Liveness in free-choice systems

Natalia Sidorova

Free-choice nets (def.1)

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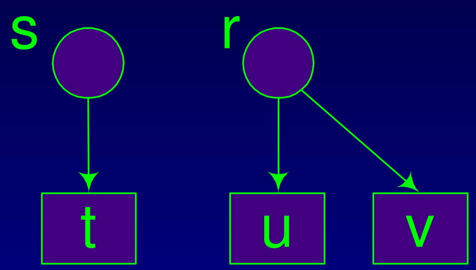

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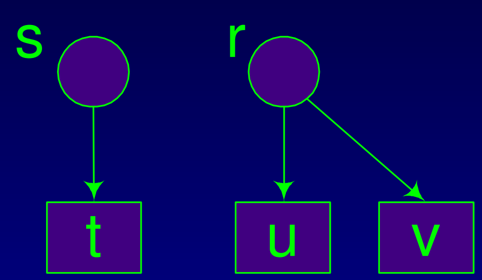

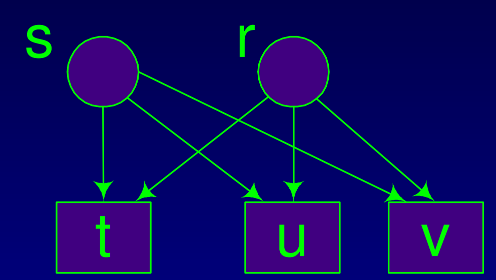

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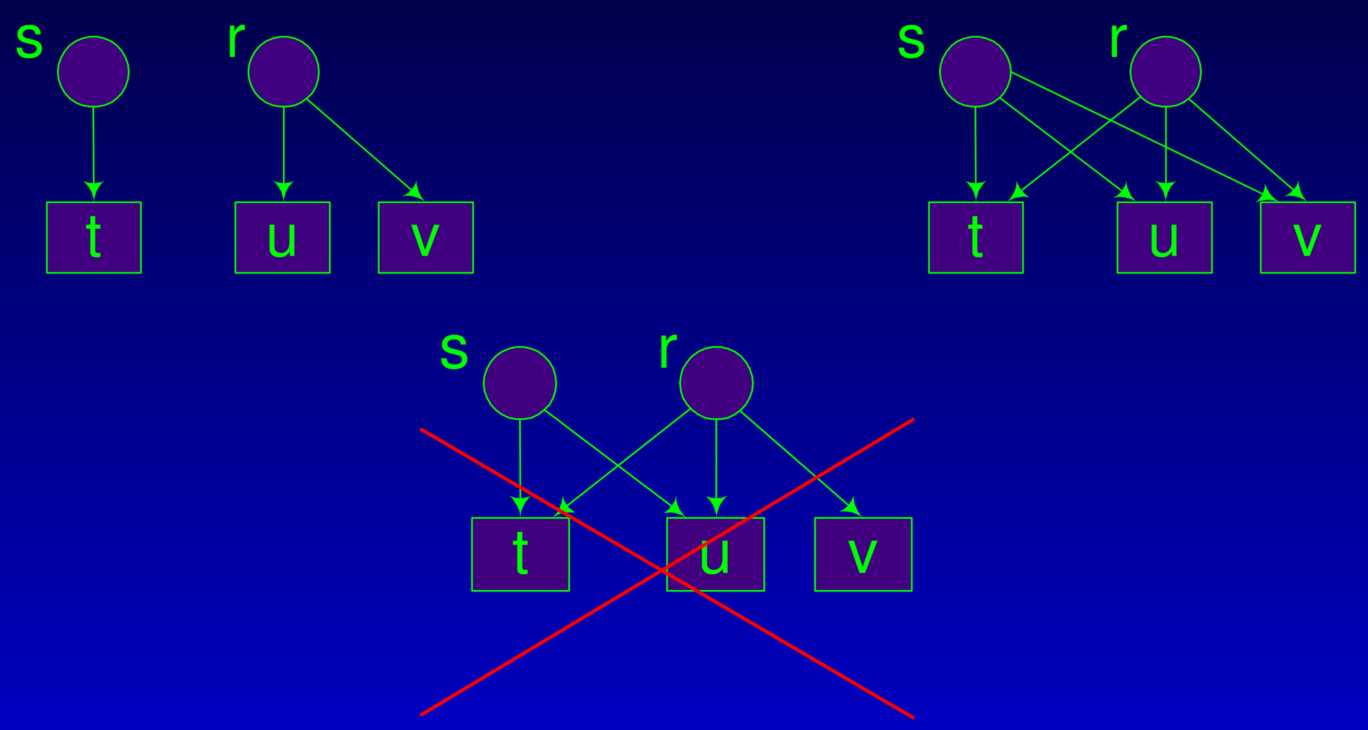

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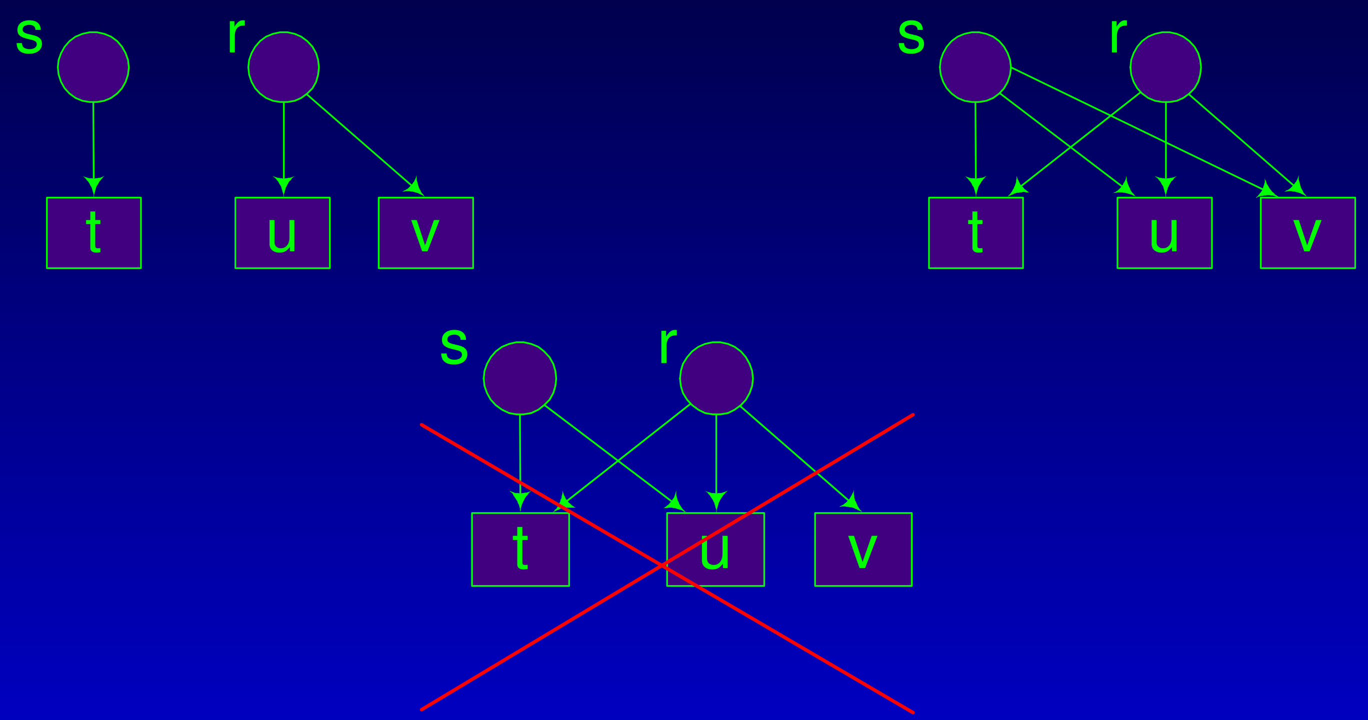

Property of free-choice nets (4.3)

If a marking M enables some transition of $s^\bullet$ then it enables every transition of $s^\bullet$.

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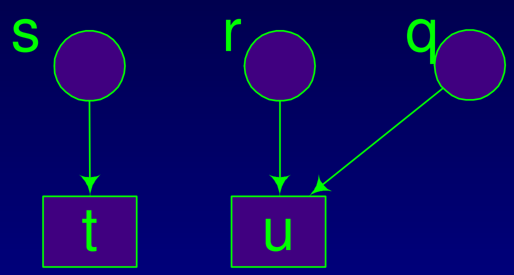

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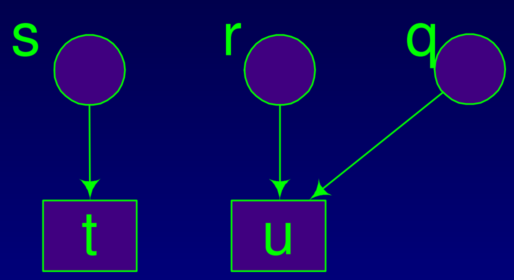

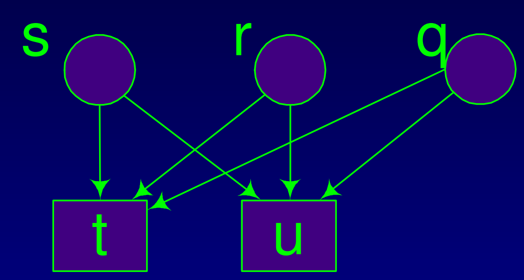

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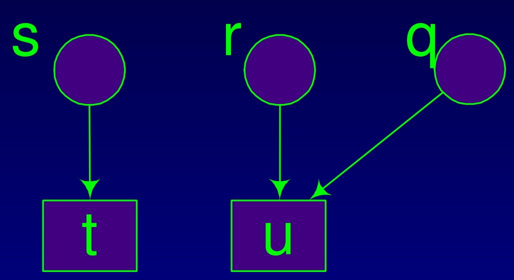

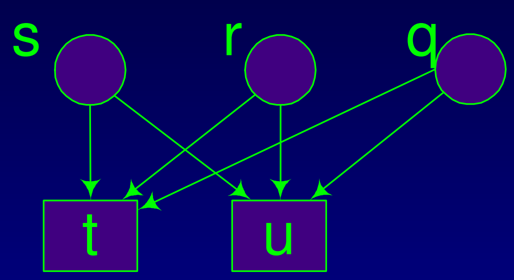

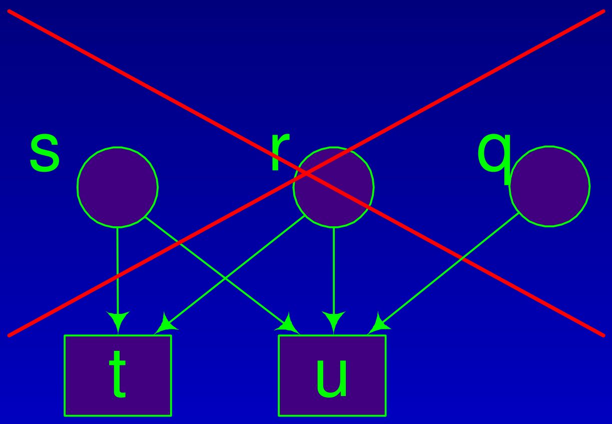

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Since $u \in s^\bullet$, $u \in r^\bullet$ as well. Hence, $r \in {}^\bullet u$ and
${}^\bullet t \subseteq {}^\bullet u$.
Proof for ${}^\bullet u \subseteq {}^\bullet t$ is similar.
Proof for ($\Leftarrow$) is similar to ($\Rightarrow$).

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A net $N = \langle S, T, F \rangle$ is free-choice iff
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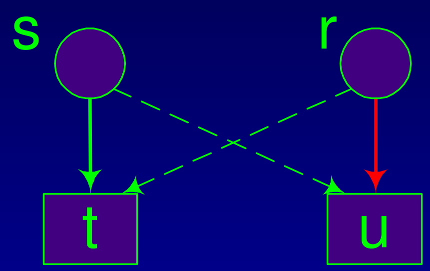

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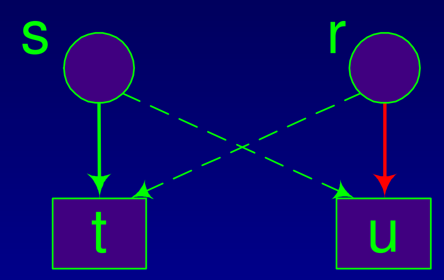

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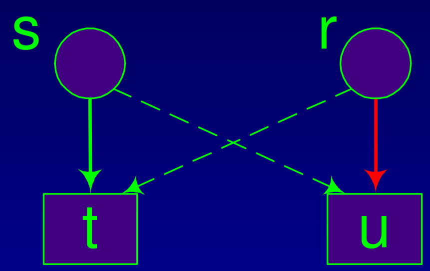

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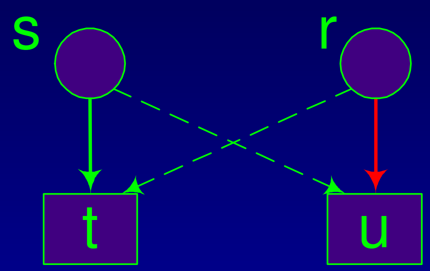

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Exercise: Prove that def.1 = def.3.

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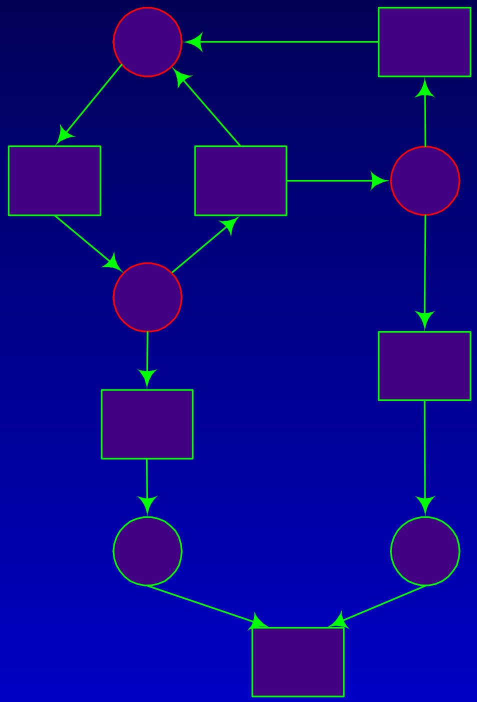

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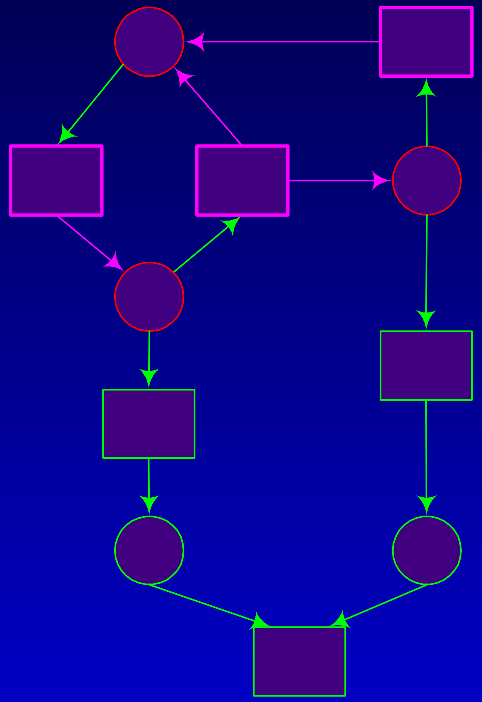

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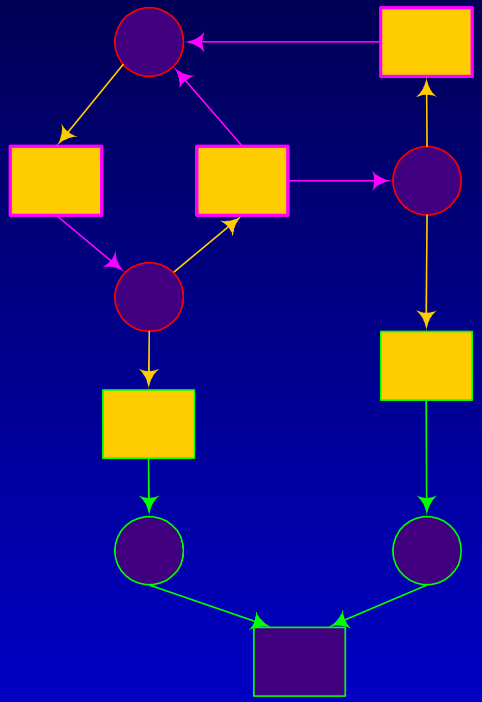

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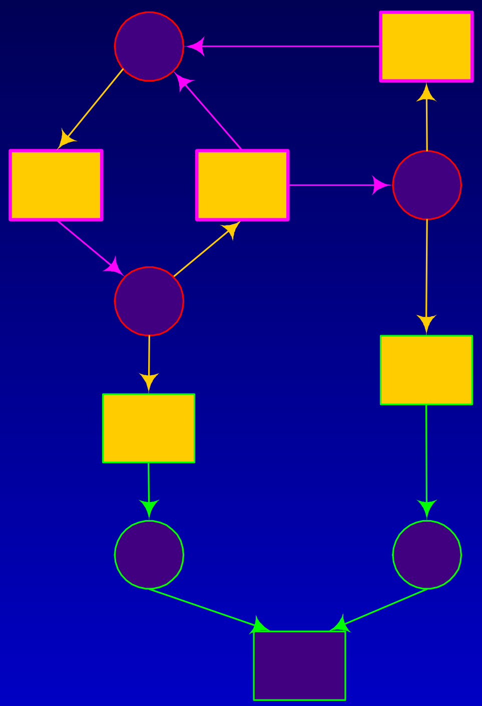

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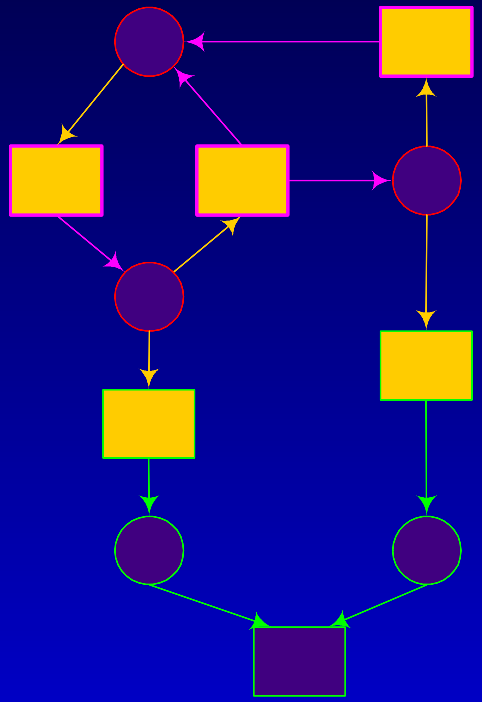

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Live systems have no unmarked siphons.

