not contain C. The line through O parallel to DA meets the line AC at I. The perpendicular bisector of OA meets S at E and at F. Prove that I is the incentre of the triangle CEF.

Problem 2 (final) Let BC be a diameter of the circle Γ with centre O. Let A be a point on Γ such that $0° < \angle AOB < 120°$. Let D be the midpoint of the arc AB not containing C. The line through O parallel to DA meets the line AC at J. The perpendicular bisector of OA meets Γ at E and at F. Prove that J is the incentre of the triangle CEF.

Problem 3 (original) Find all pairs of positive integers $m, n \geq 3$ for which there exist infinitely many positive integers a such that

$$\frac{a^m + a - 1}{a^n + a^2 - 1}$$

is itself an integer.

Problem 3 (final) Find all pairs of integers $m, n \geq 3$ such that there exist infinitely many positive integers a for which

$$\frac{a^m + a - 1}{a^n + a^2 - 1}$$

is an integer.

Problem 4 (original) Let $n \geq 2$ be a positive integer, with divisors $1 = d_1 < d_2 < \ldots < d_k = n$. Prove that $d_1 d_2 + d_2 d_3 + \ldots + d_{k-1} d_k$ is always less than n^2, and determine when it is a divisor of n^2.

Problem 4 (final) Let n be an integer greater than 1. The positive divisors of n are $d_1, d_2, \ldots, d_k$ where

$$1 = d_1 < d_2 < \cdots < d_k = n \,.$$

Define $D = d_1 d_2 + d_2 d_3 + \cdots + d_{k-1} d_k$.

(a) Prove that $D < n^2$.

(b) Determine all n for which D is a divisor of n^2.

Problem 5 (original) Find all functions f from the reals to the reals such that

$$(f(x) + f(z))\,(f(y) + f(t)) \quad = \quad f(xy - zt) + f(xt + yz)$$

for all real x, y, z, t.

Problem 5 (final) Find all functions f from the set $\mathbb{R}$ of real numbers to itself such that

$$(f(x) + f(z))\,(f(y) + f(t)) \quad = \quad f(xy - zt) + f(xt + yz)$$

for all x, y, z, t in $\mathbb{R}$.

Problem 6 (original) Let $n \geq 3$ be a positive integer. Let $C_1, C_2, C_3, \ldots, C_n$ be unit circles in the plane, with centres $O_1, O_2, O_3, \ldots, O_n$ respectively. If no line meets more than two of the circles, prove that

$$\sum_{1 \leq i < j \leq n} \frac{1}{O_i O_j} \quad \leq \quad \frac{(n-1)\pi}{4} \,.$$

Problem 6 (final) Let $\Gamma_1, \Gamma_2, \ldots, \Gamma_n$ be circles of radius 1 in the plane, where $n \geq 3$. Denote their centres by $O_1, O_2, \ldots, O_n$ respectively. Suppose that no line meets more than two of the circles. Prove that

$$\sum_{1 \leq i < j \leq n} \frac{1}{O_i O_j} \quad \leq \quad \frac{(n-1)\pi}{4} \,.$$

C Solutions

For completeness' sake, I include solutions to the problems of the IMO 2002 exam paper. I encourage you to try and solve these problems yourself first. If you just want some hints, then consult §7.2 about the marking schemes, where you can also read what ingredients in a solution are considered essential.

Let me note that there is no such thing as *the* solution to an IMO problem. Even the official *Short-listed Problems and Solutions* provides multiple solutions and variants for many of the problems. I have made a personal selection from the available solutions. Other people, no doubt, have their own favorite solutions.

A Solution to Problem 1 (From the shortlist)

Let the number of blue points with x-coordinate i be a_i, and let the number of blue points with y-coordinate i be b_i. Our task is to show that $a_0 a_1 \cdots a_{n-1} = b_0 b_1 \cdots b_{n-1}$, and to accomplish this we will show that $a_1, a_1, \ldots, a_{n-1}$ is a permutation of $b_0, b_1, \ldots, b_{n-1}$.

