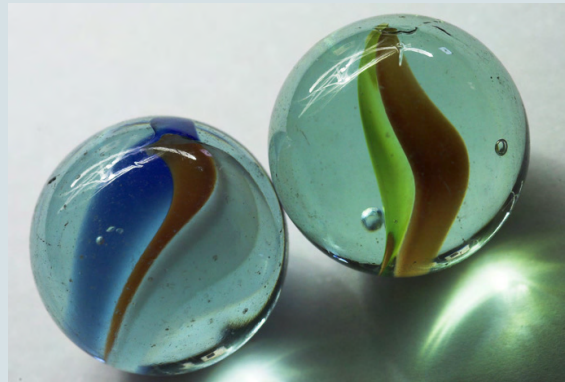


Decisions and Games

Lunch Lecture for *W.I.S.V. Christiaan Huygens* at TU Delft
28 November 2008



Tom Verhoeff



Department of Mathematics & Computer Science

First Game: Choosing Digits



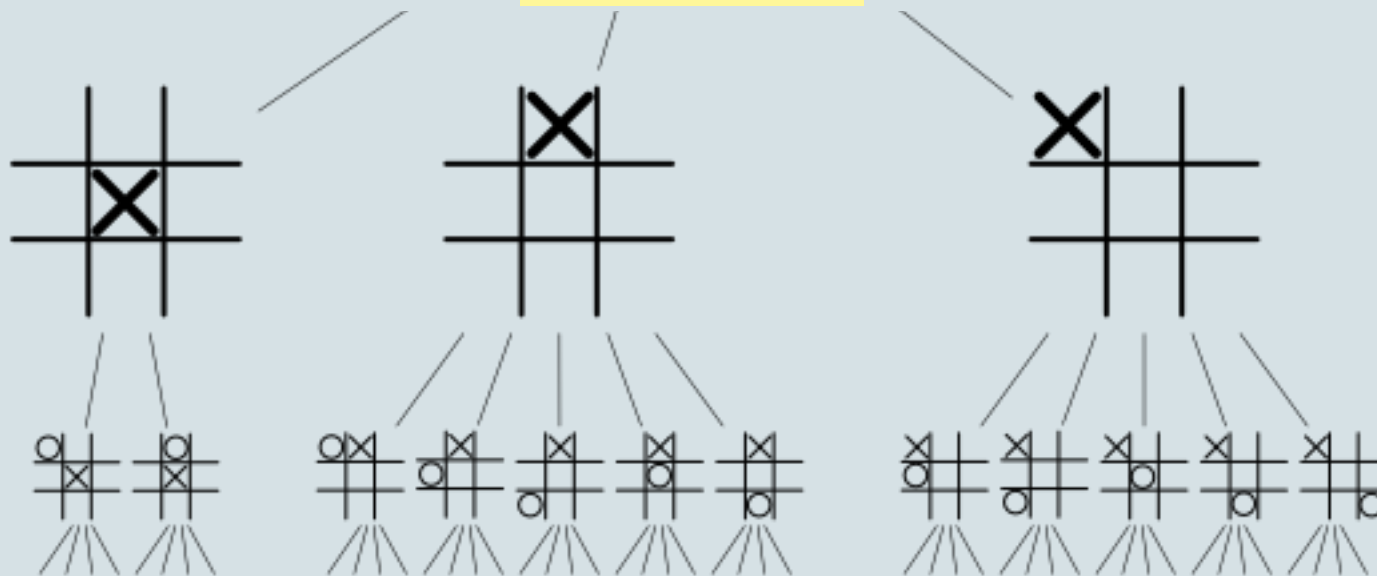
Two players: Choose a digit, taking turns.

Earlier chosen digits are no longer available.

Goal: Having chosen *three* digits that sum to **15**.

Logical Combinations

6	1	8
7	5	3
2	9	4



Analysis can be extremely hard!