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If all proper siphons are marked at every reachable marking, the system is deadlock-free.

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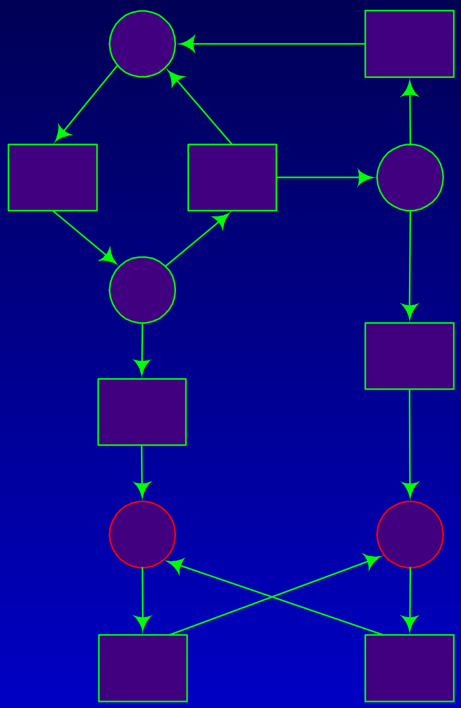

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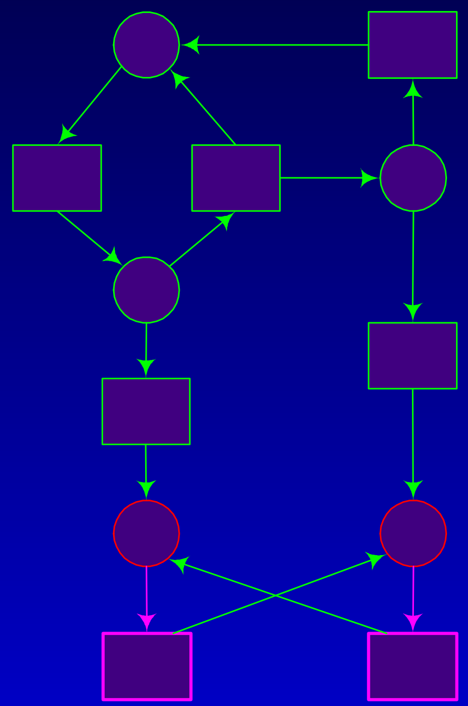

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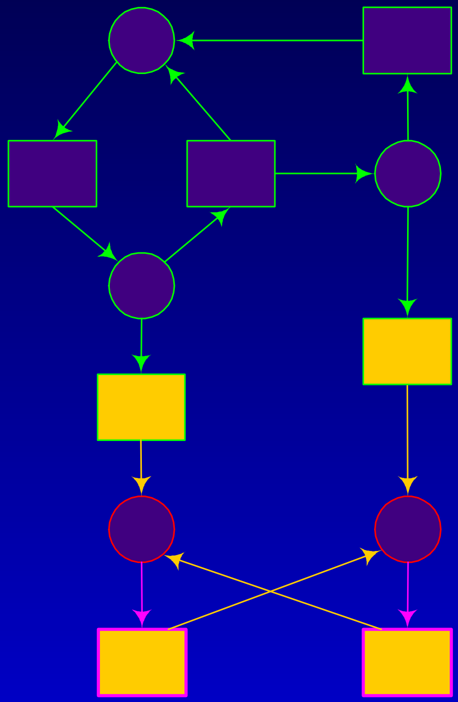

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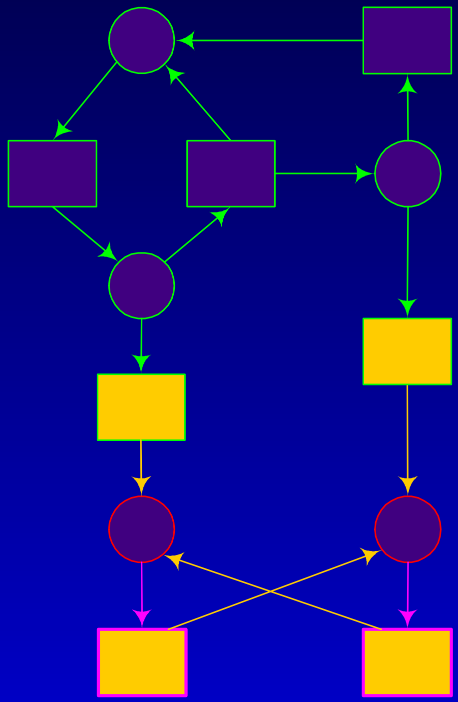

Marked traps remain marked.

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Every marked trap remains marked.

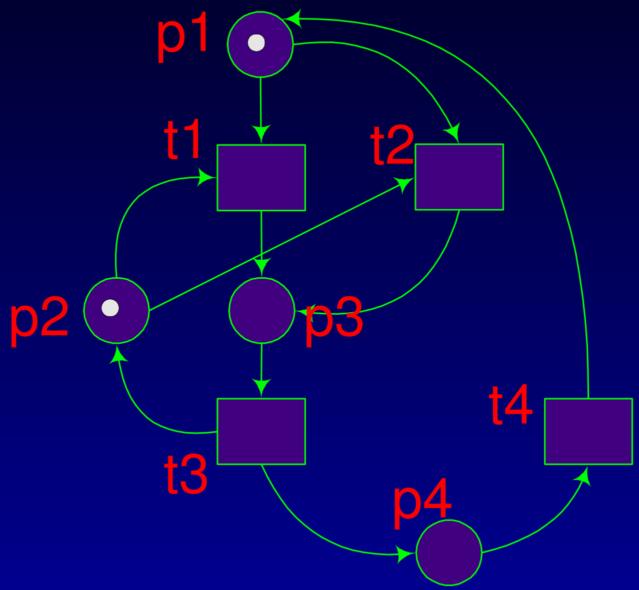
Hence, R includes no initially marked trap. $\square$

Commoner’s Theorem

A free-choice system is live if and only if every proper syphon includes an initially marked trap.

Property of free-choice systems:
Place-liveness and liveness coincide in free-choice systems.

Example



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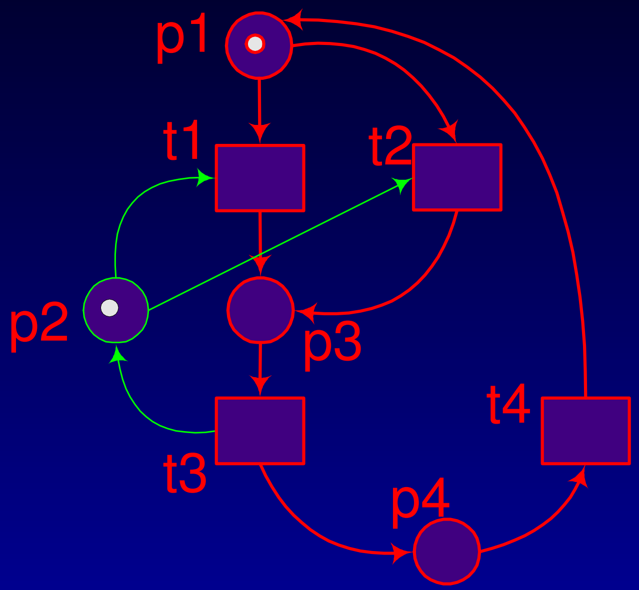

siphons ($^\bullet R \subseteq R^\bullet$):

$R_1 = \{p_1, p_4, p_3\}$,

$R_2 = \{p_1, p_2, p_3, p_4\}$

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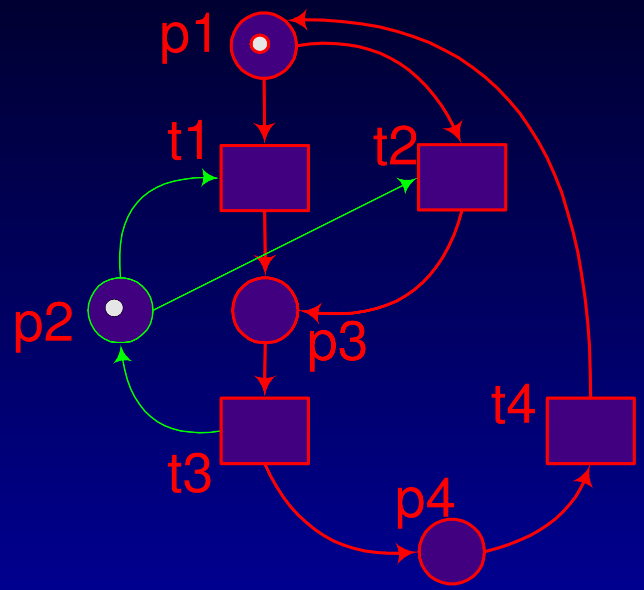

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Is there marked trap Q: $Q \subseteq R_1$?

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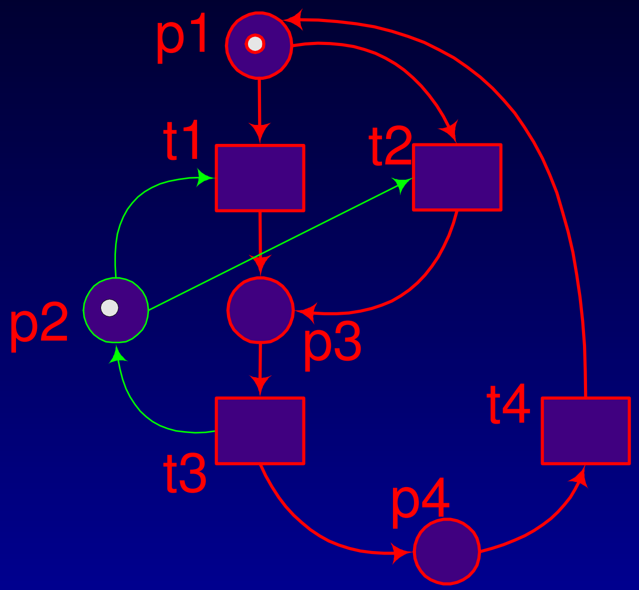

Q is a trap. Hence, $Q^\bullet \subseteq {}^\bullet Q$.

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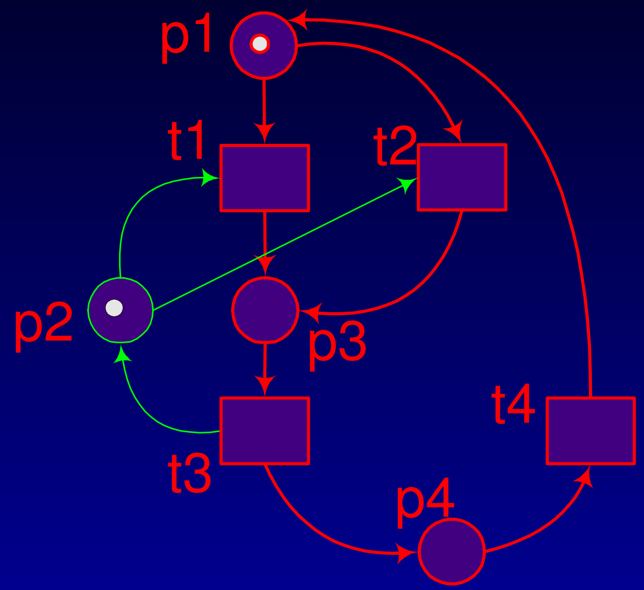

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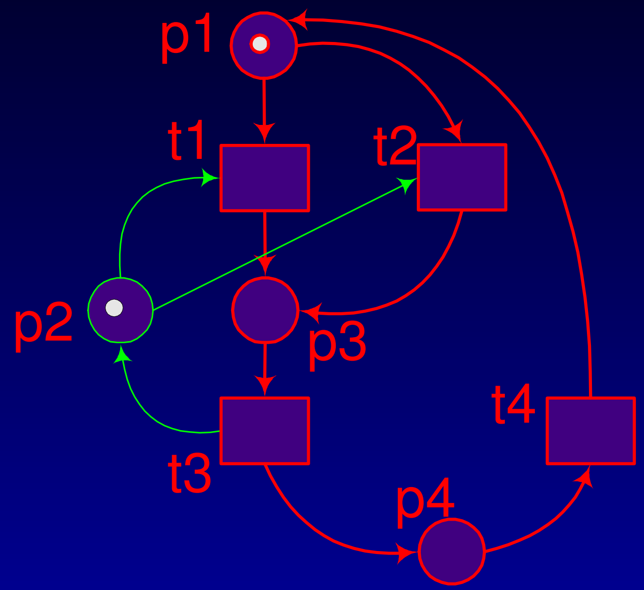

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$Q = R_1$, Q is marked, N is live.

Algorithm for deciding liveness

Given: a free-choice system (N, M_0).
Question: is N not live?

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Non-liveness problem of free-choice systems is NP-complete.

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Every proper siphon contains a minimal one.

A free-choice system is live if and only if every minimal siphon includes an initially marked trap.

Clusters

The cluster of a node x, $[x]$, is a minimal set of nodes such that

- $x \in [x]$,
- if a place $s \in [x]$ then $s^\bullet \subseteq [x]$,
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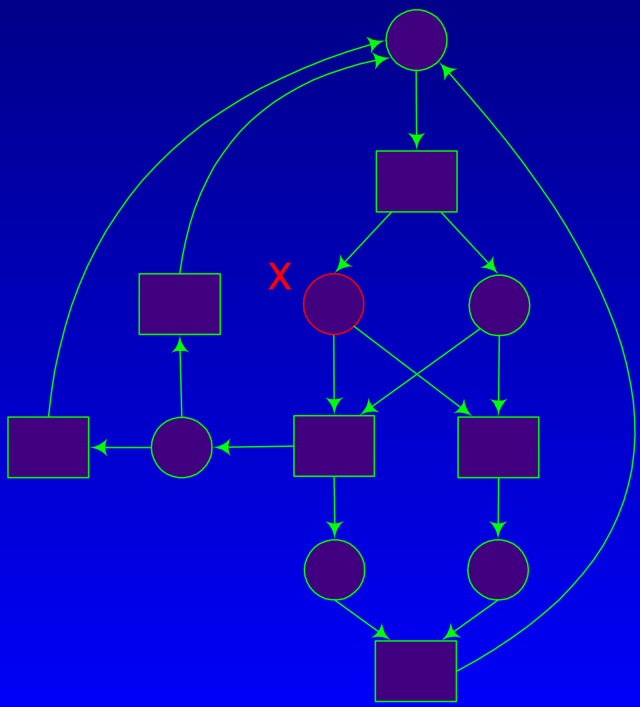

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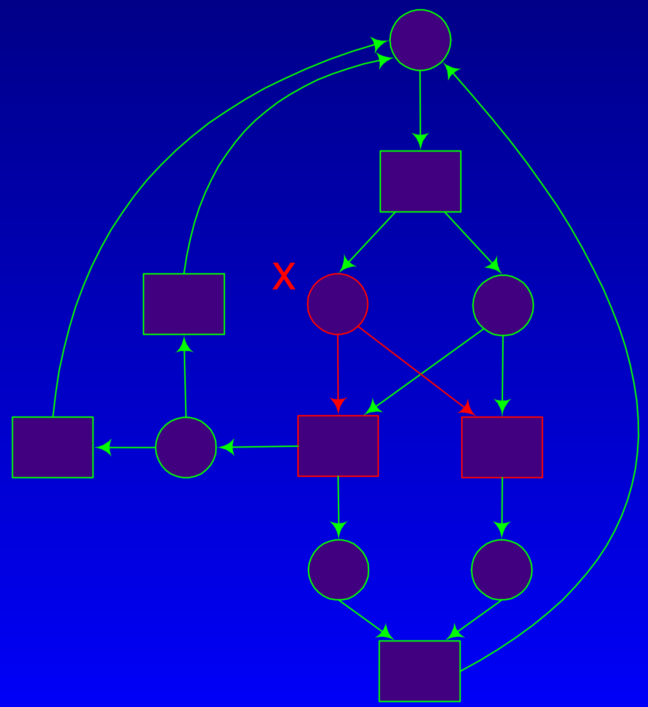

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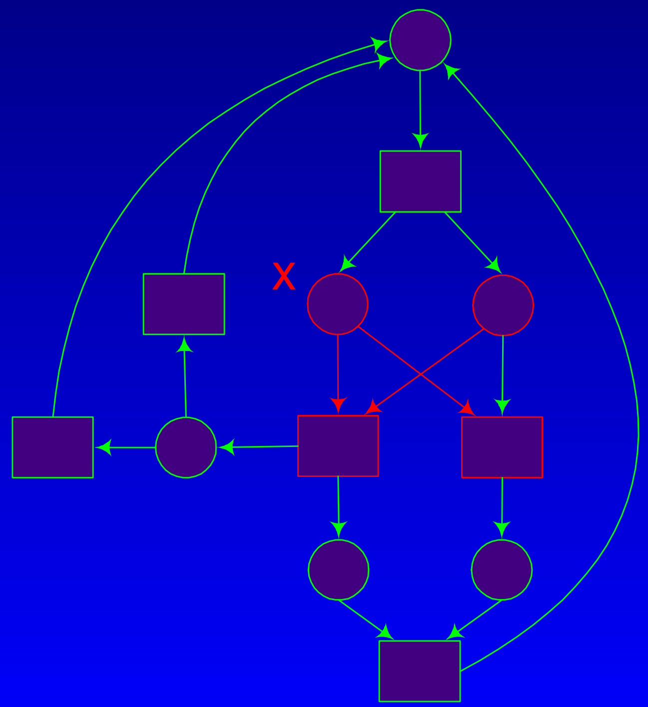

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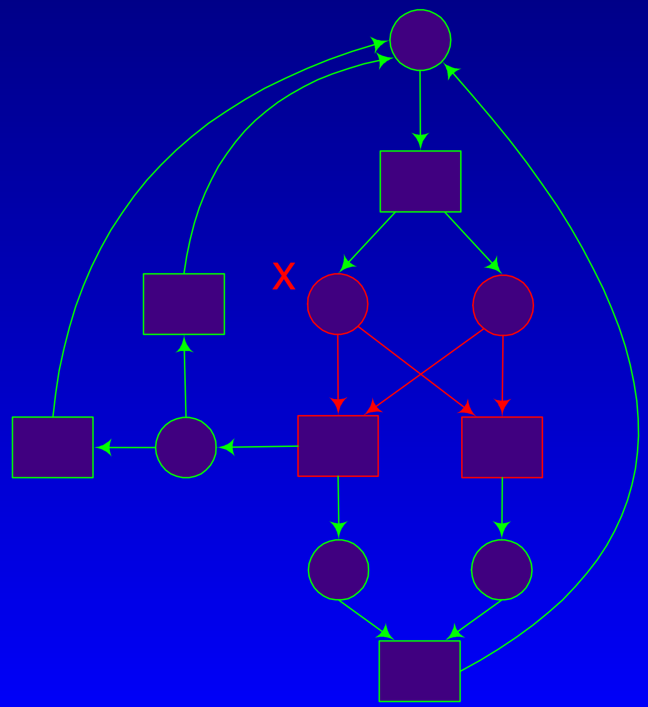