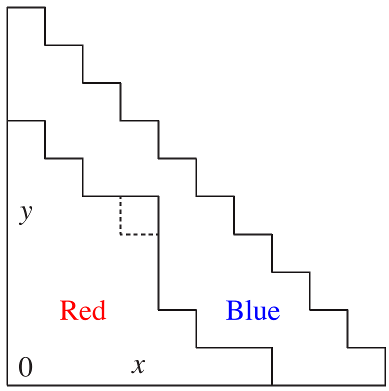

Figure 2: The induction step removes a 'maximal' red point (x, y)

We prove this by induction on the number of red points. Base case: The result is trivial when there are no red points: $a_i = b_i$. Induction step: Choose a red point (x, y) with $x + y$ maximal (see Fig. 2). Hence $a_x = b_y = n-1-x-y$. If we change this red point to blue, then we have a configuration with fewer red points, with all blue rows and columns unchanged except that the values of a_x and b_y increase by 1. So from the induction hypothesis we have that $a_0, a_1, \ldots, a_{n-1}$, with a_x replaced by $a_x + 1$, is a permutation of $b_0, b_1, \ldots, b_{n-1}$, with b_y replaced by $b_y + 1$. Since $a_x = b_y$, it follows that $a_0, a_1, \ldots, a_{n-1}$ is a permutation of $b_0, b_1, \ldots, b_{n-1}$, as required.

A Solution to Problem 2 (From the shortlist)

In Fig. 3, A is the midpoint of arc EAF, so CA bisects $\angle ECF$. Now, since $OA = OC$, $\angle AOD = \frac{1}{2}\angle AOB = \angle OAC$ so OD is parallel to IA and

$ODAI$ is a parallelogram. Hence $AI = OD = OE = AF$ since $OEAF$ (with diagonals bisecting each other at right angles) is a rhombus. Thus

$$\begin{aligned} \angle IFE &= \angle IFA - \angle EFA &= \angle AIF - \angle ECA \\ &= \angle AIF - \angle ICF &= \angle IFC. \end{aligned}$$

Therefore, IF bisects $\angle EFC$ and I is the incentre of triangle CEF. Condition $\angle AOB < 120°$ ensures that I is internal to triangle CEF.

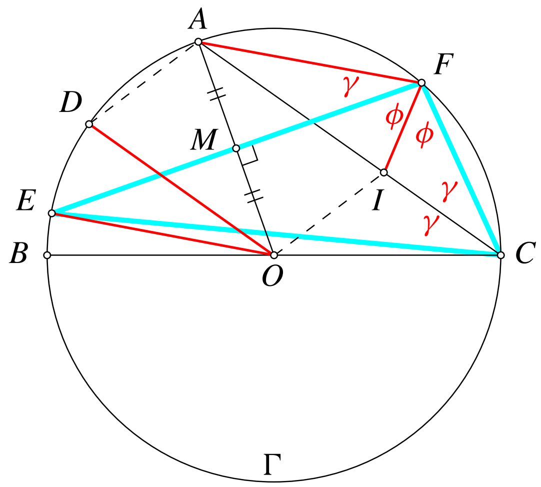

Figure 3: Given configuration and auxiliary lines

A Solution to Problem 3 (Bulgaria at IMO 2002, without number theory)

Assume m, n is such a pair. Clearly $n < m$.

Step 1. We claim that $f(x) = x^m + x - 1$ is exactly divisible in $\mathbb{Z}[x]$ by $g(x) = x^n + x^2 - 1$. Indeed, since $g(x)$ is monic, the division algorithm gives

$$f(x)/g(x) = q(x) + r(x)/g(x)$$

where $\deg(r) < \deg(g)$. The remainder term $r(x)/g(x)$ tends to zero as $x \to \infty$; on the other hand, it is, by assumption, an integer at infinitely many integers a. Thus $r(a)/g(a) = 0$ infinitely often, and so $r \equiv 0$. The claim follows; and in particular, we note that $f(a)/g(a)$ is an integer for all integers a.

Step 2. Let $m = n + k$ with $k \geq 1$. Then $f(x) = x^k g(x) + (1 - x)h(x)$ where $h(x) = x^{k+1} + x^k - 1$. So $g(x)$ also divides $(1-x)h(x)$, and because $g(1) \neq 0$, it also divides $h(x)$. Thus, $k+1 \geq n$, and since $n \geq 3$ is assumed, we even have $k \geq 2$. For any $x \in (0, 1)$ it follows that $g(x) \geq h(x)$ by considering them termwise, with equality if and only $k + 1 = n$ and $k = 2$.