Second Game: Choosing Heads or Tails

Bob chooses

0 1

Alice chooses

0 1

0	↑ 1	← 2
1	← 2	↑ 3

Payoff

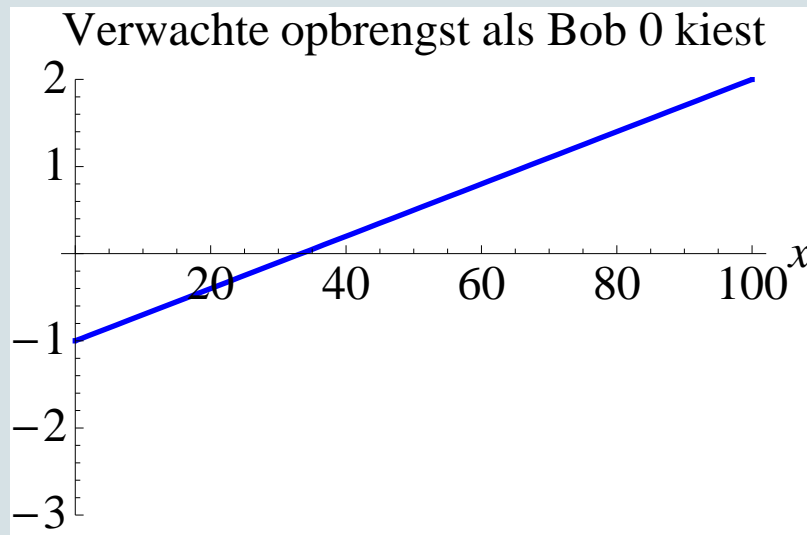
Two players: Each chooses **0** (head) or **1** (tail), *secret* the other.

Goal: Maximize total payoff under repeated play.

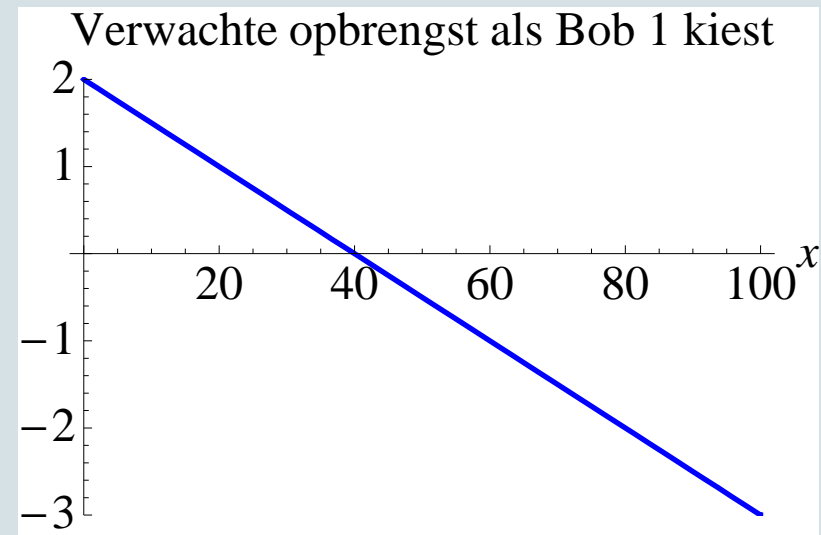
Strategic Bluffing: The Mixed Strategy

Analysis for Alice: she chooses 1 with probability $x\%$.

Alice wants to determine optimal x , not knowing Bob's choice.

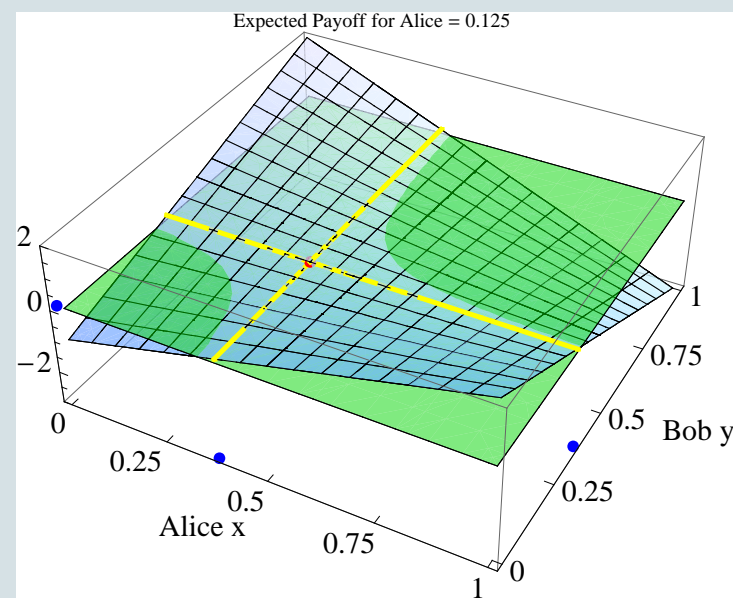
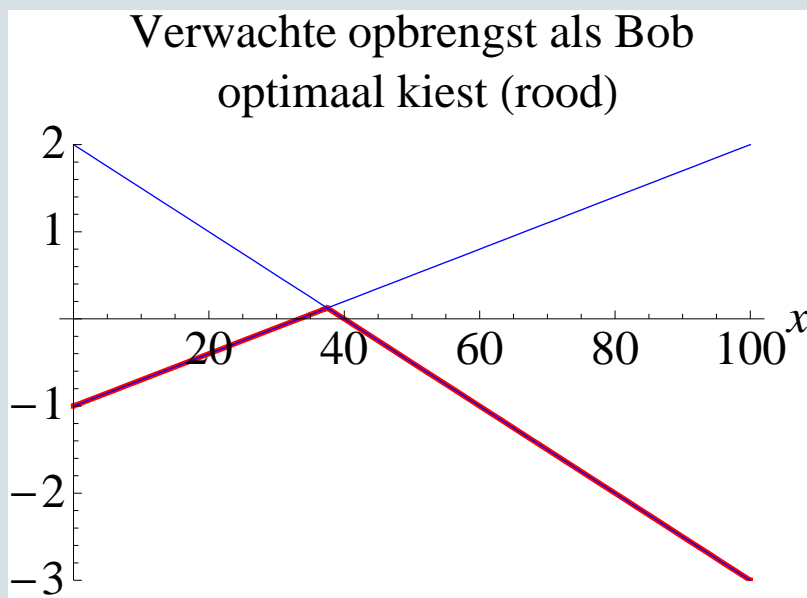