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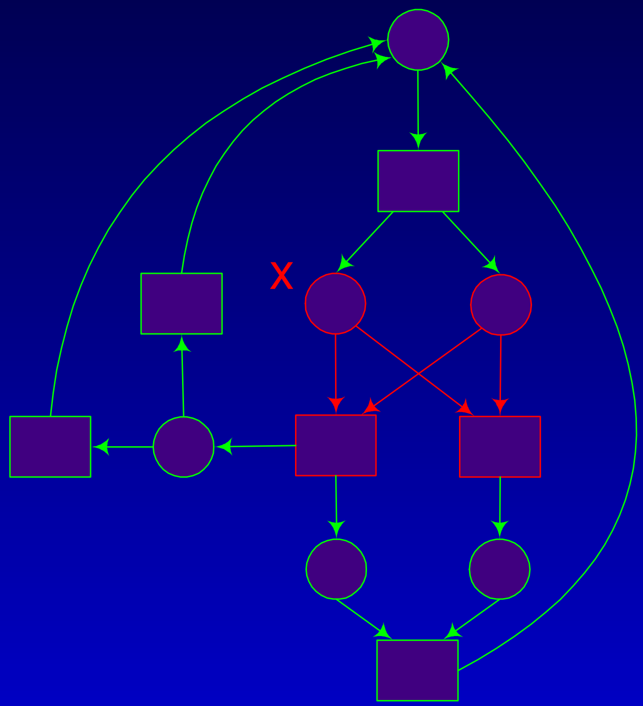

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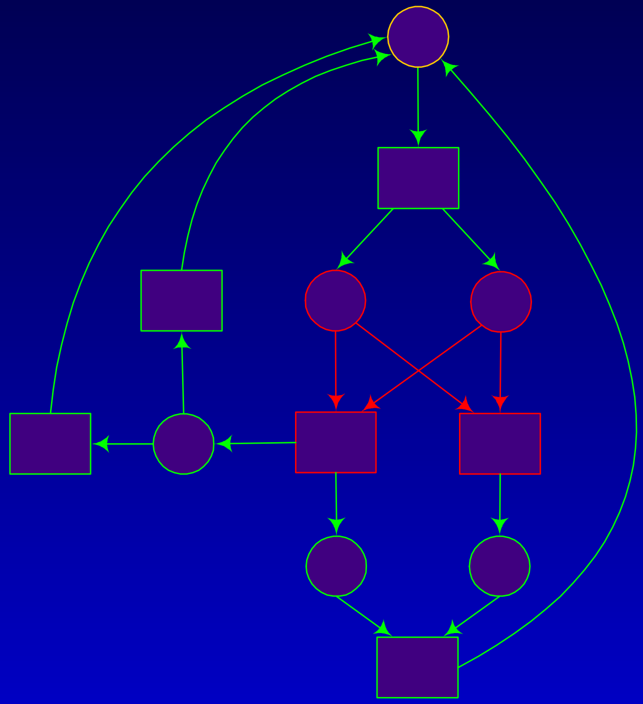

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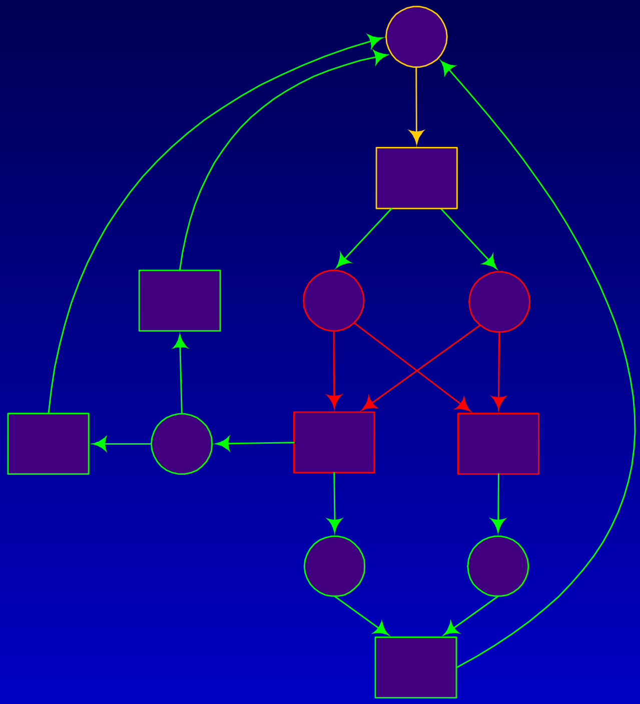

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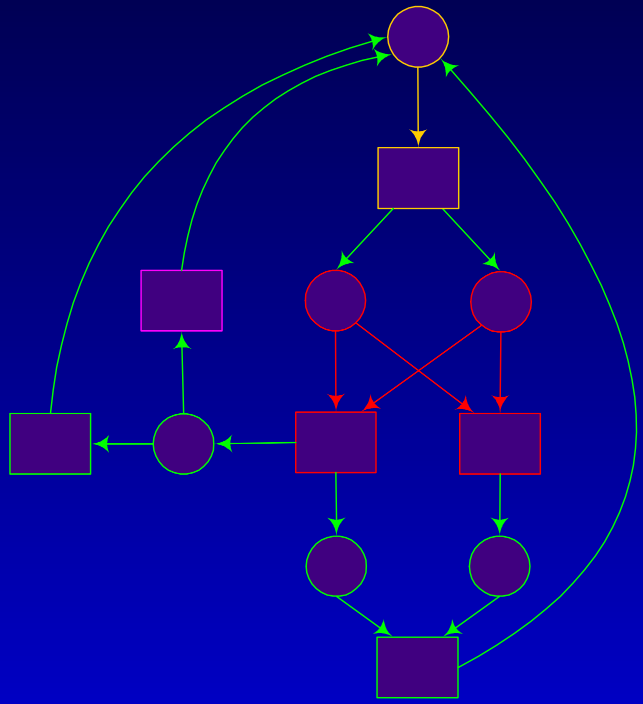

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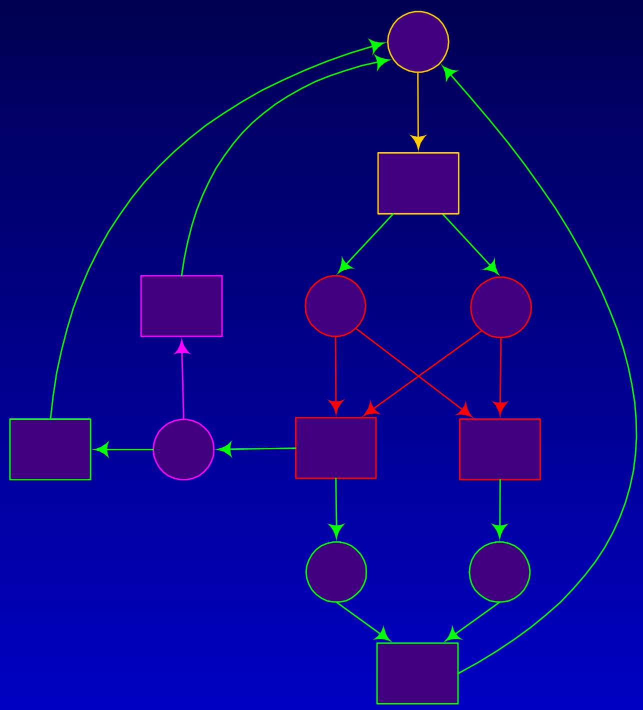

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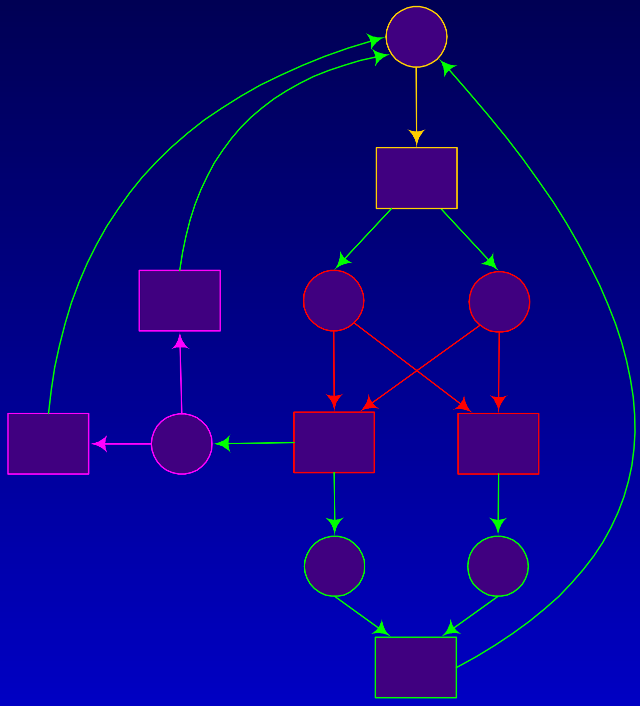

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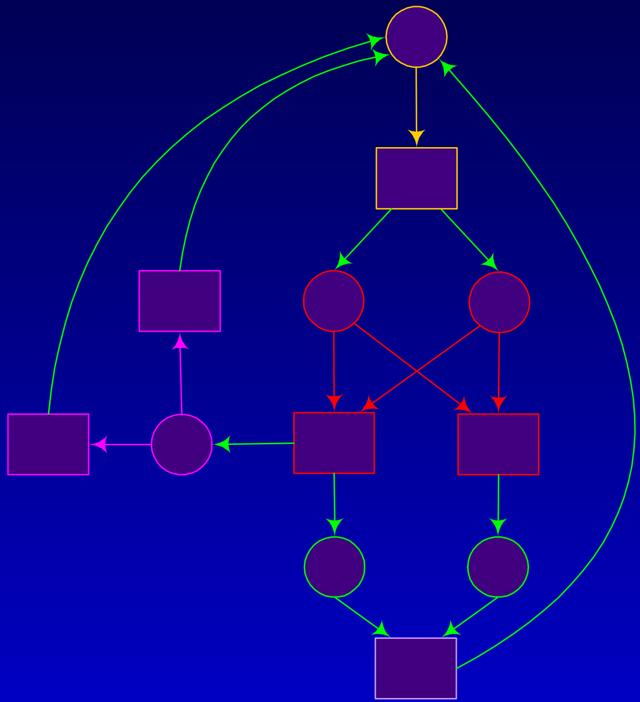

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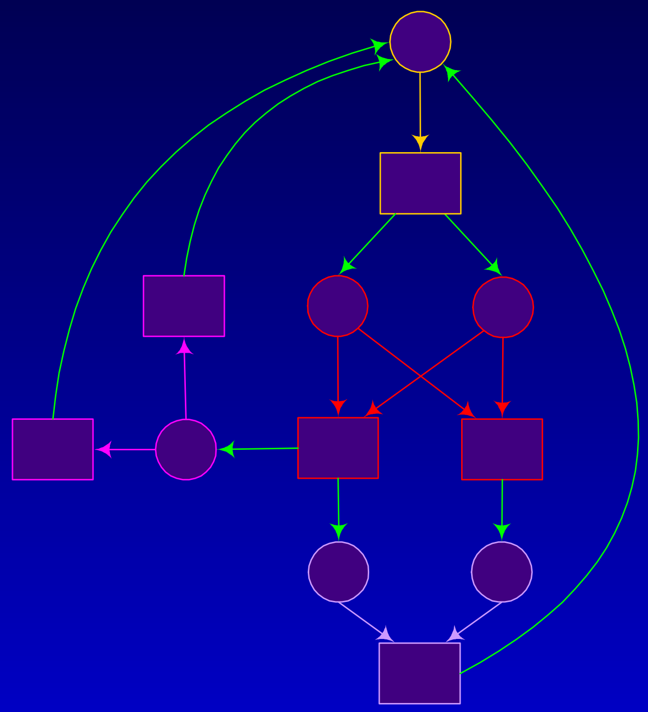

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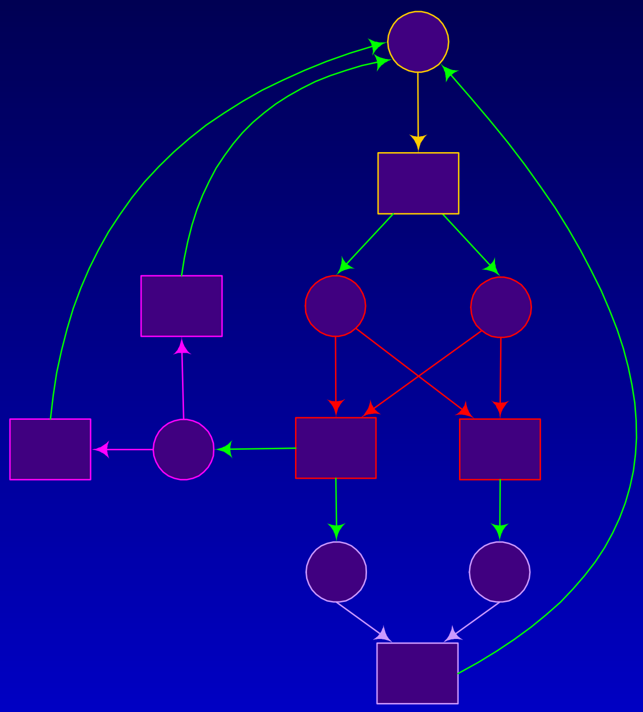

In free-choice nets, each place of a cluster c is connected to every transition t of c.

Property of clusters

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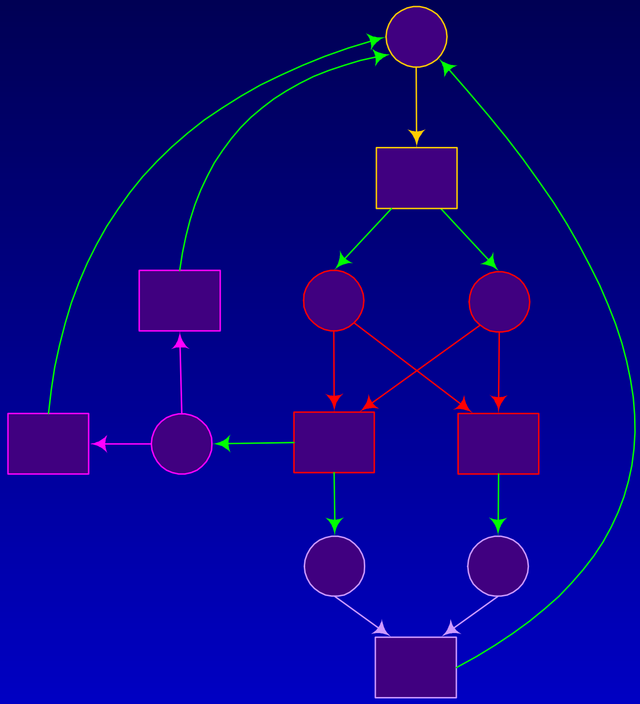

If a marking of a free-choice net enables a transition t, then it enables every transition of the cluster $[t]$.

Minimal siphons (2)

A nonempty set of places R of a free-choice net N is a minimal siphon iff:

1. every cluster c of N contains at most one place of R and
2. the subnet generated by $R \cup {}^\bullet R$ is strongly connected.

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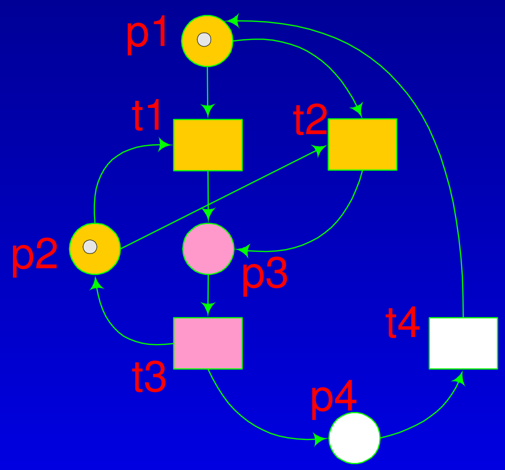


Literature

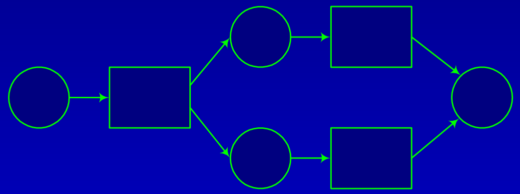
Chapter 4 in [Desel, Esparza]

Structural analysis for Workflow nets

Workflow nets

A Petri net N is a Workflow net (WF-net) iff:

- N has two special places (or transitions): an initial place (transition) i: $^{\bullet}i = \emptyset$, and a final place (transition) f: $f^{\bullet} = \emptyset$.
- For any node $n \in (P \cup T)$ there exists a path from i to n and a path from n to f.



Applications: business process modelling, software engineering, $\ldots$.

Soundness

Desired property: proper completion

A WF-net N is sound iff:

- For every marking M reachable from $[i]$, there exists a firing sequence leading to $[f]$.
- There are no dead transitions in $(N, [i])$.