Step 3. Since $g(0) = -1 < 0 < 1 = g(1)$, we know that $g(x)$ has a real root $\alpha \in (0, 1)$, which is also a root of $h(x)$ because $g(x)$ divides $h(x)$. Thus, $g(\alpha) = h(\alpha)$, which together with the preceding step implies $k + 1 = n$ and $k = 2$, yielding $(m, n) = (5, 3)$.

Finally, the identity $a^5 + a - 1 = \left(a^3 + a^2 - 1\right)\left(a^2 - a + 1\right)$ shows that $(m, n) = (5, 3)$ is indeed a solution.

A Solution to Problem 4 Note that if d is a divisor of n then so is n/d, so that the sum

$$s = \sum_{1 \le i < k} d_i\, d_{i+1} = n^2 \sum_{1 \le i < k} \frac{1}{d_i\, d_{i+1}} \le n^2 \sum_{1 \le i < k} \left(\frac{1}{d_i} - \frac{1}{d_{i+1}} \right) < \frac{n^2}{d_1} = n^2.$$

Note also that $(d_2, d_{k-1}, d_k) = (p, n/p, n)$, where p is the least prime divisor of n.

If $n = p$ then $k = 2$ and $s = p$, which divides n^2.

If n is composite then $k > 2$, and $s > d_{k-1}d_k = n^2/p$. If such an s were a divisor of n^2 then also n^2/s would be a divisor of n^2. But $1 < n^2/s < p$, which is impossible because p is the least prime divisor of n^2.

Hence, the given sum is a divisor of n^2 if and only if n is prime.

A Solution to Problem 5 (From the shortlist)

The equation has the solutions $f(x) = 0$ for all x, $f(x) = 1/2$ for all x, and $f(x) = x^2$ for all x. These make both sides equal to 0, to 1, and to $\left(x^2 + z^2\right)\left(y^2 + t^2\right)$ respectively. We claim that there are no other solutions.

Suppose the equation holds. Then setting $x = y = z = 0$ gives $2f(0) = 2f(0)\,(f(0) + f(t))$. In particular $2f(0) = 4f(0)^2$ and so $f(0) = 0$ or $f(0) = 1/2$. If $f(0) = 1/2$ we get $f(0) + f(t) = 1$ and so f is identically $1/2$.

Suppose then that $f(0) = 0$. Then setting $z = t = 0$ in the equation gives $f(xy) = f(x)f(y)$, that is f is multiplicative. In particular $f(1) = f(1)^2$ and so $f(1) = 0$ or 1. If $f(1) = 0$ then $f(x) = f(x)f(1) = 0$ for all x.

So we may suppose that $f(0) = 0$ and $f(1) = 1$. Setting $x = 0$ and $y = t = 1$, the equation gives

$$f(-z) + f(z) \quad = \quad 2f(z)$$

and so $f(-z) = f(z)$ for each z, that is f is an even function. So it suffices to show that $f(x) = x^2$ for positive x. Note that $f\left(x^2\right) = f(x)^2 \ge 0$; as f is an even function, $f(y) \ge 0$ for all y.

Now put $t = x$ and $z = y$ in the equation to get

$$f\left(x^2 + y^2\right) \quad = \quad (f(x) + f(y))^2.$$

This shows that $f\left(x^2+y^2\right) \geq f(x)^2 = f\left(x^2\right)$. Hence if $u \geq v \geq 0$ then $f(u) \geq f(v)$, that is f is an increasing function on the positive reals.

Set $y = z = t = 1$ in the equation to yield

$$f(x-1) + f(x+1) \quad = \quad 2\,(f(x)+1)\,.$$

By induction on n, it readily follows from this that $f(n) = n^2$ for all non-negative integers n. As f is even, $f(n) = n^2$ for all integers n, and further, as f is multiplicative, $f(a) = a^2$ for all rationals a. Suppose that $f(x) \neq x^2$ for some positive x. If $f(x) < x^2$ take a rational a with $x > a > \sqrt{f(x)}$. Then $f(a) = a^2 > f(x)$, but $f(a) \leq f(x)$ as f is increasing. This is a contradiction. A similar argument shows that $f(x) > x^2$ is impossible. Thus $f(x) = x^2$ for all positive x, and since f is even, $f(x) = x^2$ for all real x.