Expected payoff = $+0,5$
voor $x = 50\%$



Expected payoff = $-0,5$
voor $x = 50\%$

Strategic Bluffing: The Nash Equilibrium



Optimal for Alice:

choose 0 with probability $5/8$ & choose 1 with probability $3/8$

Expected payoff for Alice: $+0,125$ or $6\frac{1}{4}\%$ of 2

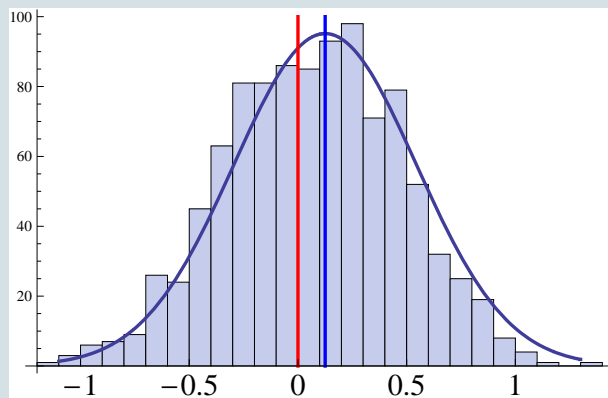
Moreover, this is independent of Bob's choice.

The Effect of Variance

Expected payoff $\mu = 0,125$ with standard deviation of $\sigma = 1,884$

Three times 1000 experiments of N repetitions each

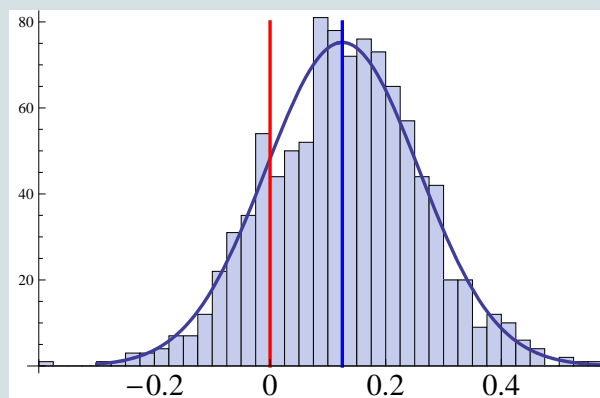
N = 20



$\sigma = 0,42$

38% chance to lose

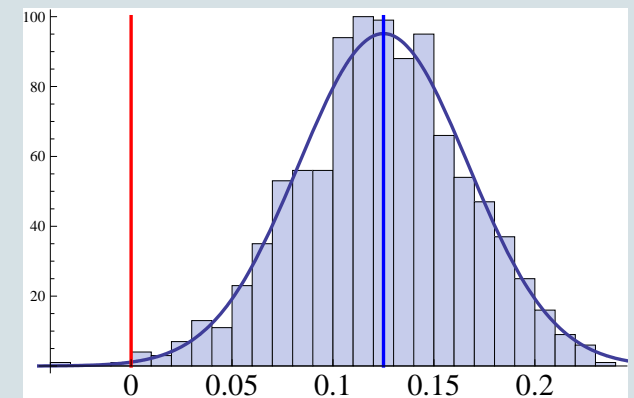
N = 200



$\sigma = 0,13$

17% chance to lose

N = 2000



$\sigma = 0,04$

0,1% chance to lose

Third Game: Choosing Categories

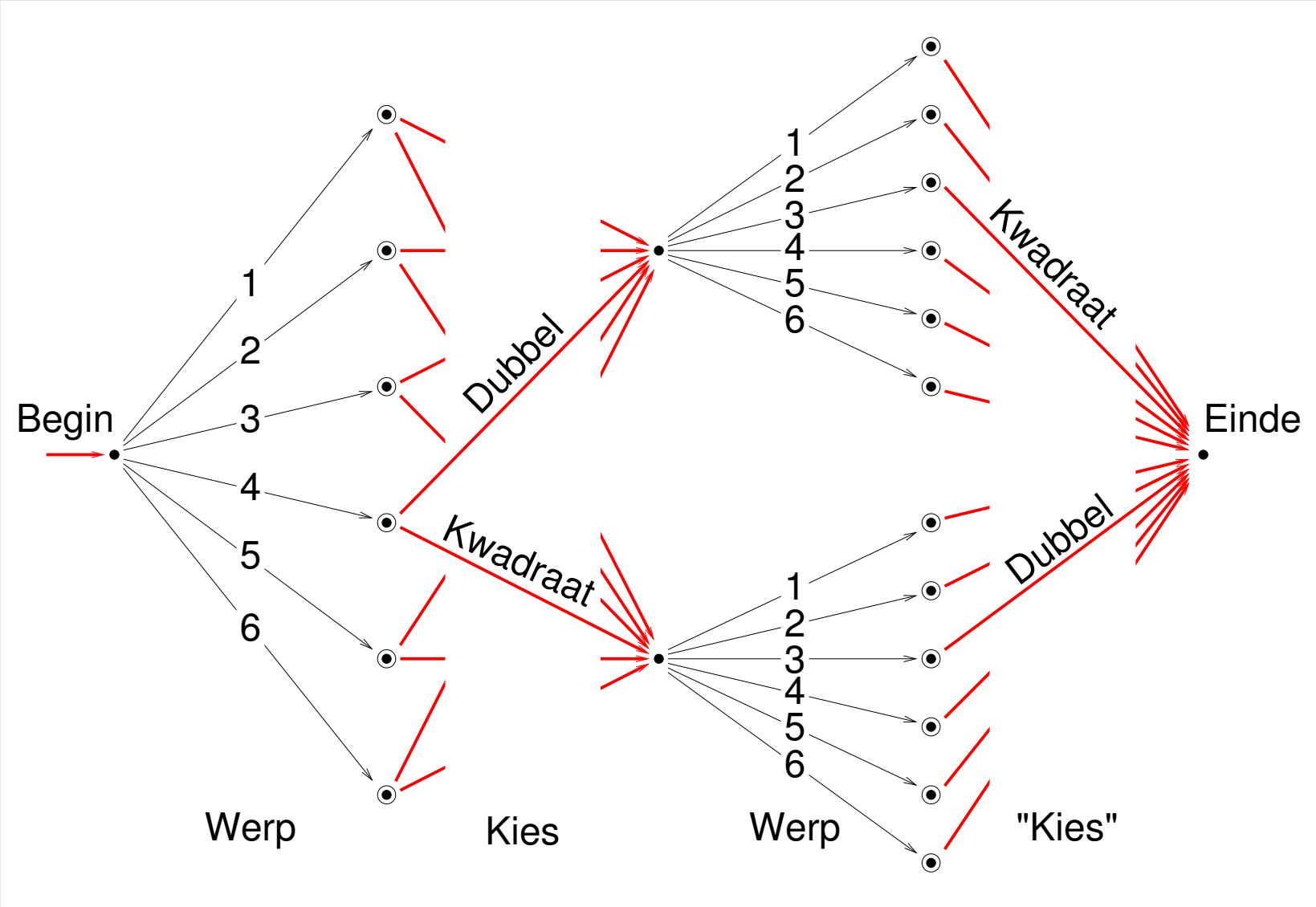


Category	Score
<i>Double</i> (value + value)	...
<i>Square</i> (value × value)	...
<i>Total</i>	...