Refinement of Workflow Nets

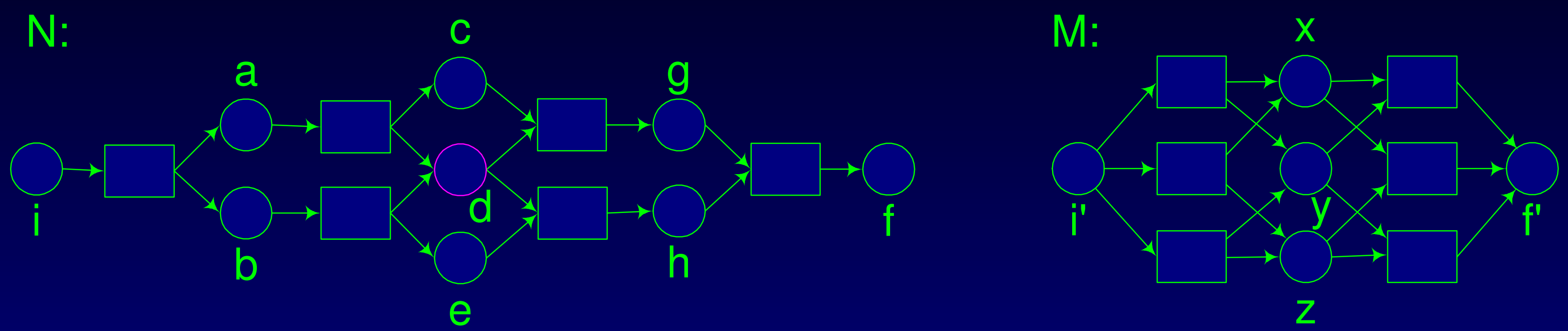
Place refinement: $N = L \otimes_p M$

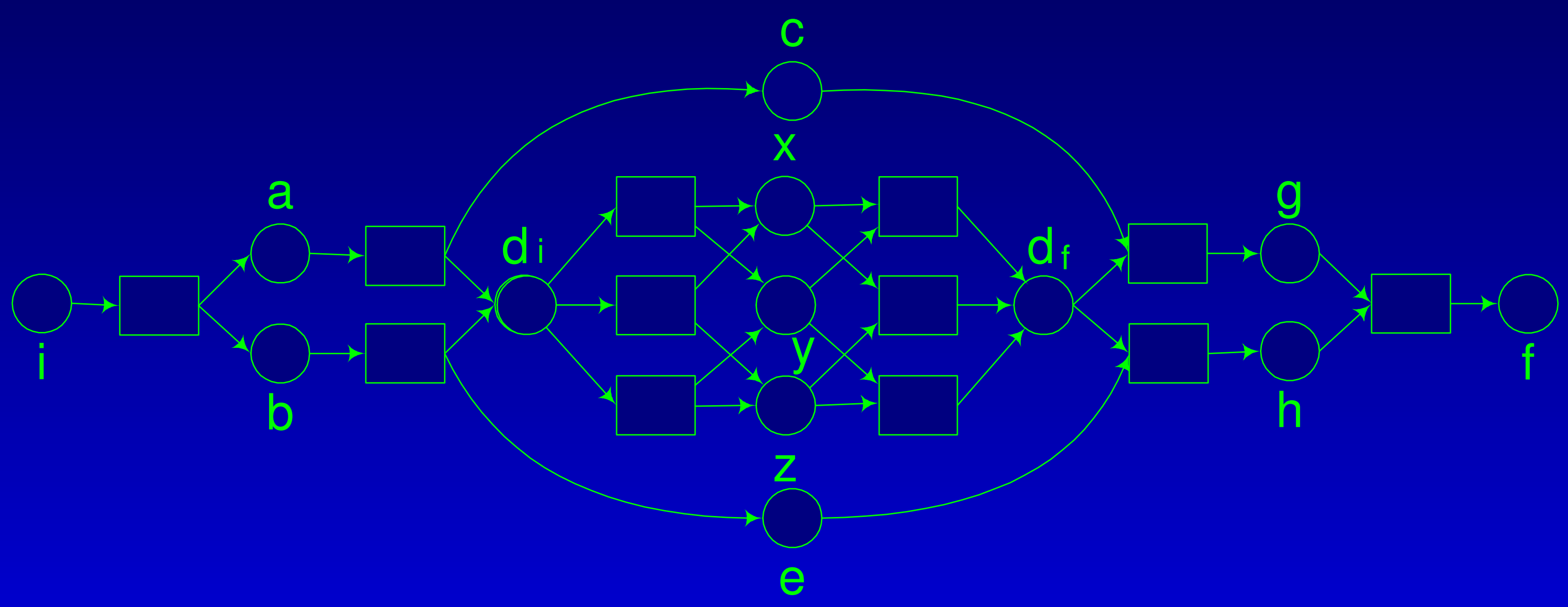
Being at some location (place of the net) resources (tokens) undergo a number of operations.

Transition refinement: $N = L \otimes_t M$

A single task on a higher level becomes a sequence of subtasks also involving choice and parallelism.

Refinements and soundness





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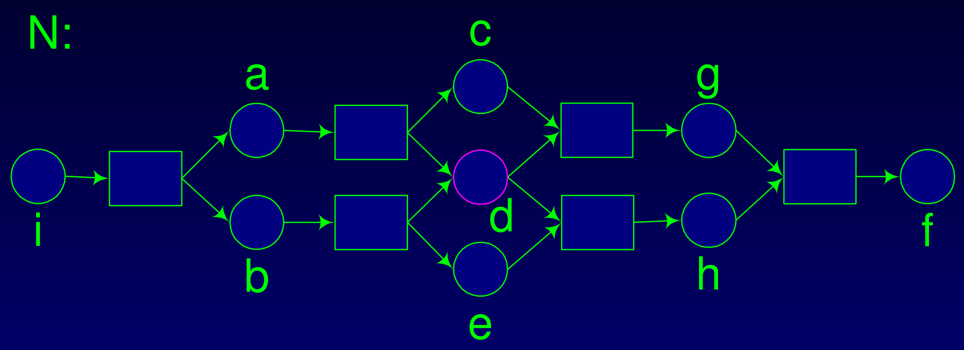

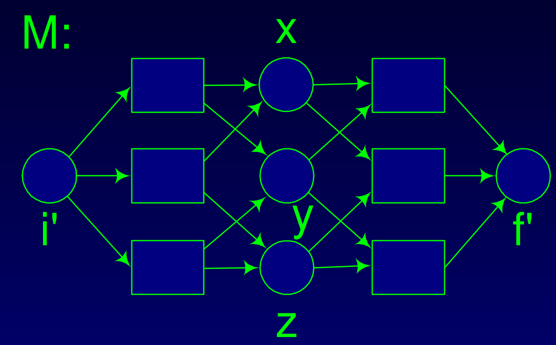

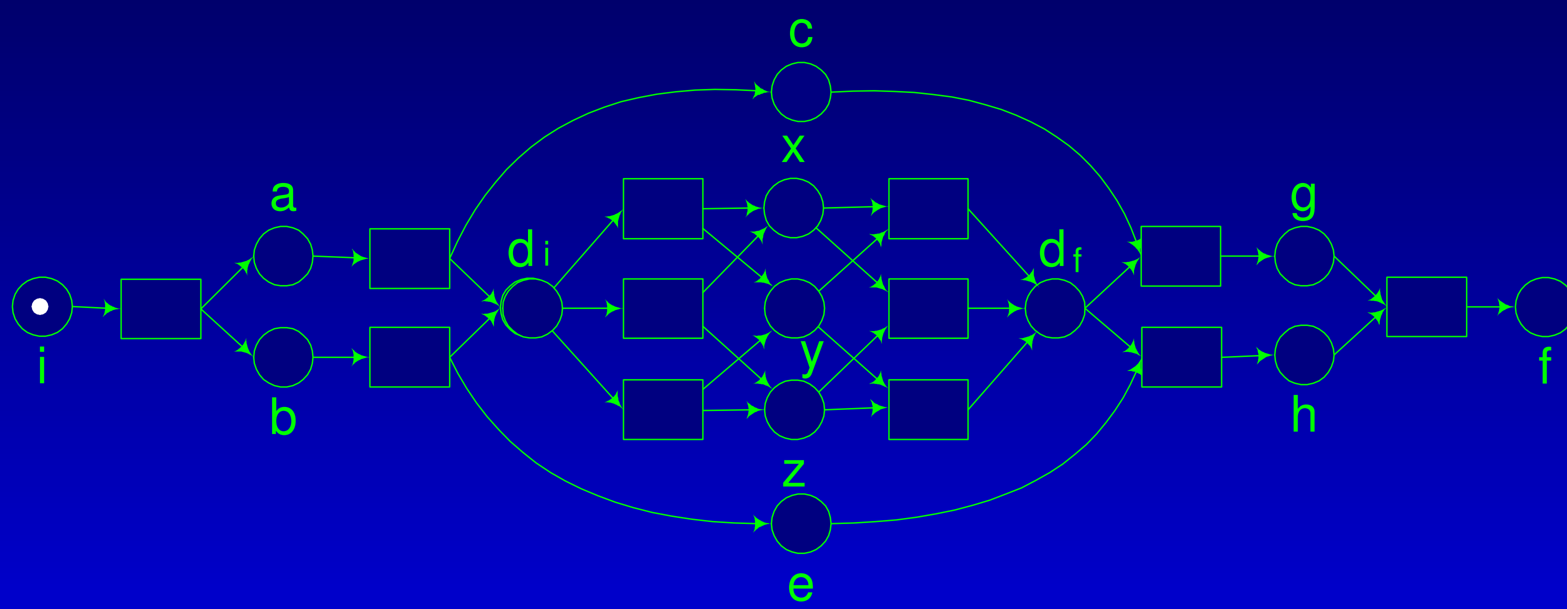

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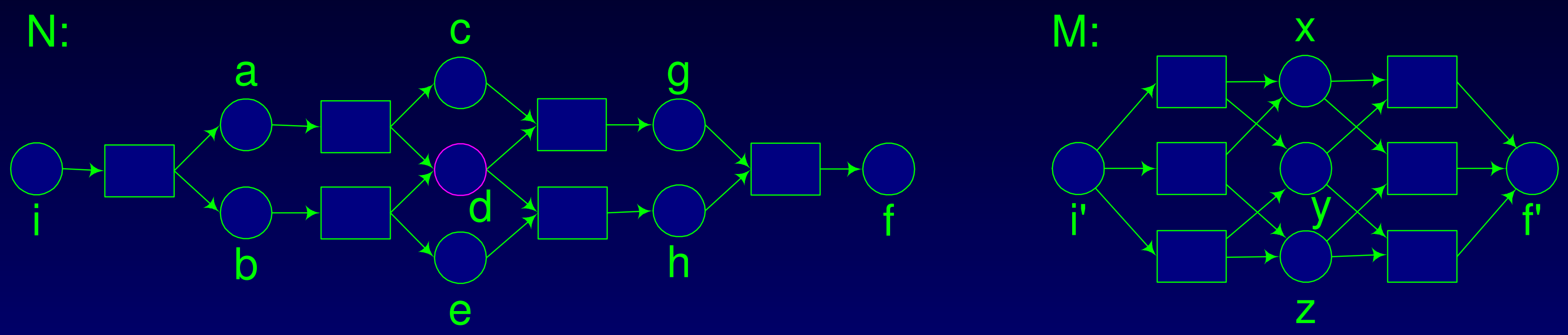

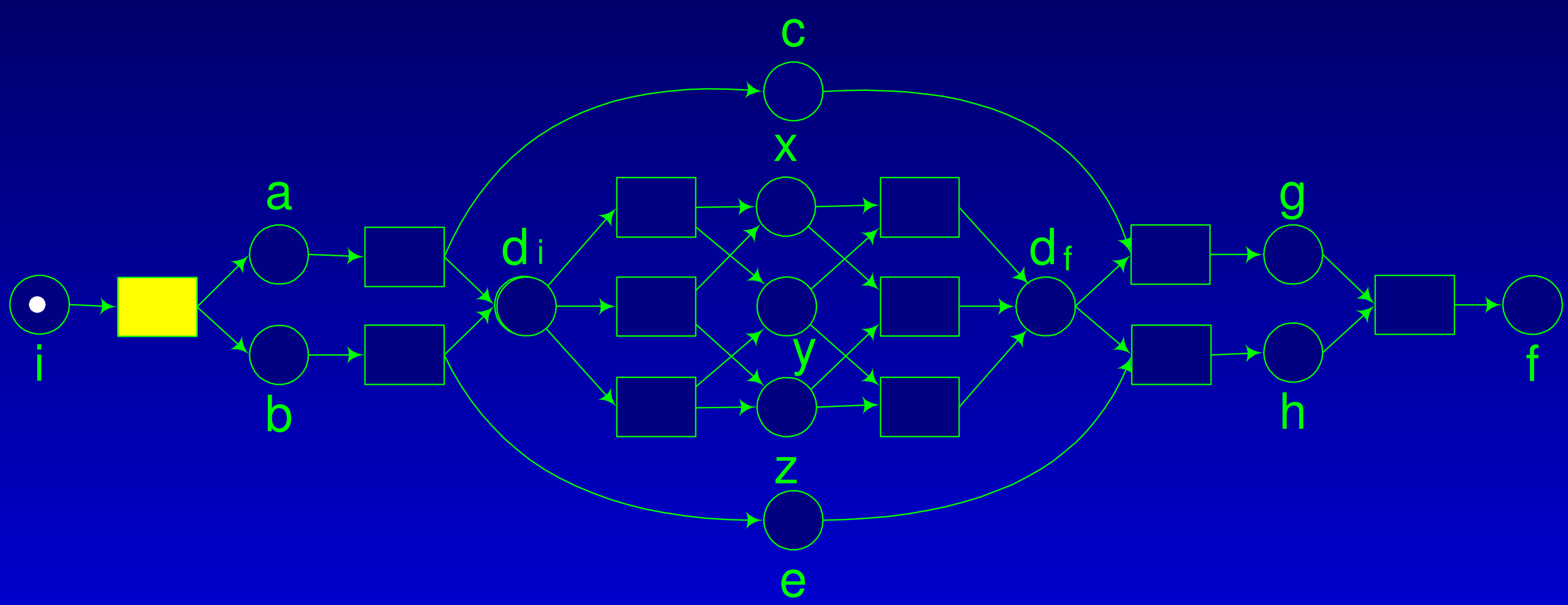

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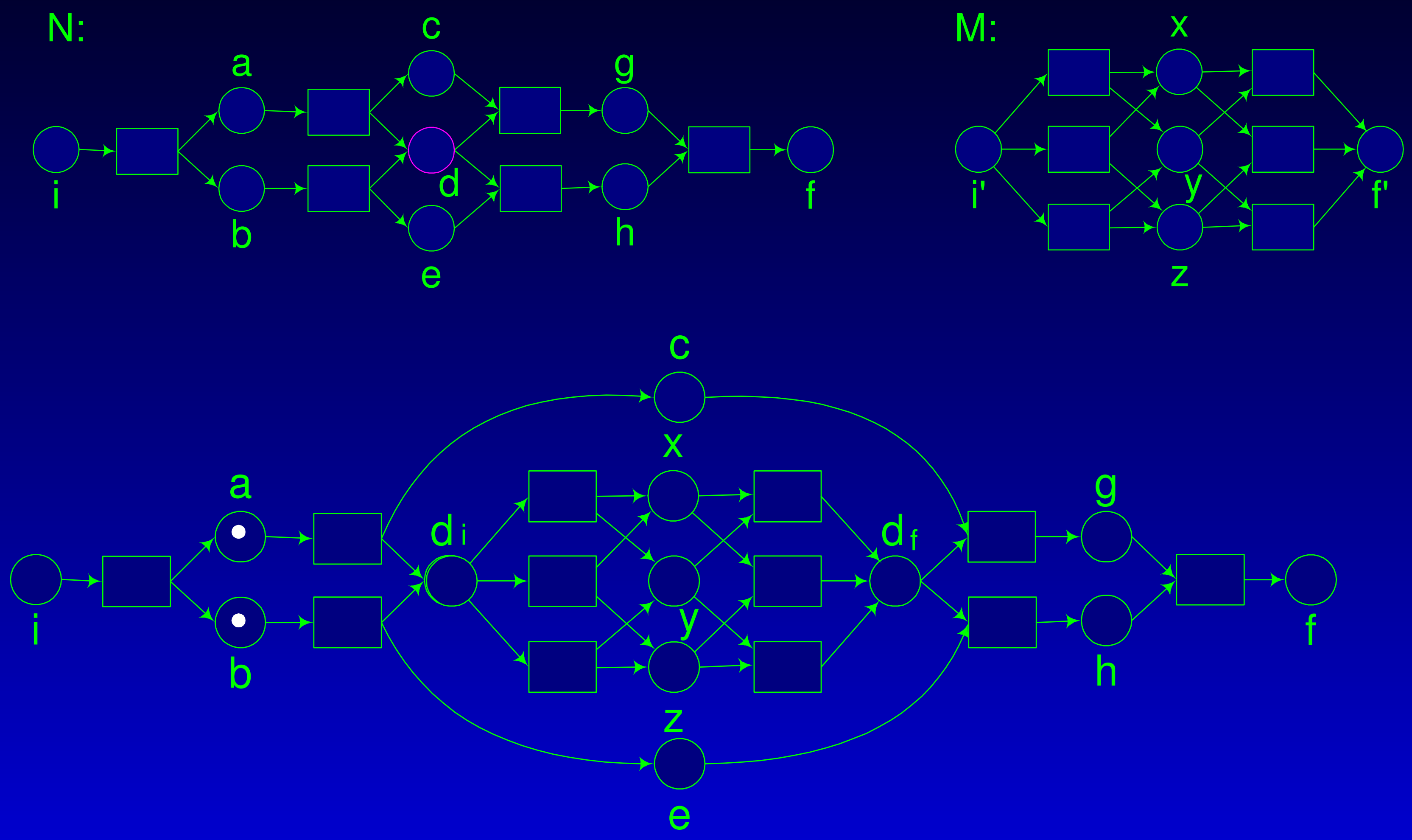

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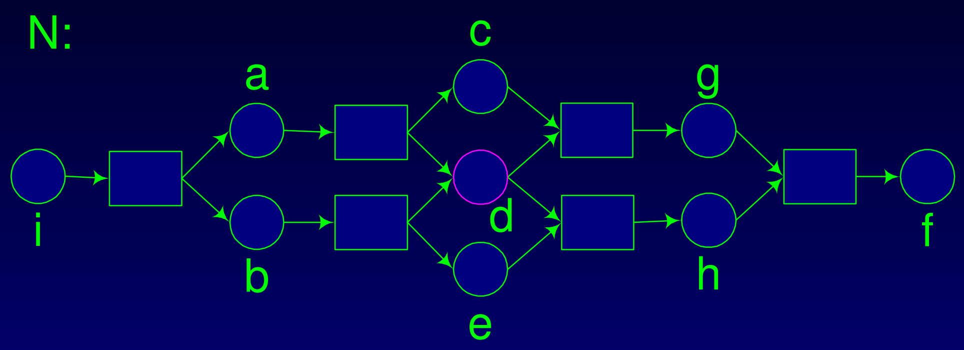