A Solution to Problem 6 (Columbia at IMO 2002)

In Fig. 4, we consider two circles Γ_i and Γ_j with centers O_i and O_j respectively ($i \neq j$). Let M_{ij} be the midpoint of O_iO_j. From the given conditions ($n \geq 3$ and no line meets more than two of the circles), Γ_i and Γ_j do not intersect. Consider the points P on the circumference of Γ_i for which the tangent to Γ_i through P cuts Γ_j. They form two arcs of length θ_{ij} (the arcs P_1P_2 and P_3P_4 in Fig. 4). Because the circles have radius 1, we find

$$\sin\theta_{ij} = \frac{1}{O_iM_{ij}} = \frac{2}{O_iO_j}$$

Since these tangents cut at most one circle C_j, these arcs are disjoint for fixed i while varying j.

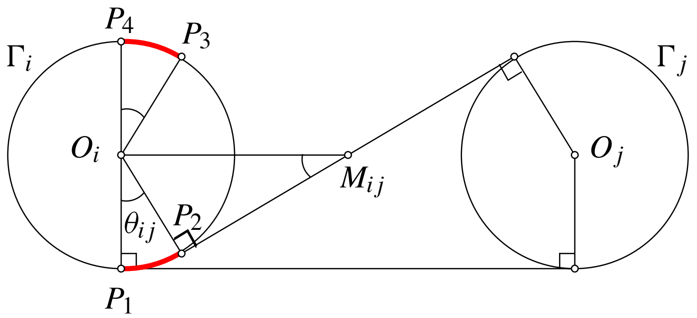

Figure 4: The arcs generated by tangents on Γ_i cutting Γ_j

Define α_i as the total length of arcs of Γ_i consisting of points P for which the tangent to Γ_i through P cuts another circle:

$$\alpha_i \quad = \quad \sum_{j\neq i} 2\theta_{ij}$$

Now consider the points Q on Γ_i for which the tangent to Γ_i lies on one side of all the circles. These points form an arc β_i disjoint from the θ_{ij}. The β_i are nonzero only on the "outer" circles of the configuration, as part of the convex hull (see Fig. 5).

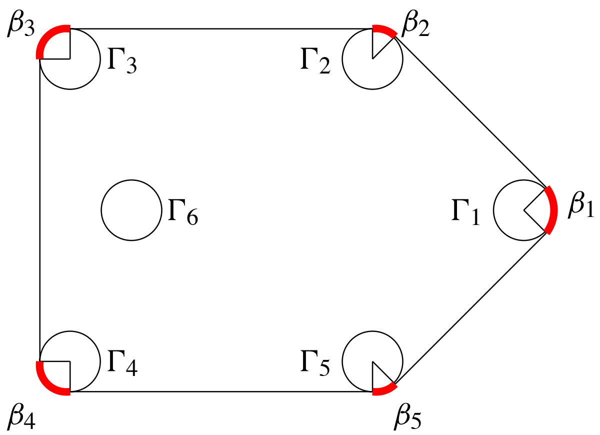

Figure 5: The arcs on the convex hull of $n = 6$ unit circles

Hence,

$$\begin{aligned}\sum_{i=1}^{n} \alpha_i + \beta_i &\leq n(2\pi) \\ \sum_{i=1}^{n} \beta_i &= 2\pi\end{aligned}$$

Thus,

$$\sum_{i=1}^{n} \alpha_i \leq 2(n-1)\pi$$

Finally, we calculate

$$\begin{aligned}\sum_{1\leq i<j\leq n} \frac{8}{O_i O_j} &= \sum_{1\leq i<j\leq n} 4\sin\theta_{ij} \\ &\leq \sum_{1\leq i<j\leq n} 4\theta_{ij} \\ &= \sum_{i=1}^{n} \sum_{j\neq i} 2\theta_{ij} \\ &= \sum_{i=1}^{n} \alpha_i \\ &\leq 2(n-1)\pi\end{aligned}$$