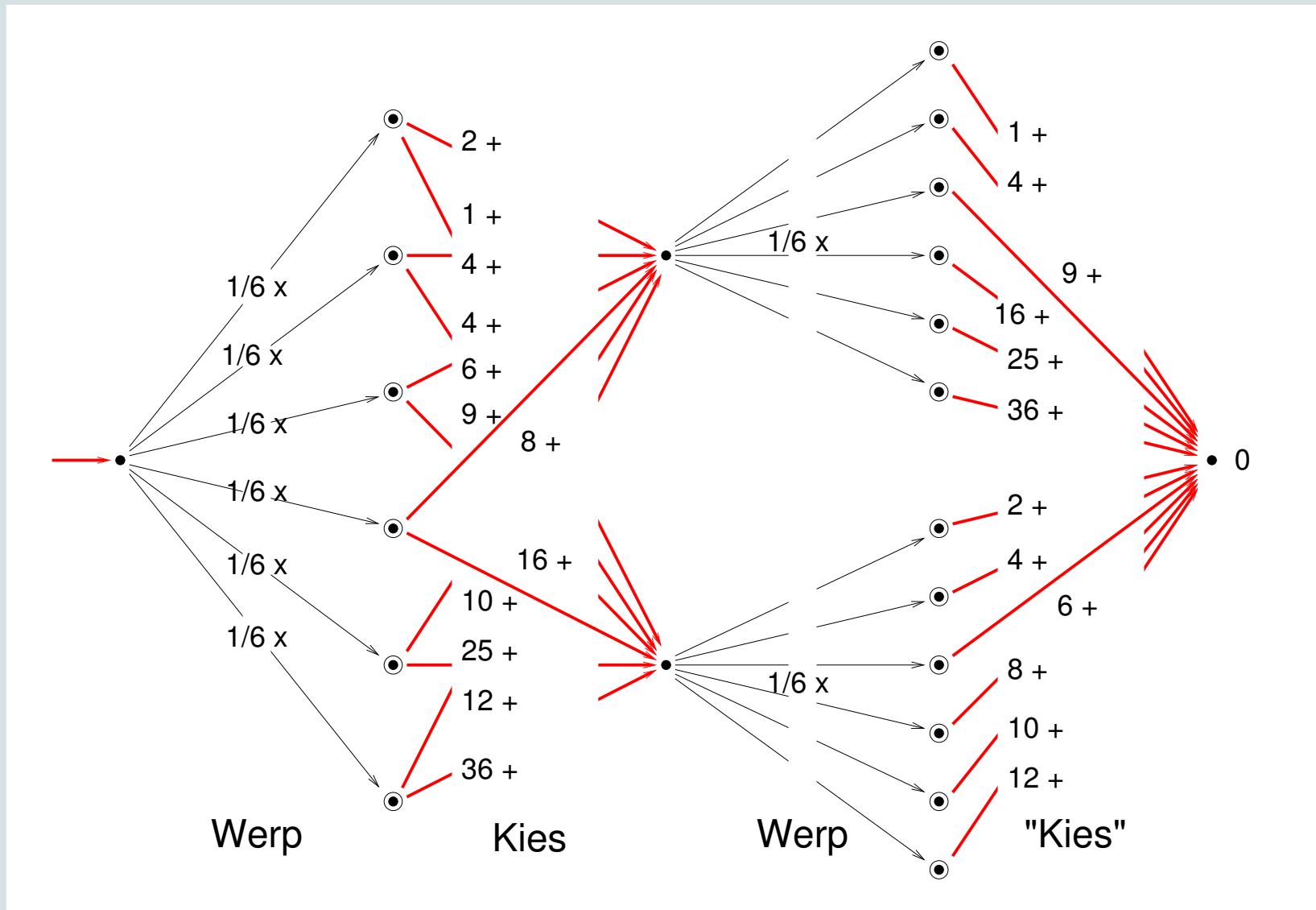
One player: Roll dice, choose *unscored* category, repeat.

Goal: Maximize total score under repeated play.