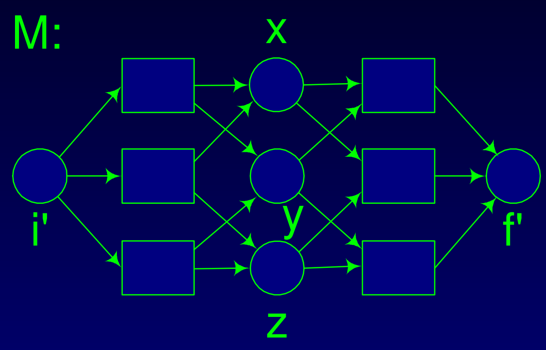

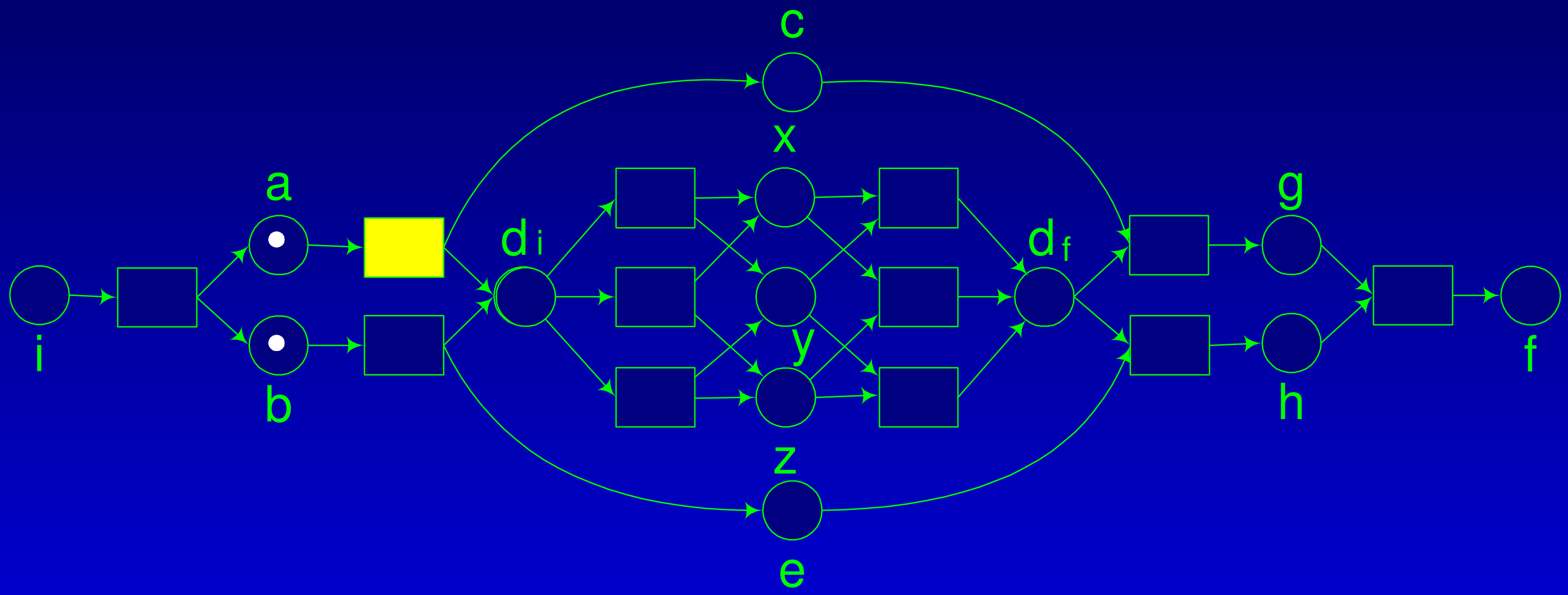

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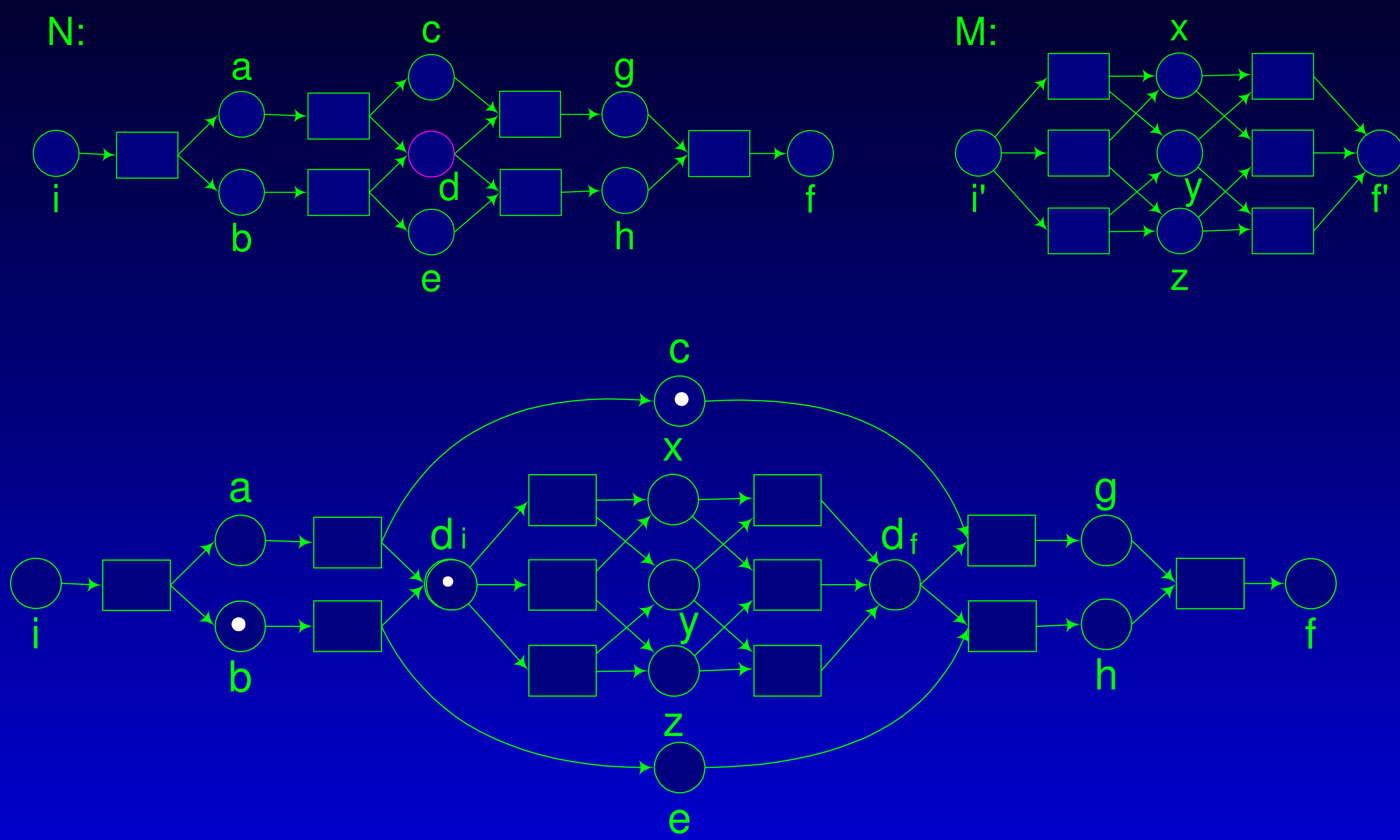

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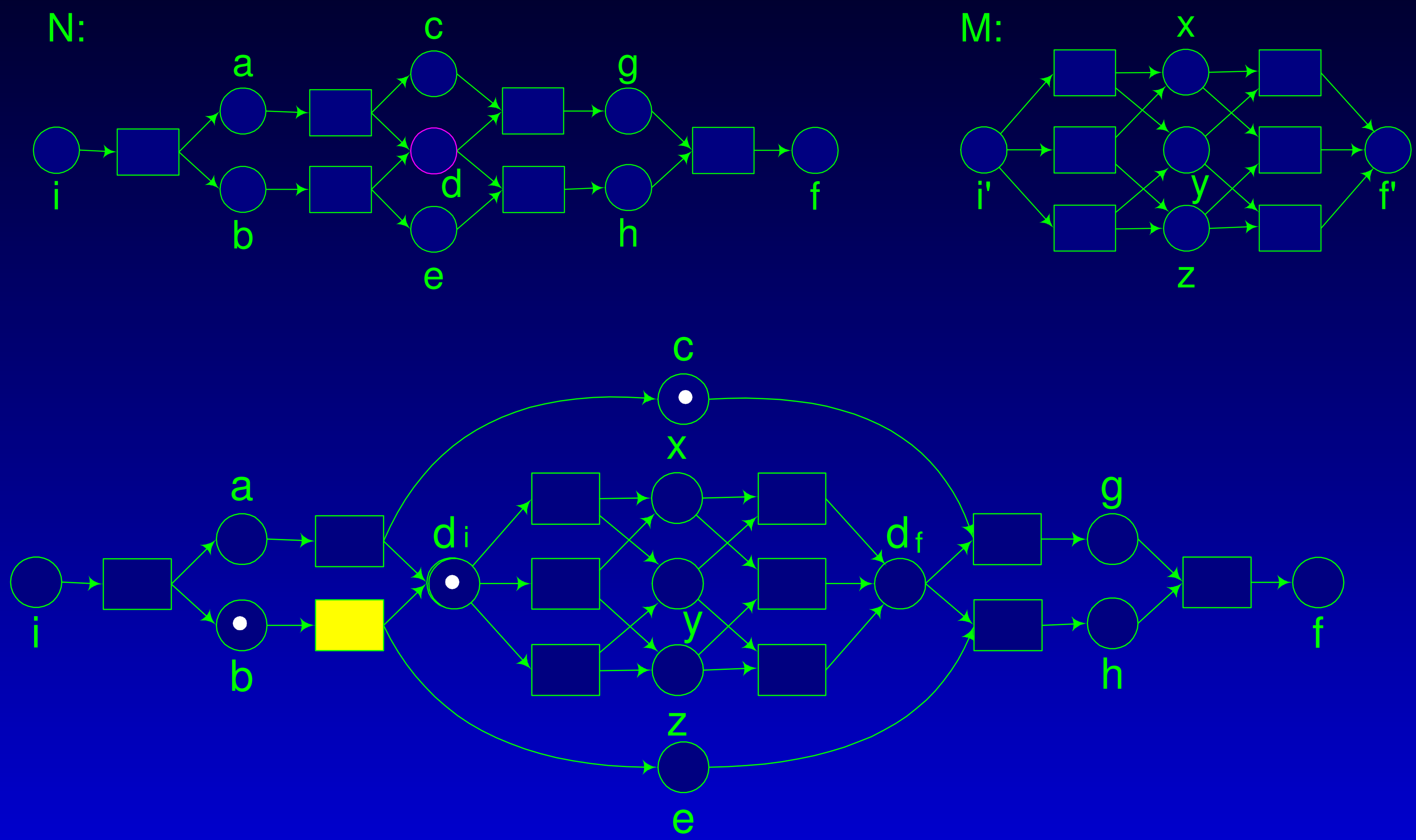

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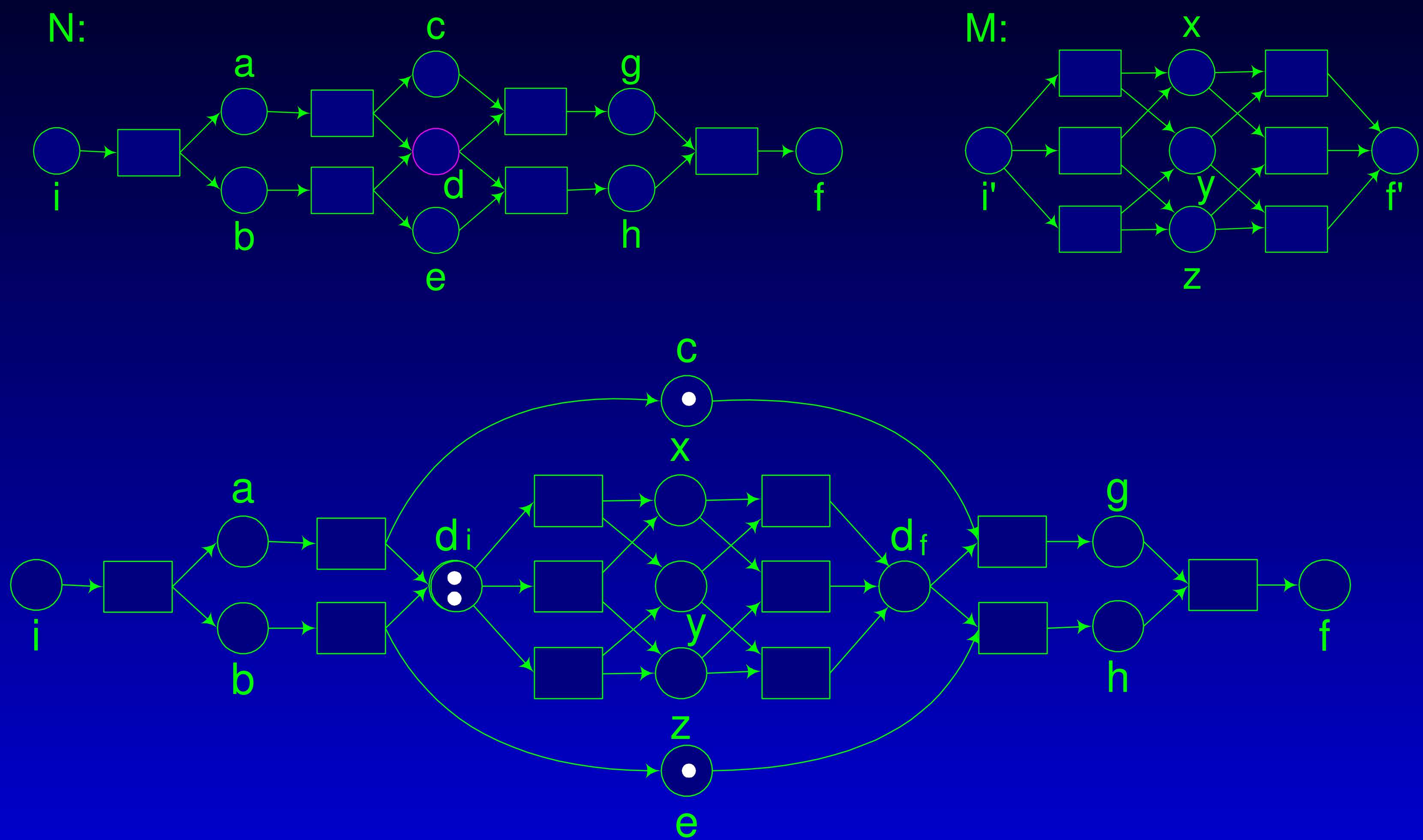

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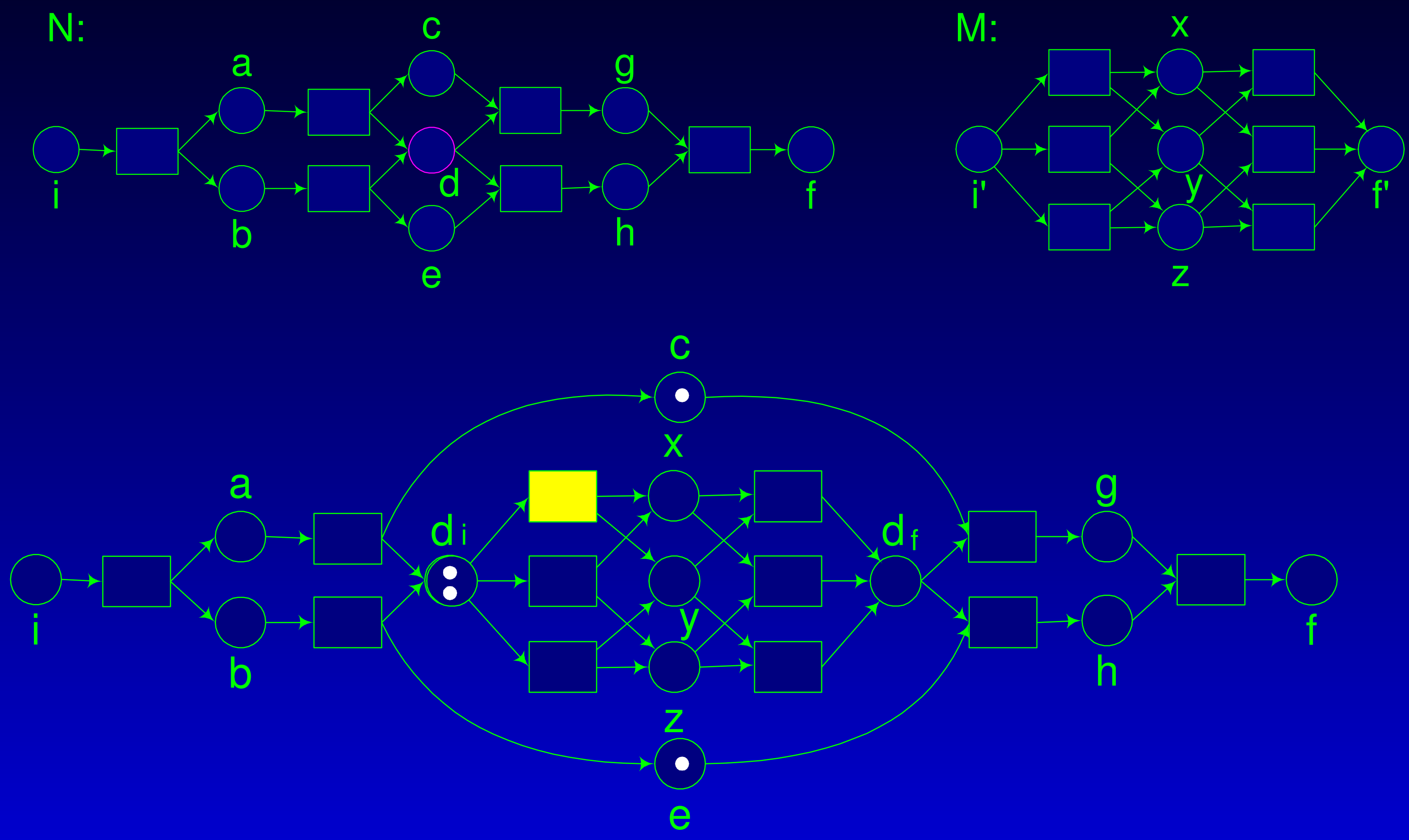