D Statistics

Score	Freq.	Rel.freq.	Cum.freq	Cum.rel.freq.	Medal
42	3	0.63%	3	0.63%	
41	0		3	0.63%	
40	0		3	0.63%	
39	0		3	0.63%	
38	0		3	0.63%	
37	0		3	0.63%	**G**
36	6	1.25%	9	1.88%	**o**
35	2	0.42%	11	2.30%	**l**
34	3	0.63%	14	2.92%	**d**
33	0		14	2.92%	
32	4	0.84%	18	3.76%	
31	5	1.04%	23	4.80%	
30	5	1.04%	28	5.85%	
29	11	2.30%	39	8.14%	
28	18	3.76%	57	11.90%	**S**
27	6	1.25%	63	13.15%	**i**
26	14	2.92%	77	16.08%	**l**
25	8	1.67%	85	17.75%	**v**
24	13	2.71%	98	20.46%	**e**
23	14	2.92%	112	23.38%	**r**
22	20	4.18%	132	27.56%	
21	12	2.51%	144	30.06%	
20	15	3.13%	159	33.19%	**B**
19	8	1.67%	167	34.86%	**r**
18	9	1.88%	176	36.74%	**o**
17	14	2.92%	190	39.67%	**n**
16	10	2.09%	200	41.75%	**z**
15	19	3.97%	219	45.72%	**e**
14	13	2.71%	232	48.43%	
13	16	3.34%	248	51.77%	
12	17	3.55%	265	55.32%	
11	15	3.13%	280	58.46%	
10	13	2.71%	293	61.17%	
9	14	2.92%	307	64.09%	
8	26	5.43%	333	69.52%	**N**
7	18	3.76%	351	73.28%	**o**
6	8	1.67%	359	74.95%	**n**
5	18	3.76%	377	78.71%	**e**
4	15	3.13%	392	81.84%	
3	17	3.55%	409	85.39%	
2	21	4.38%	430	89.77%	
1	28	5.85%	458	95.62%	
0	21	4.38%	479	100.00%	

Table 1: Frequencies of final scores

Award	**Number of problems scoring ≥ 5**							**Total**
	6	**5**	**4**	**3**	**2**	**1**	**0**	
Gold	3	12	24					39
	8%	31%	62%					
Silver			31	42				73
			42%	58%				
Bronze				38	77	5		120
				32%	64%	4%		
None					8	112	127	247
					3%	45%	51%	
Total	3	12	55	80	85	117	127	479
	1%	3%	11%	17%	18%	24%	27%	

Table 2: Frequency of number of problems "solved" (score ≥ 5) per award

Problem	**Score**							
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	179	39	11	11	8	15	61	155
	37%	8%	2%	2%	2%	3%	13%	32%
2	173	46	1	4	0	6	129	120
	36%	10%	0%	1%	0%	1%	27%	25%
3	311	145	1	2	0	4	2	14
	65%	30%	0%	0%	0%	1%	0%	3%
4	106	45	15	38	70	9	20	176
	22%	9%	3%	8%	15%	2%	4%	37%
5	97	159	101	21	0	10	25	66
	20%	33%	21%	4%	0%	2%	5%	14%
6	408	21	25	12	0	1	0	12
	85%	4%	5%	3%	0%	0%	0%	3%
Total	1274	455	154	88	78	45	237	543
	44%	16%	5%	3%	3%	2%	8%	19%

Table 3: Frequency of each score per problem for 479 contestants

E Glossary

Unless stated otherwise, the terminology below refers to the IMO.

Advisory Board (AB) The long-term standing committee which supervises the IMO. It consists of 3 members from past, present, and future IMO hosts, 3 elected members-at-large, and an elected secretary and chair. Members are elected from the *Jury*.

Brigadoon The nickname for the secretive place where the (IMO 2002) *Jury* meets, in total isolation from deputy leaders and contestants, to select and translate the *problems* for the *paper*.

Coordination The process of determining points for the *scripts* written by the contestants. This is a joint effort of the *problem coordinators* and the leaders, who assess the *scripts* of their own contestants, in accordance with the pre-established *marking schemes*.

Coordination Session A meeting at which two *problem coordinators* and up to two representatives of a *team* agree on the scores of the *team*'s contestants for one *problem* of the *paper*. Coordination sessions typically last from five minutes to one hour.

At IMO 2002, all coordination sessions were scheduled on two consecutive days for 45 minutes per session. At past IMOs, three days have often been allocated.