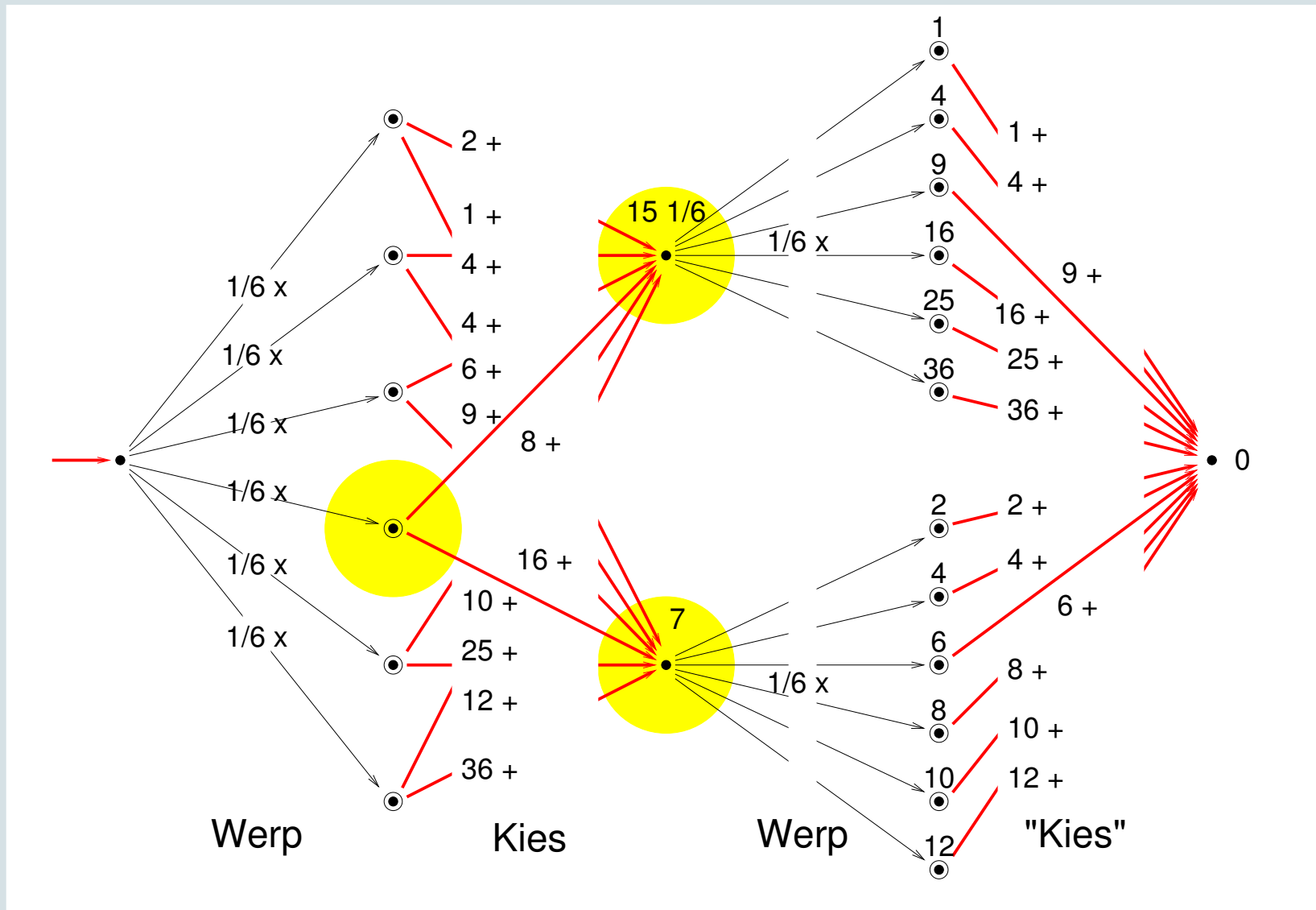
Weighing Chances: MicroYahtzee Game Graph