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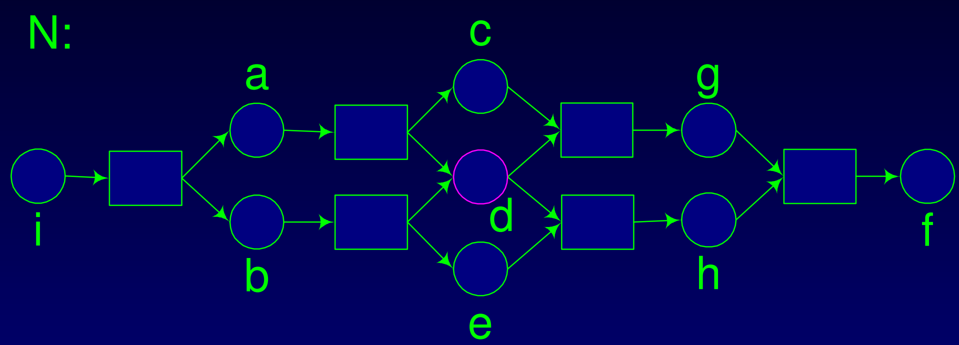

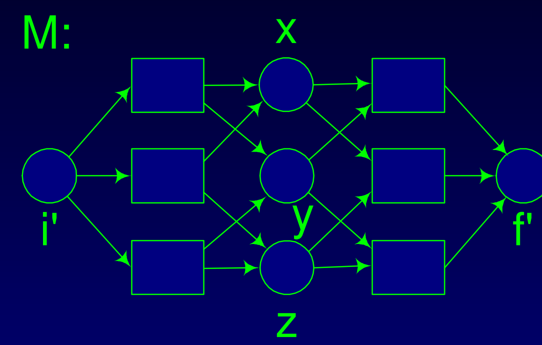

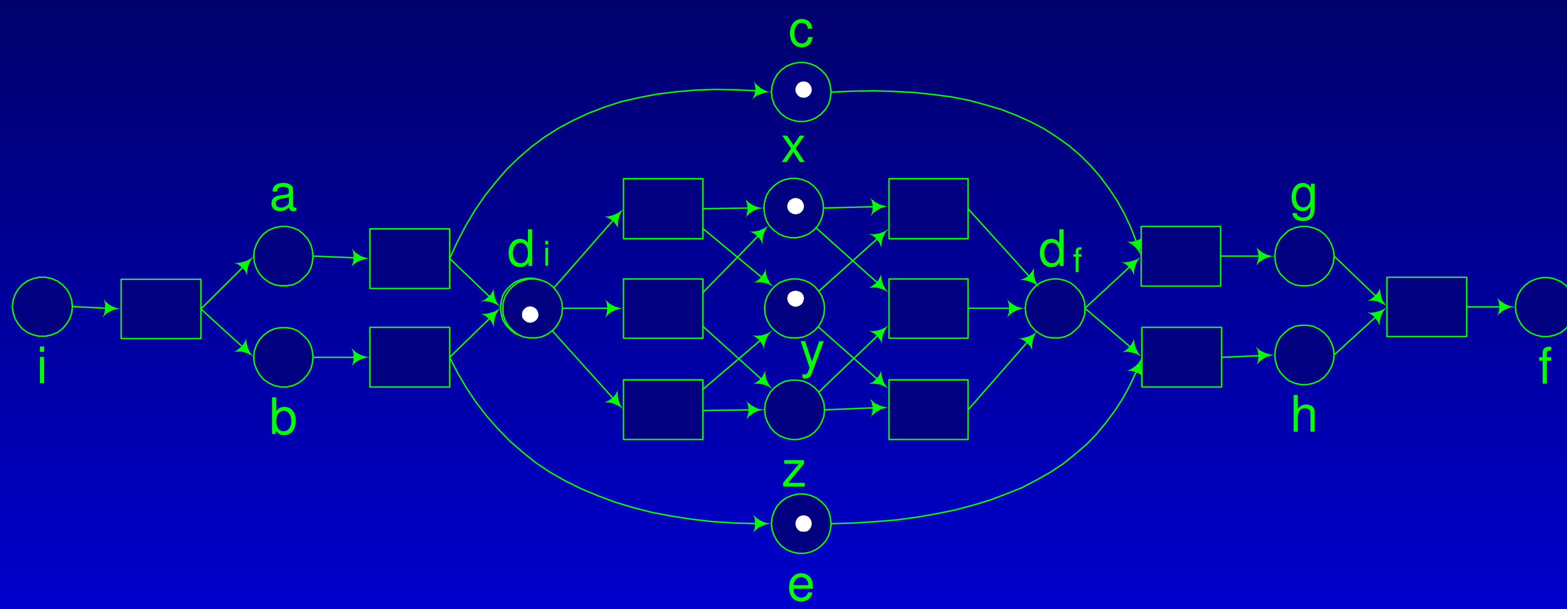

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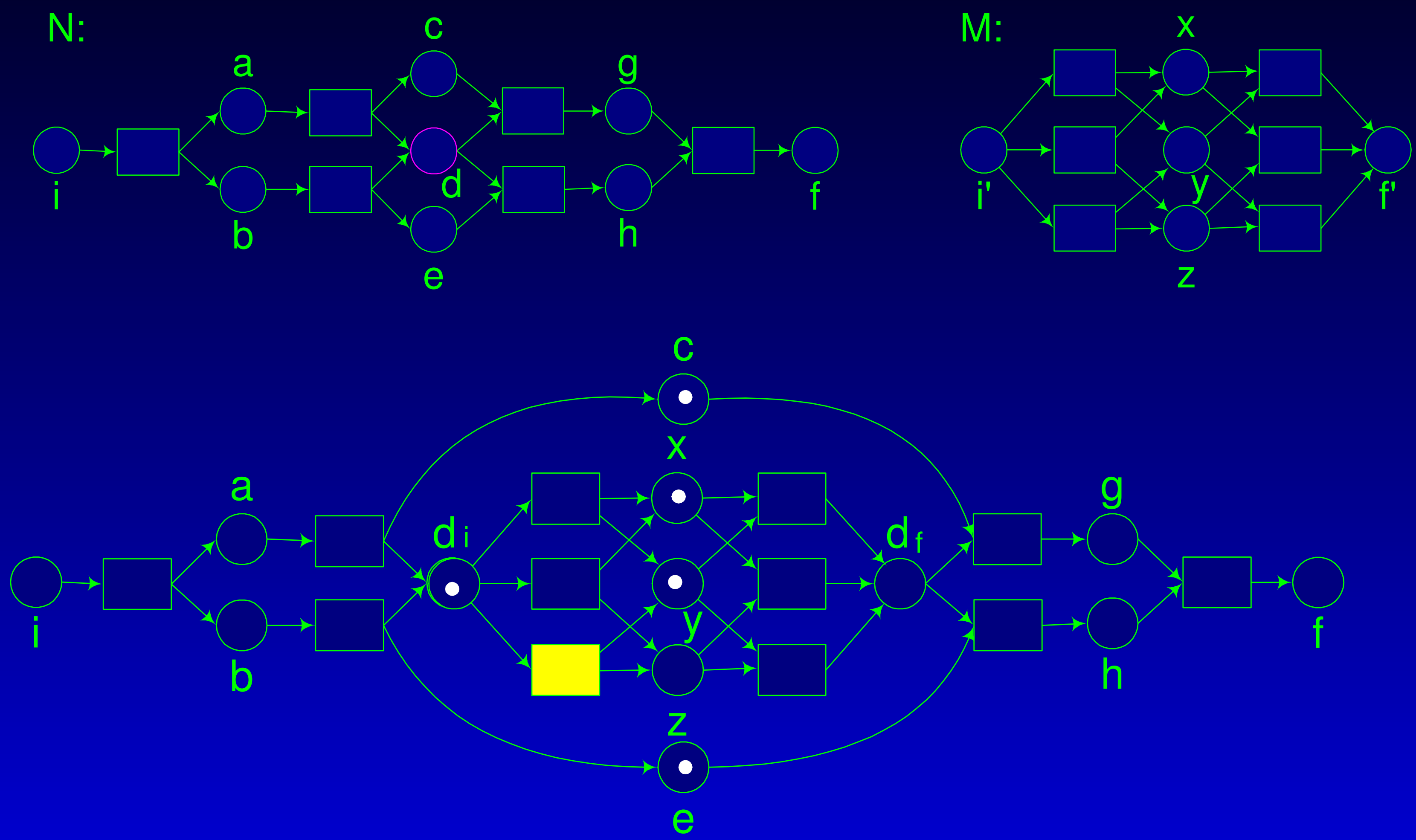

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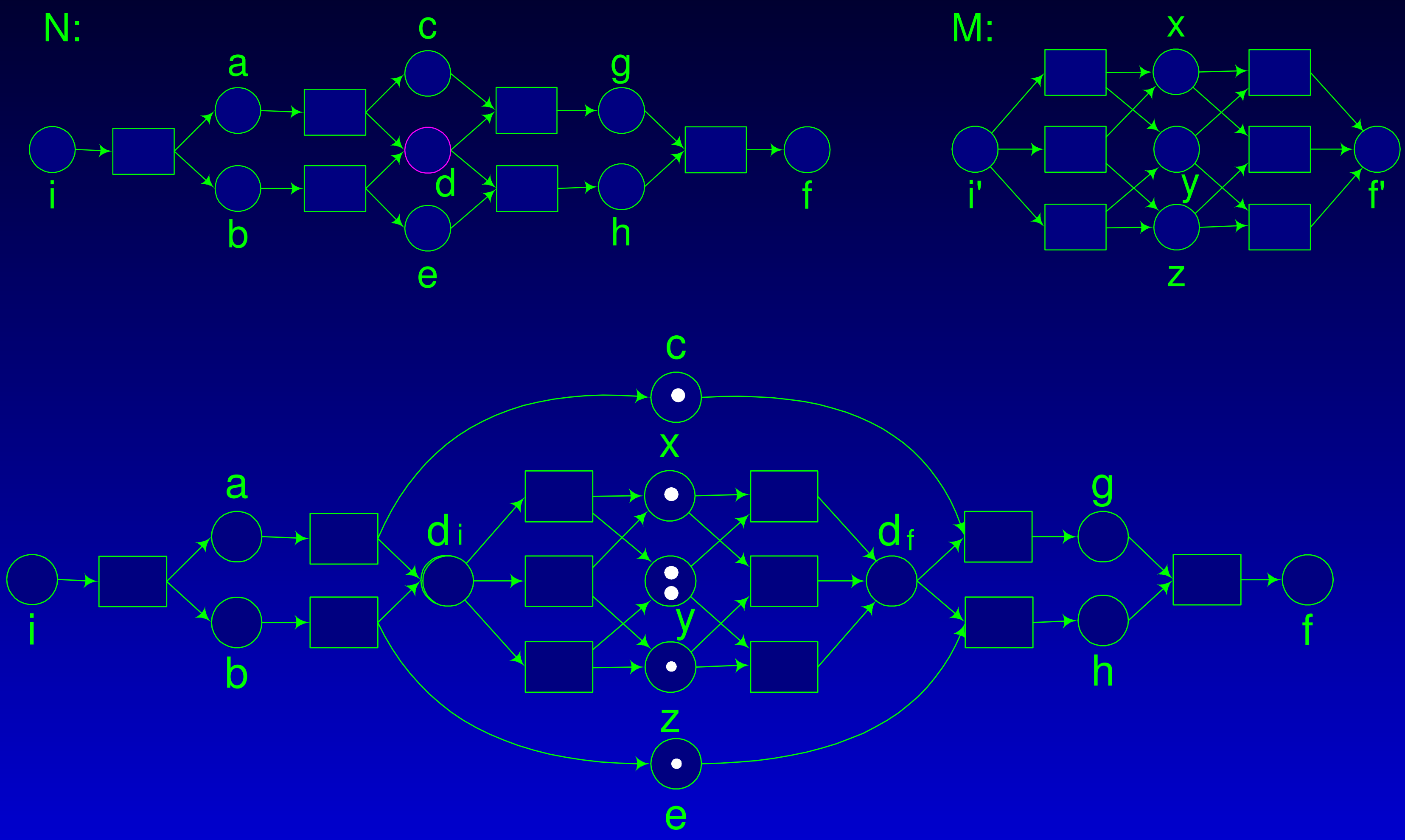

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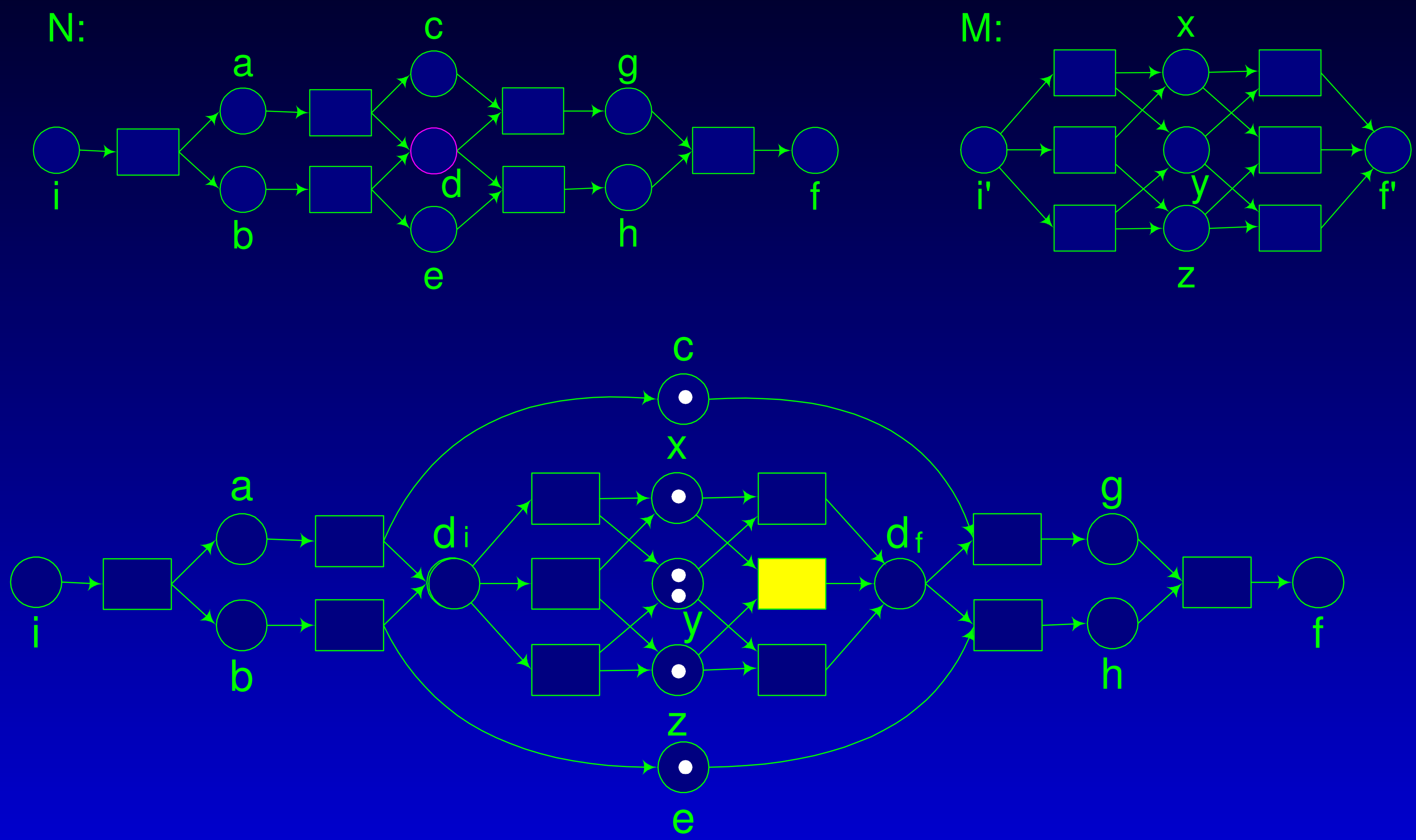

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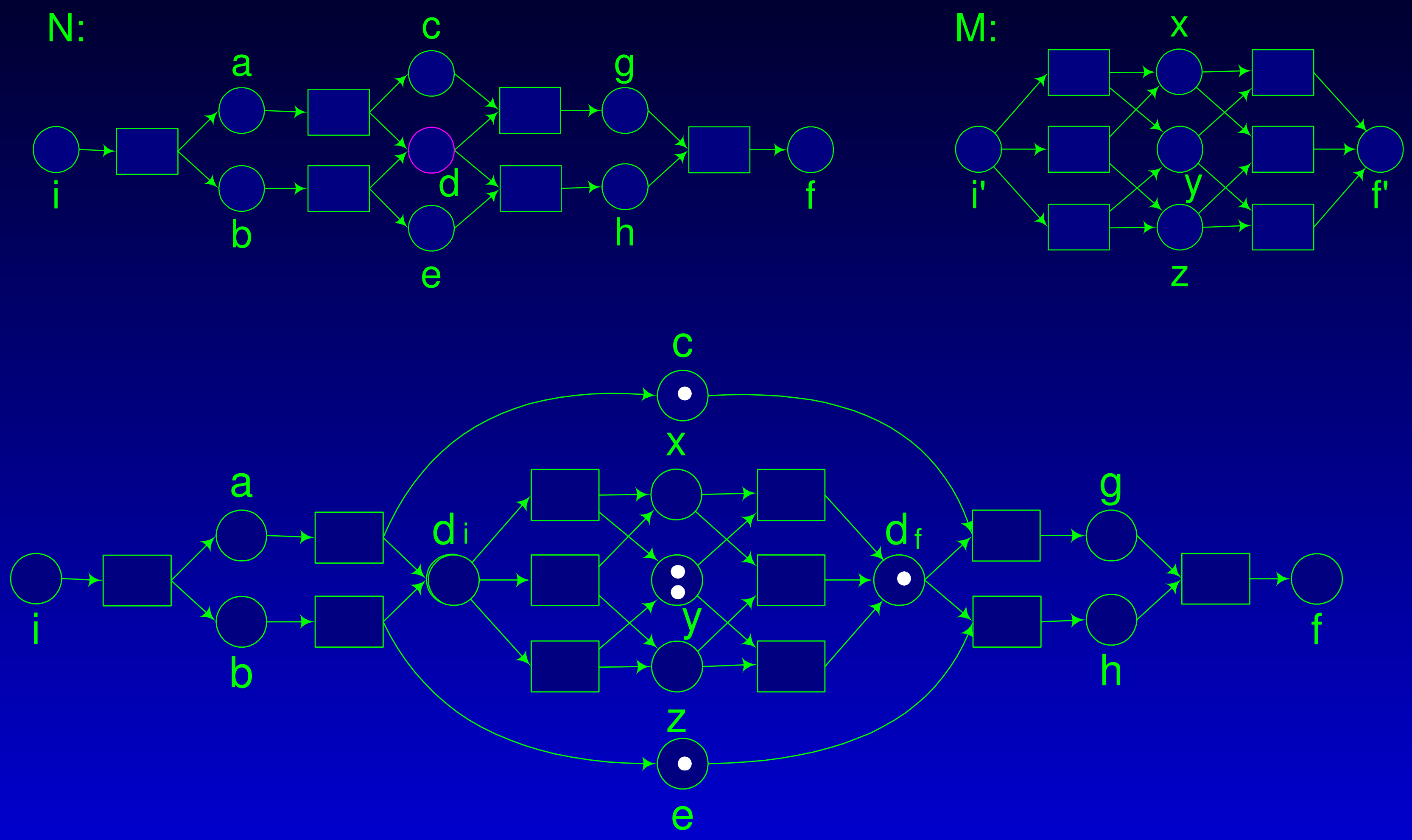