Examination One of the two sessions in which contestants attempt to solve the *problems* of the *paper*. Each examination lasts for four and a half hours. The examinations take place on two consecutive days.

Guide Person accompanying the contestants of a designated *team* on excursions.

At IMO 2002, there were 80 guides and 6 Deputy Chief Guides, headed by Chief Guide Peter Covey-Crump.

Honorable Mention A certificate awarded to a contestant who obtained a perfect score (7 points) on a *problem*, but who did not receive a medal.

At IMO 2002, there were 66 Honorable Mentions.

IC International Committee of the IOI.

IMO International Mathematical Olympiad.

Invigilator Person watching over the proper conduct of the *examinations*.

At IMO 2002, there were 35 invigilators headed by Chief Invigilator Terry Heard, assisted by a Deputy Chief Invigilator.

IOI International Olympiad in Informatics.

ISC IOI Scientific Committee.

Jury The temporary short-term committee that "owns" the IMO. It consists of the leaders of all *teams*, and a chairperson. It debates issues, proposes *motions*, and makes decisions by voting.

At IMO 2002, Adam McBride chaired the Jury of 84 leaders. Adam was assisted by a deputy chair, a secretary, and two extra members, who also happen to be on the *Problem Selection Committee*.

Marking Scheme The guidelines for allocating from 0 to 7 points to a *script*. For each *problem* of the *paper*, a marking scheme is prepared by the *problem coordinators*.

At IMO 2002, the marking schemes were also presented to the *Jury* for approval.

Motion A proposal made by a member of the *Jury* to be decided by voting. A motion must be seconded in order to qualify for voting. All motions are numbered and tracked. Before voting, motions are translated orally into French, German, Russian, and Spanish.

At IMO 2002, about 30 motions were put forward.

Paper One of the two papers with three *problems* for each *examination*. Also known as **exam paper** or **question paper**. Often 'paper' refers to the set of all six *problems* of both *examinations* together. The *problems* cover various fields and degrees of difficulty. Also see *shortlist*.

Problem An assignment in pre-university mathematics to be solved in the *examination*. Also see *shortlist*.

Problem Coordinators The committee established by the host in charge of *coordination*. Typically, four kinds of coordinators are distinguished:

Chief Coordinator person in charge of all problem coordinators;

Senior Coordinator person in charge of the group of coordinators responsible for one of the *problems* in the *paper*; also known as **problem captain**;

Coordinator person who participates in a *coordination session* to agree on the scores of each contestant of a *team* for one of the *problems* of the *paper*;

Coordination Consultant person who can be consulted during *coordination sessions* on specific issues, often in the role as interpreter;

At IMO 2002, Imre Leader was Chief Coordinator, there were 6 Senior Coordinators (4 of them were also in the *Problem Selection Committee*), and 42 coordinators. Each problem group consisted of a Senior Coordinator and

7 coordinators. There were 9 Coordination Consultants, covering Russian, Chinese, Bulgarian, Serbian, Macedonian, Farsi, Greek, Spanish, and Hungarian; German and Romanian were covered by two of the coordinators.

Problem Selection Committee The committee established by the host in charge of collecting the proposed *problems* and preparing a *shortlist* with solutions and comments.

For IMO 2002, this committee had 10 renowned members, with Imre Leader as chair.

Shortlist The list of about thirty *problems* from which the *Jury* will select the *problems* for the *paper*. This list is prepared by the *Problem Selection Committee*, based on submissions from the participating countries. The shortlist must remain confidential until the next IMO.

In case of IMO 2002, 130 *problems* were submitted by 41 countries. The shortlist contained 27 of them, categorized into the four fields of Number Theory, Geometry, Algebra, and Combinatorics.

Script What a contestant has written during the *examination* to solve the *problems* of the *paper*. This must be written on special sheets, which afterwards are photocopied and given to the leaders for assessment.

At IMO 2002, approximately 15 000 pages of script were written, amounting to about 5.2 pages per *problem* per contestant on average.

Team All participants from one invited country, consisting of a **leader**, a **deputy leader** and up to 6 **contestants**. In spite of the term 'team', all contestants compete individually. There is no official team ranking.

At IMO 2002, 84 teams participated with a total of 479 contestants.