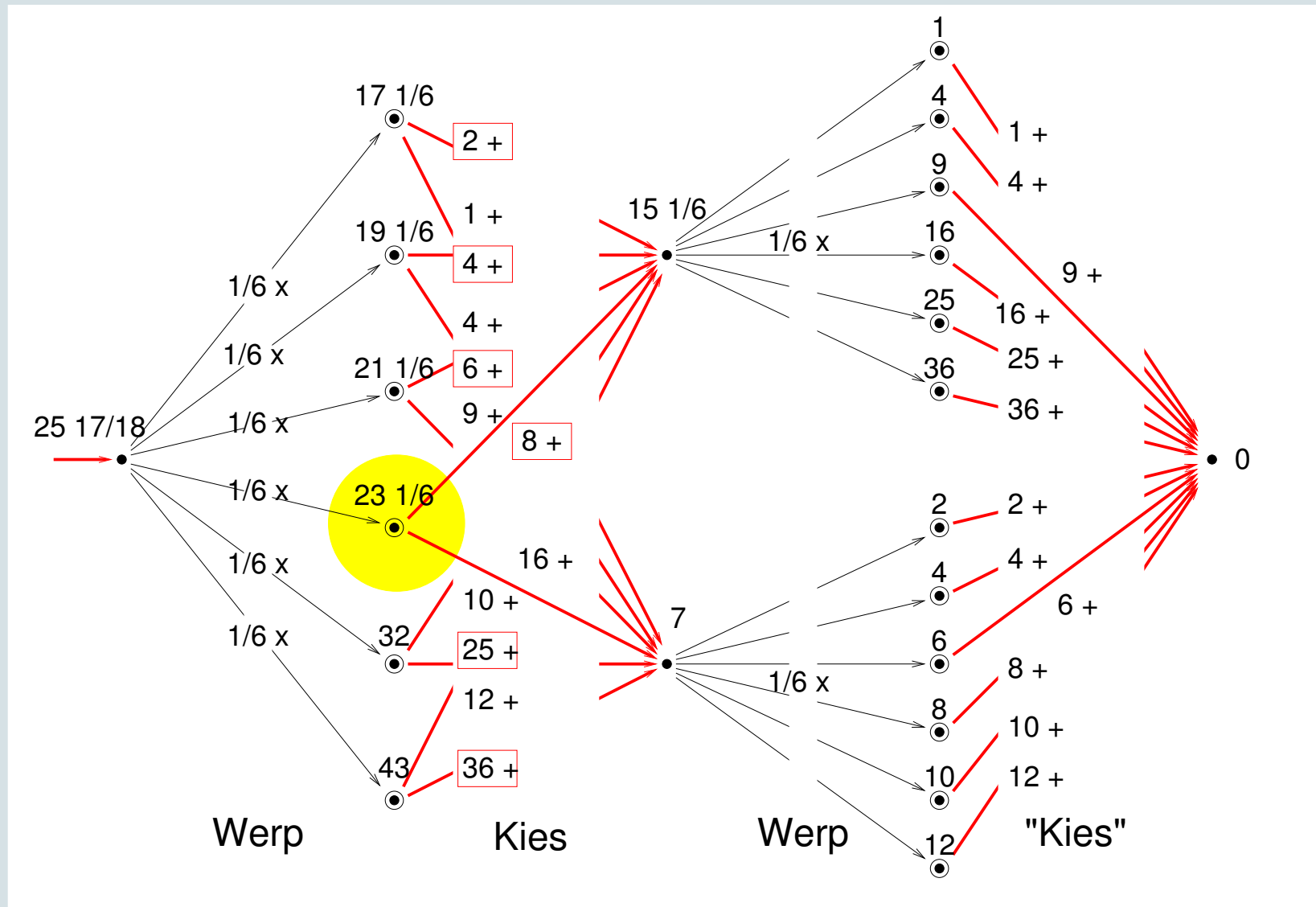
Weighing Chances: MicroYahtzee Expected Scores