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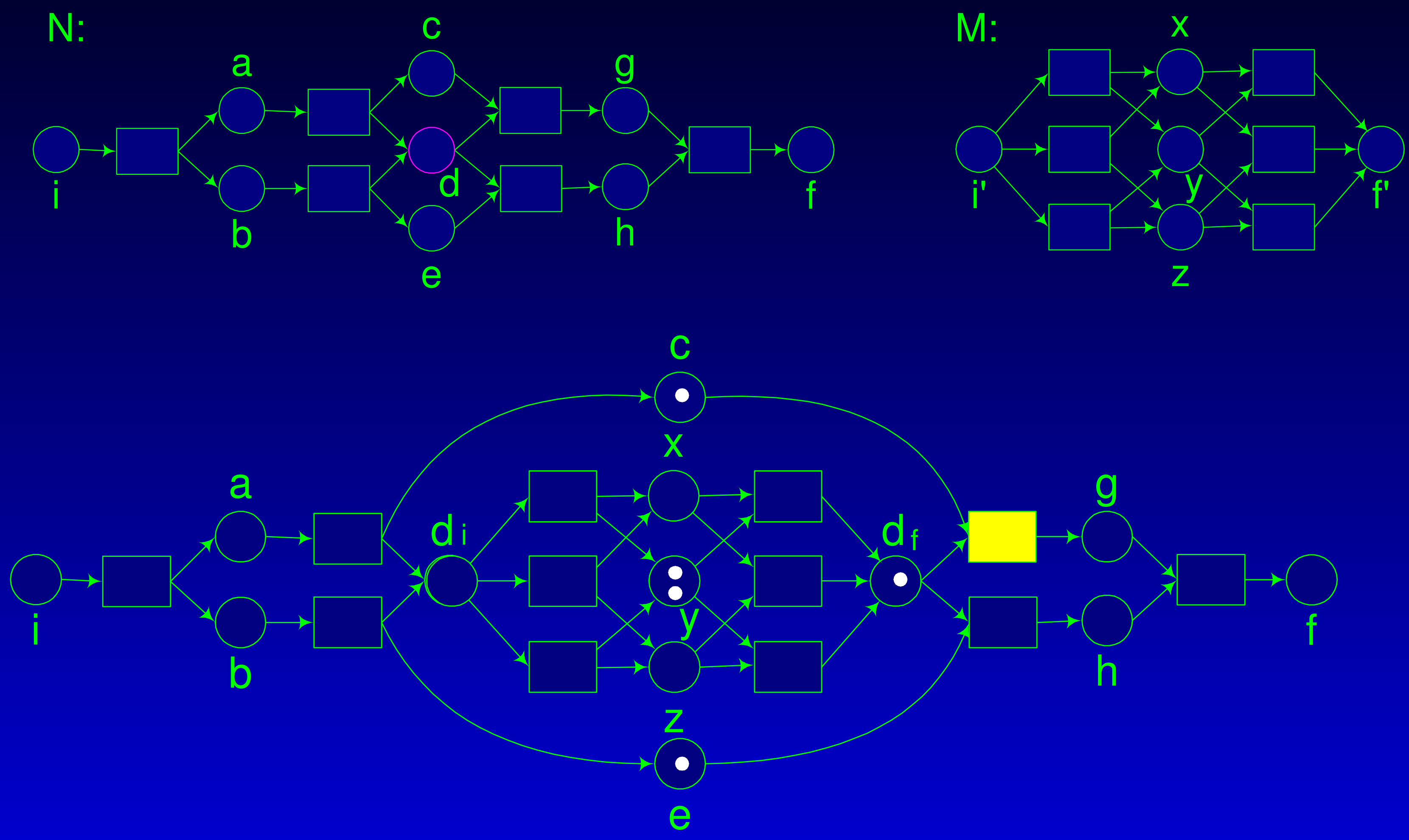

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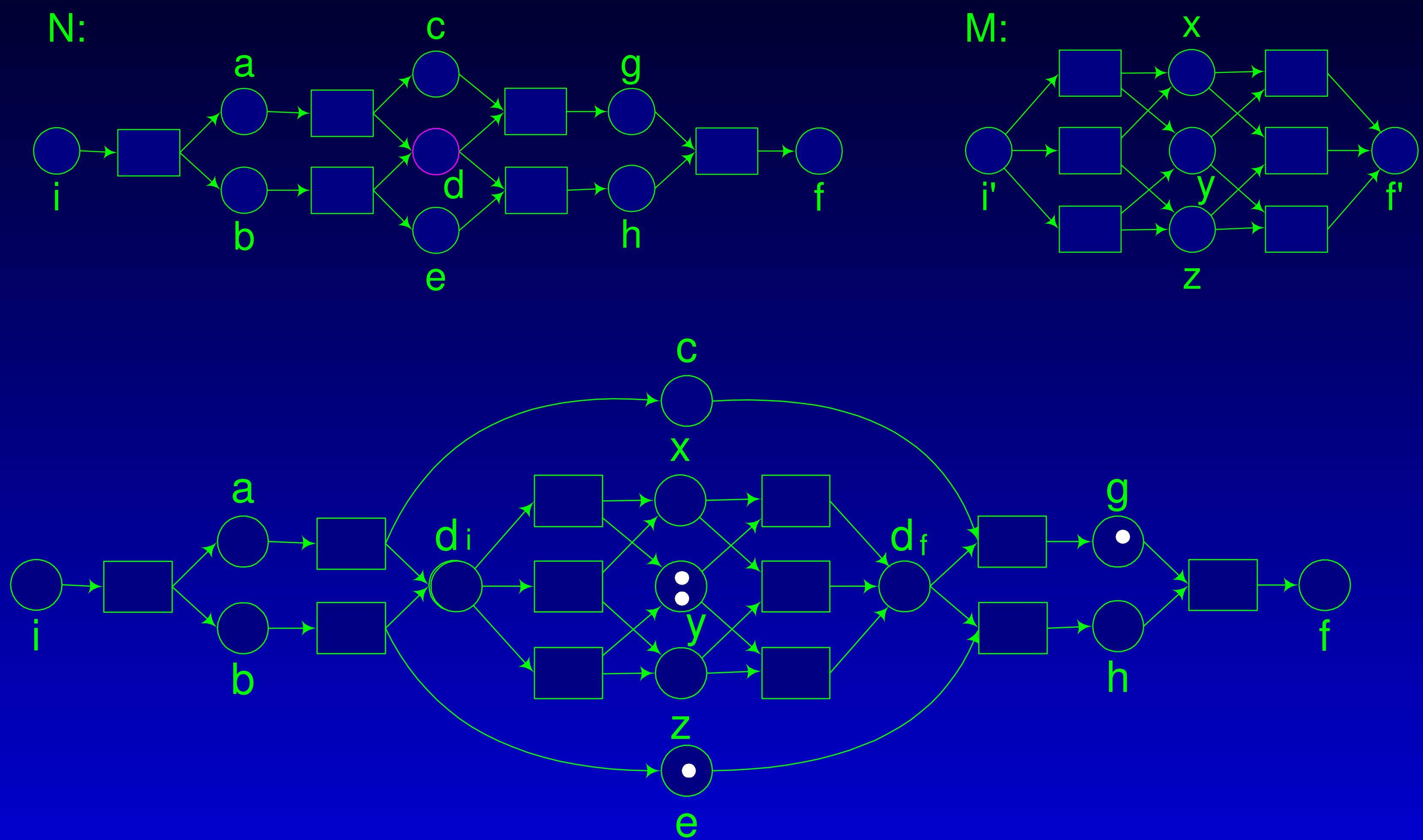

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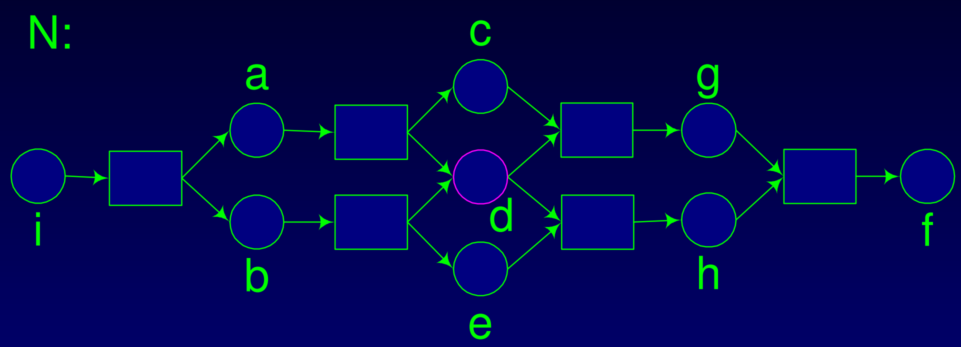
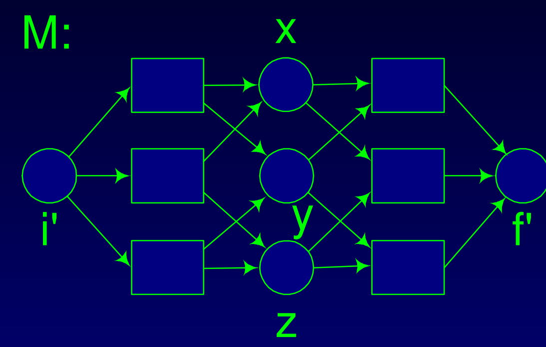
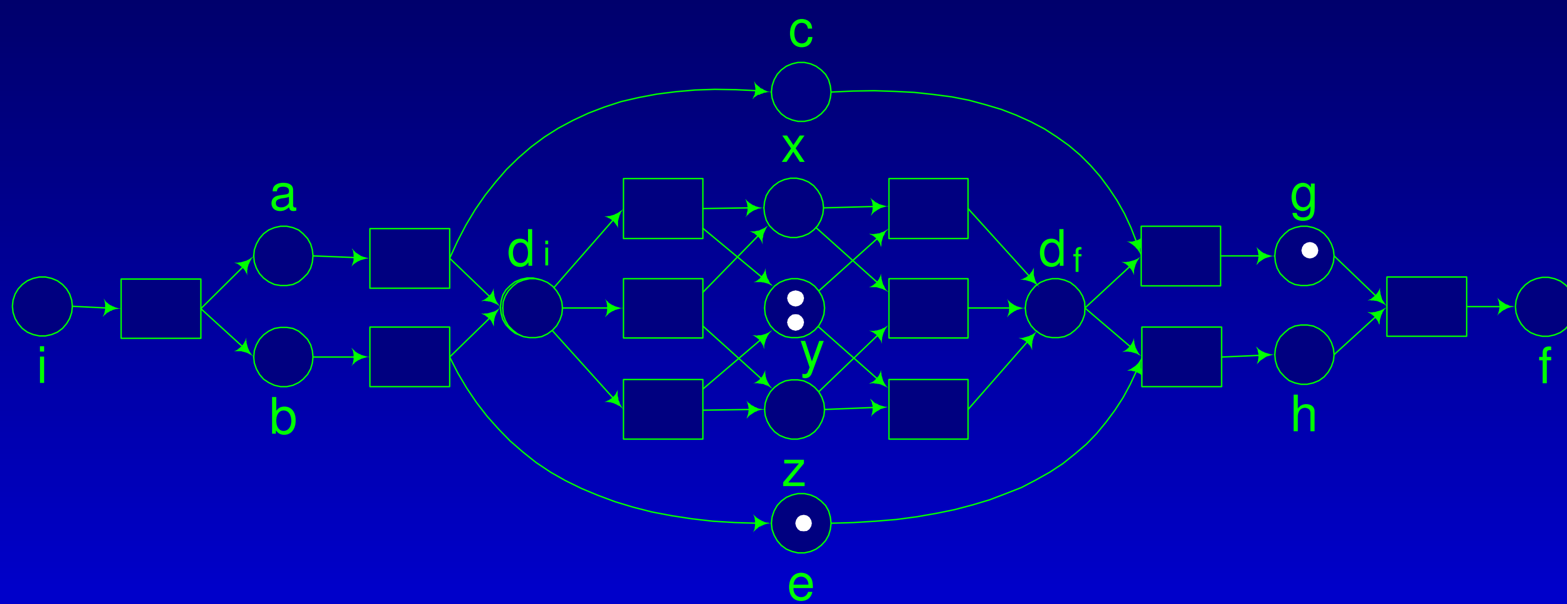






N and M are "sound", but $N \otimes_d M$ is not!

Generalised soundness

A sWF-net N with initial and final places i and f resp. is *k-sound* for $k \in \mathbb{N}$ iff $[f^k]$ is reachable from all markings m from $\mathcal{M}(N, [i^k])$.

A tWF-net N with initial and final transitions t_i, t_f respectively is k-sound iff the sWF-net formed by adding to S_N places p_i, p_f with $^\bullet p_i = \emptyset, p_i^\bullet = [t_i], ^\bullet p_f = [t_f], p_f^\bullet = \emptyset$ is k-sound.

A WF-net is *sound* iff it is k-sound for every natural k.

Refinements and generalised soundness

Soundness preservation

Let $N = L \otimes_n M$ be a refinement built of sound WF-nets L, M. Then N is sound.

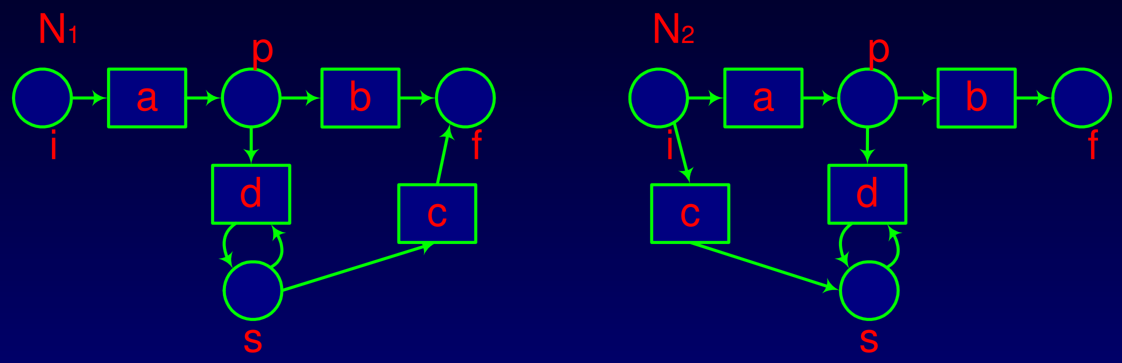
Old vs. new soundness

A WF-net N is sound iff:

- $[f]$ is reachable from any marking m from $\mathcal{M}(N, [i])$.
- There are no dead transitions in $(N, [i])$.

A WF-net N is sound iff $[f^k]$ is reachable from all markings m from $\mathcal{M}(N, [i^k])$, for any for $k \in \mathbb{N}$.

Structural non-redundancy



- Non-redundancy: every transition can potentially fire and every place can potentially obtain tokens, provided that there are enough tokens on the initial place.
- Persistency: it should be possible for every place (except for f) to become unmarked again—otherwise the net is guaranteed to be not sound.

Siphons

A set R of places is a siphon if ${}^\bullet R \subseteq R^\bullet$.
A siphon is a proper siphon if it is not empty.

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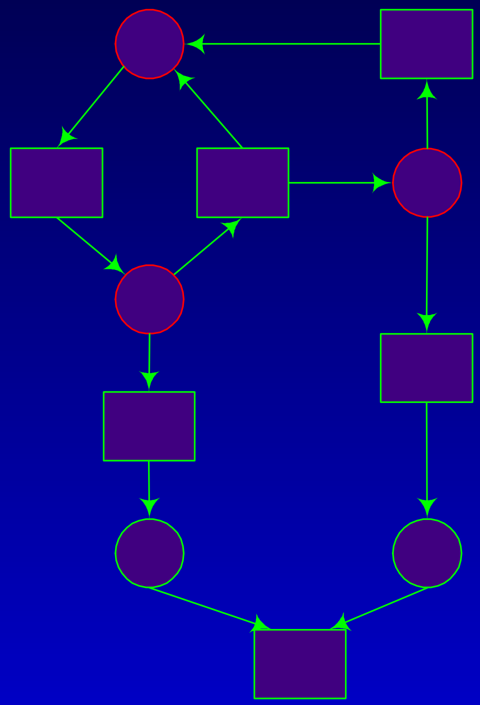

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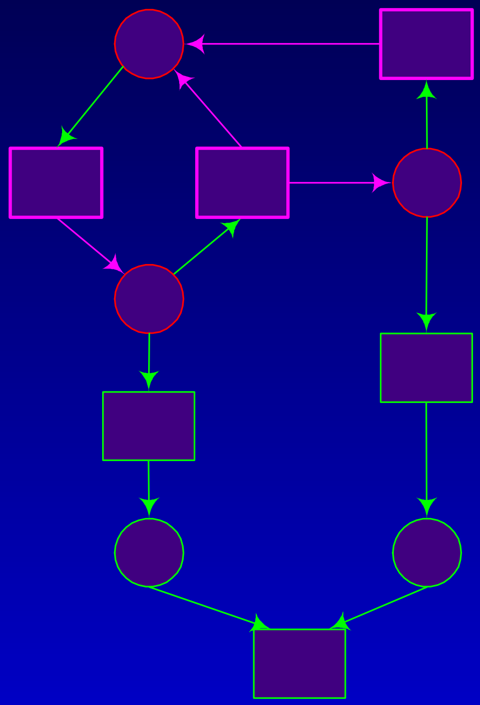

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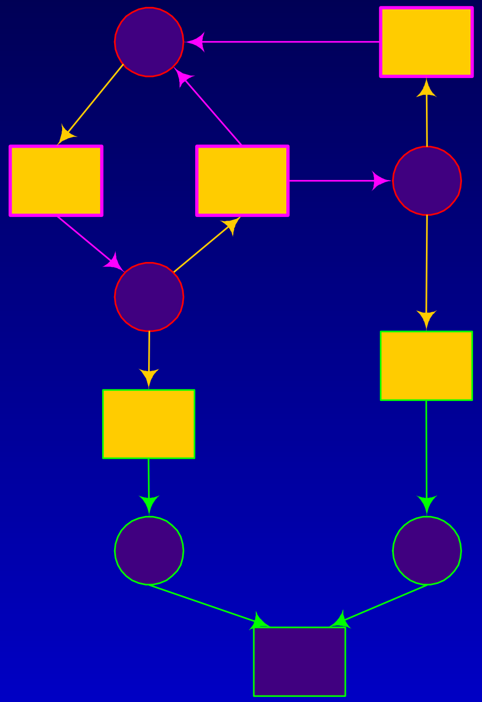