Weighing Chances: MicroYahtzee Optimal Scores



Weighing Chances: MicroYahtzee Dilemma Resolved



Weighing Chances: 'Real' Yahtzee







- Score card with 13 categories and 5 dice
- Choose $39\times$: more than 10^9 game states
- Expected total score under optimal play: $254, \dots$ [Verhoeff, 1999]

But variance is high: $\sigma = \pm 60$ (70% in 200–300)

- On-line advice and practicing:

www.win.tue.nl/~wstomv/misc/yahtzee

Yahtzee[®]

UPPER SECTION	HOW TO SCORE	GAME #1	GA #
Aces  = 1	Count and Add Only Aces		
Twos  = 2	Count and Add Only Twos		
Threes  = 3	Count and Add Only Threes		
Fours  = 4	Count and Add Only Fours		
Fives  = 5	Count and Add Only Fives		
Sixes  = 6	Count and Add Only Sixes		
TOTAL SCORE	→		
BONUS <small>If total score is 63 or over</small>	SCORE 35		
TOTAL <small>Of Upper Section</small>	→		
LOWER SECTION			
3 of a kind	Add Total Of All Dice		
4 of a kind	Add Total Of All Dice		
Full House	SCORE 25		
Sm. Straight <small>Sequence of 4</small>	SCORE 30		
Lg. Straight <small>Sequence of 5</small>	SCORE 40		
YAHTZEE <small>5 of a kind</small>	SCORE 50		
Chance	Score Total Of All 5 Dice		
YAHTZEE BONUS	<input type="checkbox"/> FOR EACH BONUS		
	SCORE 100 PER <input type="checkbox"/>		
TOTAL <small>Of Lower Section</small>	→		
TOTAL <small>Of Upper Section</small>	→		
GRAND TOTAL	→		

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Optimal Solitaire Yahtzee: Final Scores per Category

Category	E	SD	% 0
<i>Aces</i>	1.88	1.22	10.84
<i>Twos</i>	5.28	2.00	1.80
<i>Threes</i>	8.57	2.71	0.95
<i>Fours</i>	12.16	3.29	0.60
<i>Fives</i>	15.69	3.85	0.50
<i>Sixes</i>	19.19	4.64	0.53
<i>U. S. Bonus</i>	23.84	16.31	31.88
<i>Three of a Kind</i>	21.66	5.62	3.26
<i>Four of a Kind</i>	13.10	11.07	36.34
<i>Full House</i>	22.59	7.38	9.63
<i>Small Straight</i>	29.46	3.99	1.80
<i>Large Straight</i>	32.71	15.44	18.22
<i>Yahtzee</i>	16.87	23.64	66.26
<i>Chance</i>	22.01	2.54	0.00
<i>Extra Y. Bonus</i>	9.58	34.08	91.76
GRAND TOTAL	254.59	59.61	0.00
<i>Yahtzees Rolled</i>	0.46	0.69	63.24
<i>Jokers Applied</i>	0.04	0.19	96.30