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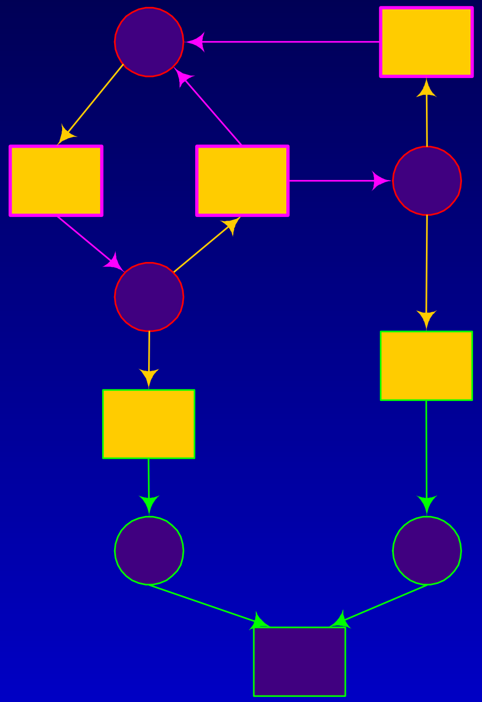
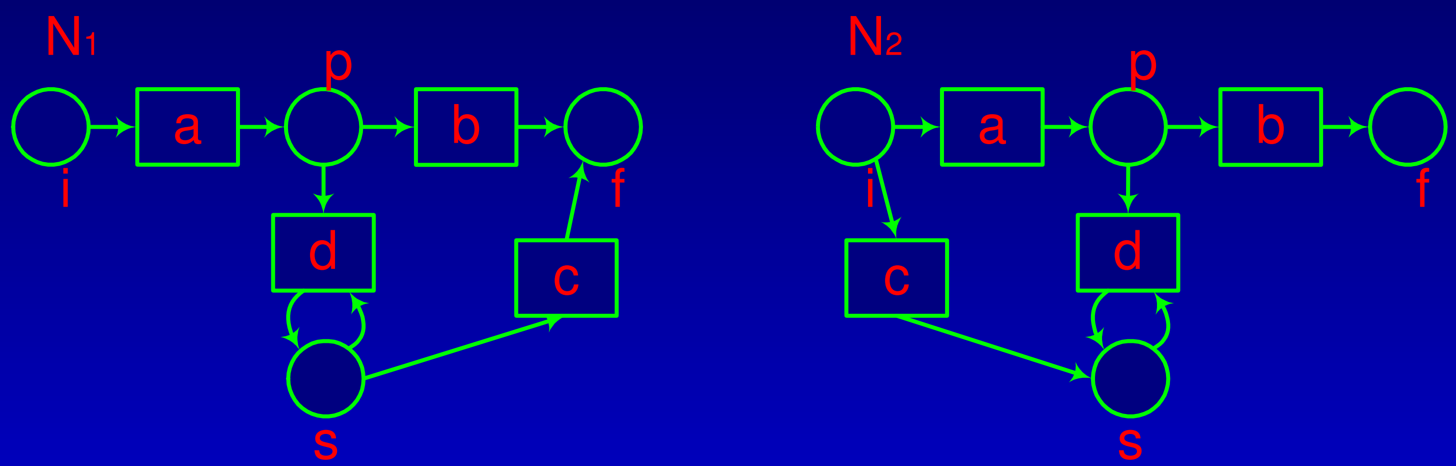


Unmarked siphons remain unmarked

Non-redundancy criterion

- A WF-net has no redundant places iff $P \setminus \{i\}$ contains no proper siphon.
- A WF-net has no redundant places iff it has no redundant transitions.



Non-redundancy check

Compute the largest siphon X in $P \setminus \{i\}$ in a standard manner [Starke]:

---

input : A Petri net $N = (P, T, F^+, F^-)$ and $S \subseteq P$;
output: $X \subseteq S$;
$X = S$;
while there exist $p \in X$ and $t \in {}^\bullet p$ such that $t \notin X^\bullet$
do $X = X \setminus \{p\}$;
return(X);

Traps

A set R of places is a trap if $R^\bullet \subseteq {}^\bullet R$.
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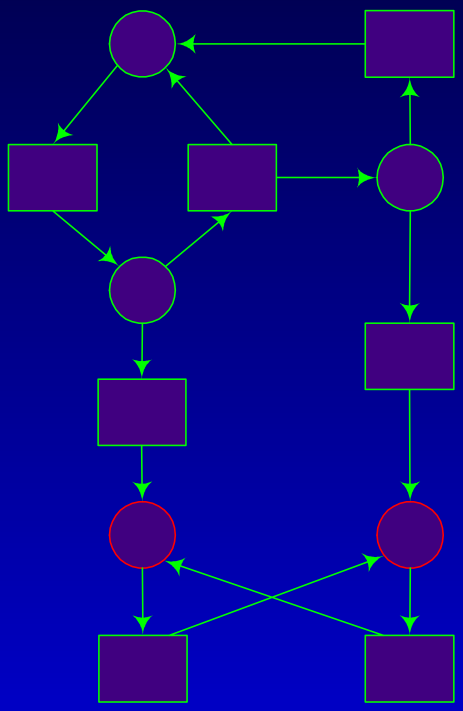

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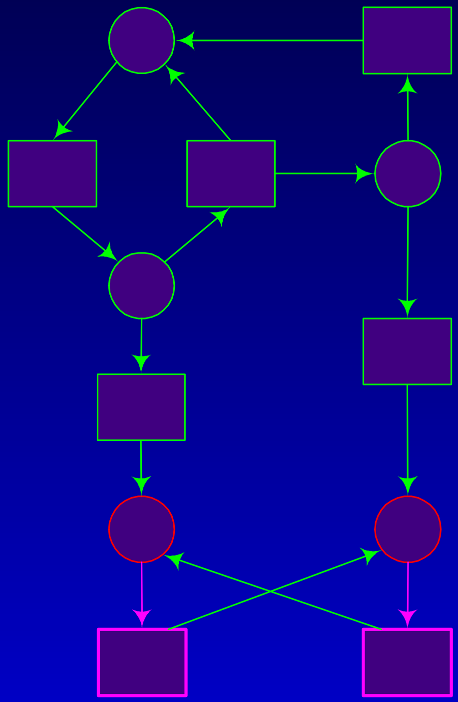

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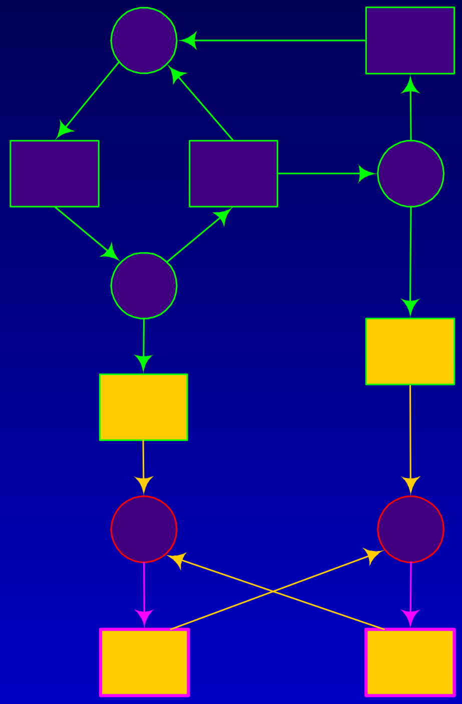

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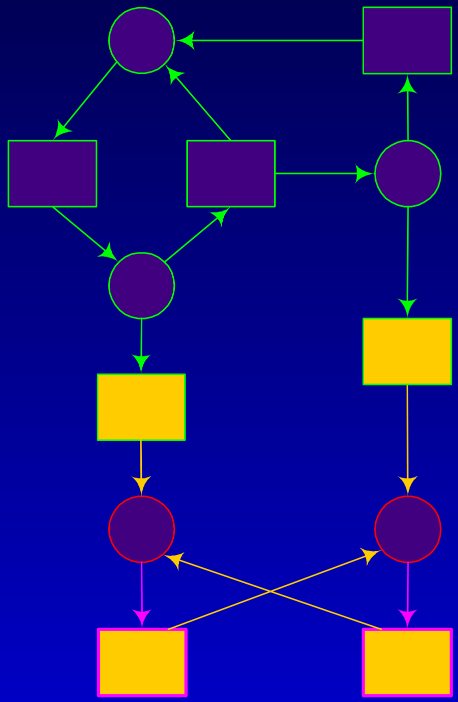
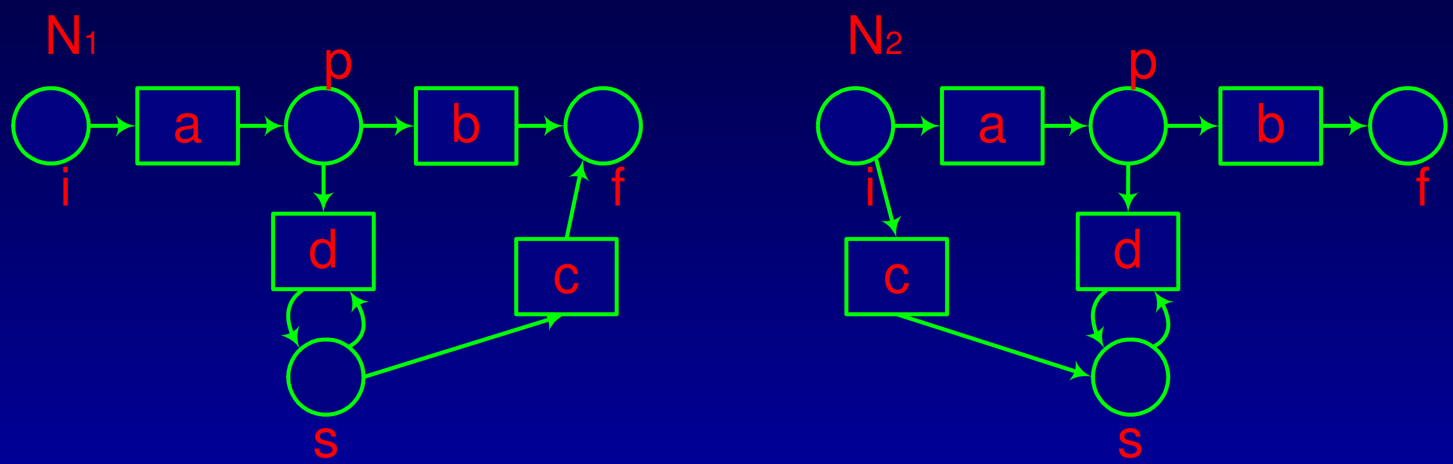


Marked traps remain marked.

Non-persistency criterion

A WF-net has no persistent places iff $P \setminus \{f\}$ contains no proper trap.



Correcting workflow nets

Let a WF-net N be given.

First, find a maximal siphon X in $P \setminus \{i\}$.
All places from X are redundant. $\Rightarrow$
Transitions from $X^\bullet$ are redundant as well. $\Rightarrow$

$(N_1, k[i])$ obtained by removing places from X and transitions from $X^\bullet$ is WF-bisimilar to $(N, k[i])$ for any k.

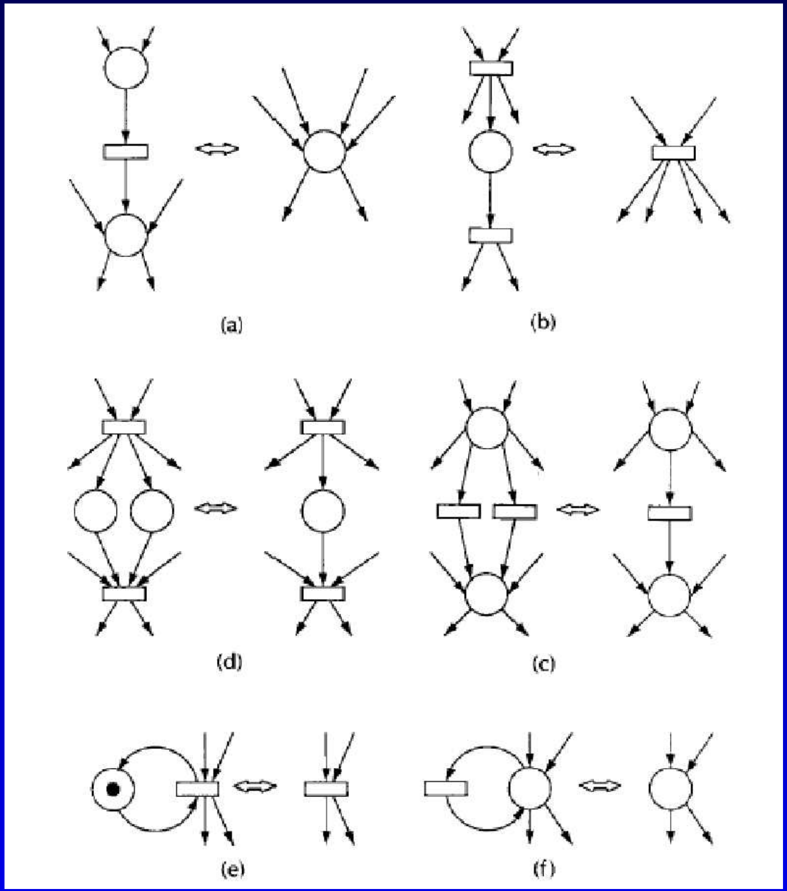
N_1 is either not a WF-net any more and so N was ill-designed,
or N_1 is a WF-net, which is an improved version of N.

Check whether N_1 has persistent places. If yes, N_1 is not a sound WF-net. Otherwise, we can work with N_1 instead of N.

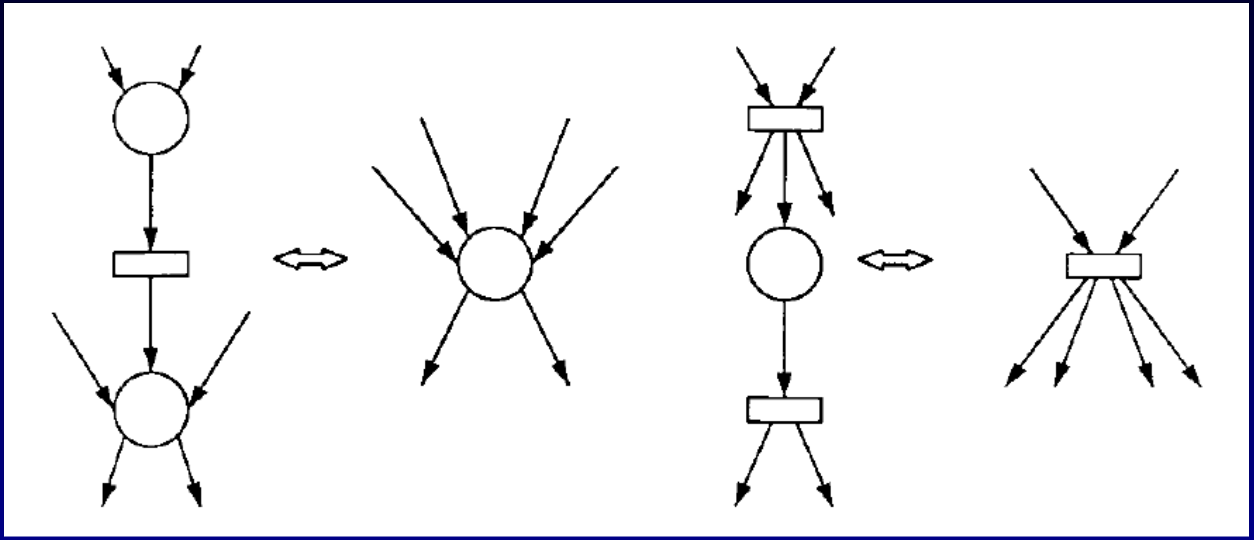
Petri net reduction techniques

Goal: to preserve such Petri net properties as liveness, safeness and boundedness.

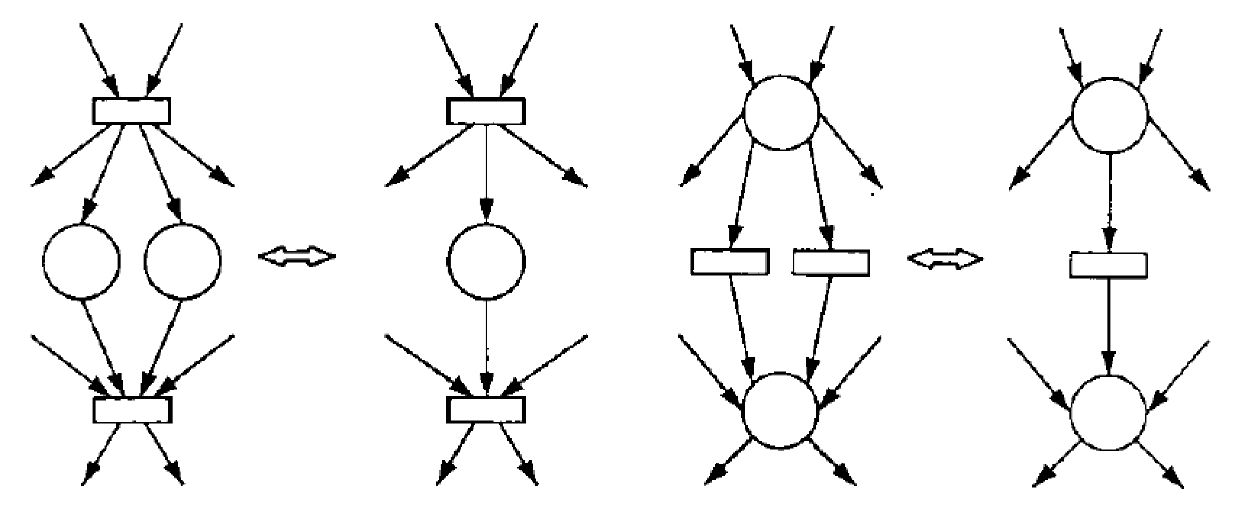
The simplest transformations: (see [Murata1989])



Fusion of series places/transitions



Fusion of parallel places/transitions



Elimination of self-loop places/trans.