Optimal Solitaire Yahtzee: Distribution of Final Score

Score range	%	Cum.%	
100 – 119	0 %	0 %	
120 – 139	0 %	0 %	
140 – 159	2 %	2 %	■
160 – 179	3 %	5 %	■
180 – 199	9 %	14 %	■
200 – 219	13 %	27 %	■
220 – 239	14 %	41 %	■
240 – 259	20 %	60 %	■
260 – 279	19 %	80 %	■
280 – 299	6 %	86 %	■
300 – 319	5 %	90 %	■
320 – 339	2 %	92 %	■
340 – 359	1 %	93 %	■
360 – 379	1 %	94 %	■
380 – 399	2 %	96 %	■
400 – 419	2 %	98 %	■
420 – 439	1 %	99 %	■
440 – 459	0 %	99 %	
460 – 479	0 %	99 %	
480 – 499	0 %	99 %	

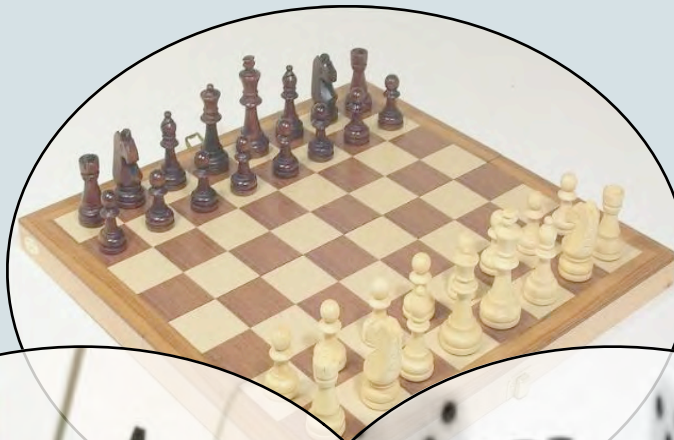
Results based on **simulation** of 10^5 games

Optimal Solitaire Yahtzee: Game with Minimum Score

Turn	Third Roll	Score	in Category
1	1 4 4 5 5	1	<i>Aces</i>
2	1 2 3 5 5	2	<i>Twos</i>
3	1 1 2 2 6	0	<i>Four of a Kind</i>
4	1 2 2 4 6	0	<i>Yahtzee</i>
5	1 1 2 2 6	0	<i>Threes</i>
6	1 2 2 3 3	0	<i>Fours</i>
7	1 2 2 3 3	0	<i>Fives</i>
8	1 2 2 3 3	0	<i>Full House</i>
9	1 2 2 3 3	0	<i>Sixes</i>
10	1 1 2 3 3	0	<i>Large Straight</i>
11	1 1 2 2 3	9	<i>Chance</i>
12	4 5 5 6 6	0	<i>Three of a Kind</i>
13	5 6 6 6 6	0	<i>Small Straight</i>
		12	GRAND TOTAL

Three Kinds of Situations for Making Decisions

Logical combinations (complete information)

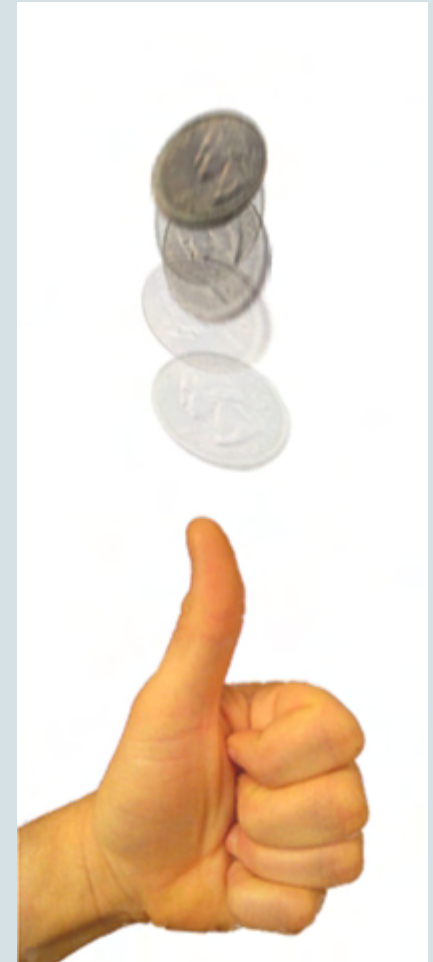


Strategic bluffing (secrets)

Weighing chances (fortune)

Wise Lessons

1. 'Playful' mathematical techniques may help at making good choices in diverse situations.
2. For repeated strategical choices it can be optimal to toss a (well-chosen) 'coin' (cf. the so-called *Mixed Nash Strategy*, preventing predictability and exploitation).
3. For repeated tests of fortune it can be optimal to make a (well-chosen) fixed choice (cf. the so-called *Markov Decision Processes*).
4. Large variance requires patience for increased certainty (due to the factor $1/\sqrt{N}$).



Decide Playfully, Consult a Mathematician!

TU/e Technische Universiteit
Eindhoven
University of Technology



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