Exercises Algorithms for Model Checking

1 CTL*

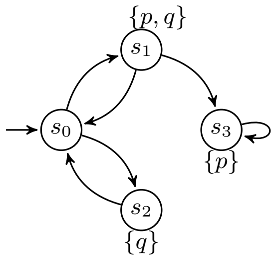

Figure 1:

1. For each of the CTL^* formulae below, indicate whether it is (syntactically) a formula in LTL and/or CTL. Determine for each formula in which states of the Kripke Structure of Fig. 1 it holds.

   (a) p,

   (b) $\mathsf{E}\ [q\ \mathsf{R}\ p]$,

   (c) $\mathsf{E}\ \mathsf{F}\ \mathsf{G}\ p$,

   (d) $\mathsf{A}\ \mathsf{G}\ \mathsf{F}\ p$,

   (e) $\mathsf{A}\ \mathsf{G}\ \mathsf{E}\ \mathsf{F}\ p$,

   (f) $\mathsf{A}\ \mathsf{G}\ \mathsf{F}\ (p \wedge \mathsf{X}\ q)$,

   (g) $\mathsf{A}\ \mathsf{G}\ (\neg q \vee \mathsf{F}\ p)$,

   (h) $\mathsf{A}\ ((\mathsf{G}\ p) \vee (\mathsf{F}\ q))$

2. For each pair of CTL^* formulae below, if possible, give a Kripke Structure in which both are valid, a Kripke Structure in which both are not valid, and a Kripke Structure in which only one of them is valid.

   (a) p and $\mathsf{A}\ \mathsf{F}\ p$

   (b) $\mathsf{A}\ \mathsf{F}\ \mathsf{A}\ \mathsf{G}\ p$ and $\mathsf{A}\ \mathsf{G}\ \mathsf{A}\ \mathsf{F}\ p$

   (c) $\mathsf{A}\ \mathsf{F}\ \mathsf{A}\ \mathsf{X}\ p$ and $\mathsf{A}\ \mathsf{F}\ \mathsf{X}\ p$

   (d) $\mathsf{A}\ \mathsf{X}\ \mathsf{E}\ \mathsf{X}\ p$ and $\mathsf{A}\ \mathsf{X}\ \mathsf{X}\ p$

   (e) $\mathsf{A}\ \mathsf{X}\ \mathsf{A}\ \mathsf{X}\ p$ and $\mathsf{A}\ \mathsf{X}\ \mathsf{X}\ p$

   (f) $\mathsf{A}\ [p\ \mathsf{U}\ q]$ and $\mathsf{A}\ [\neg q\ \mathsf{R}\ \neg p]$

3. Consider LTL, CTL and CTL^*. State for each of the claims below whether they hold or not. Motivate your answer by providing counterexamples or a formal justification.

(a) Every CTL^* formula is equivalent to either an LTL formula or a CTL formula.

(b) The language LTL is more expressive than CTL.

(c) The language CTL is more expressive than LTL.

(d) On deterministic Kripke Structures (i.e., Kripke Structures with a single initial state in which each state has exactly one successor), LTL and CTL are equally expressive; that is, every LTL formula has an equivalent CTL formula and vice versa.

4. Express that along all paths, proposition p holds infinitely often and $\neg p$ holds infinitely often.

5. Express that along all paths, proposition p holds infinitely often and $\neg p$ only holds finitely often.

6. Prove using the semantics of CTL^*, or disprove using a Kripke Structure, the following equivalences:

   (a) $\mathsf{A}\ [\phi\ \mathsf{U}\ \psi] \equiv \neg(\mathsf{E}\ [\neg\psi\ \mathsf{U}\ \neg(\phi \vee \psi)] \vee \mathsf{E}\ \mathsf{G}\ \neg\psi)$

   (b) $\mathsf{A}\ \mathsf{G}\ \mathsf{A}\ \mathsf{F}\ p \equiv \mathsf{A}\ \mathsf{G}\ \mathsf{F}\ p$

2 Model Checking CTL and Fair CTL

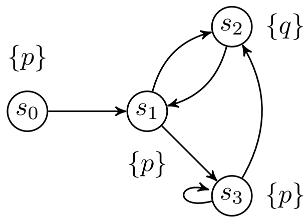

Figure 2:

1. For each of the CTL formulae below, draw a Kripke Structure in which the formula holds, a Kripke Structure in which it does not hold, but in which it does hold fairly with an appropriate fairness constraint. Also provide this fairness constraint.

   (a) $\mathsf{A}\ \mathsf{G}\ \mathsf{A}\ \mathsf{F}\ (\neg p \vee q)$

   (b) $q \wedge \mathsf{A}\ \mathsf{F}\ q \wedge \neg(\mathsf{E}\ [\neg q\ \mathsf{R}\ \neg p])$

   (c) $\neg A\mathsf{F}\ p \vee \mathsf{E}\ \mathsf{G}\ (\neg p \vee q)$

   (d) $(p \vee \mathsf{A}\ \mathsf{F}\ p) \wedge \neg \mathsf{E}\ \mathsf{G}\ p$

2. Determine for each of the following CTL formulae in which states of the Kripke Structure of Fig. 1 it holds using the labelling algorithm. Repeat the exercise using the symbolic model checking algorithm for CTL, using explicit set notation to represent sets of states, rather than BDDs.

   (a) p,

   (b) $\mathsf{E}\ [q\ \mathsf{R}\ p]$,

   (c) $\mathsf{A}\ \mathsf{G}\ \mathsf{E}\ \mathsf{F}\ p$,

   (d) $\mathsf{A}\ \mathsf{G}\ p \vee \mathsf{A}\ \mathsf{F}\ q$

   (e) $\mathsf{A}\ \mathsf{F}\ q$

   (f) $\mathsf{A}\ [q\ \mathsf{R}\ p]$

3. Extend the Kripke Structure of Fig. 1 with the Fairness constraints $F = \{\ \{s_1\}, \{s_2\}\ \}$. In which states do the formulae of exercise 2 *fairly* hold? Repeat the exercise using fairness constraint $F = \{\ \{s_3\}\ \}$.

4. Answer Exercises 2 and 3 for the Kripke Structure in Fig. 2 instead of the Kripke Structure of Fig. 1.

5. Prove that $\mathsf{A}\ \mathsf{F}\ f = \mu Z. f \cup \mathsf{A}\ \mathsf{X}\ Z$.

3 Counterexamples and Witnesses

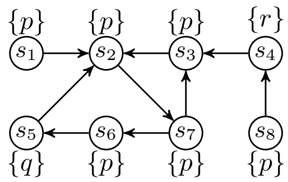

Figure 3:

1. Consider the Kripke Structure in Fig. 3.

   (a) Fairness constraint: $\neg r$ and q. Check that $s_1 \models_F \mathsf{E}\ \mathsf{G}\ (p \vee q)$.

   (b) Construct a witness for $s_1 \models_F \mathsf{E}\ \mathsf{G}\ (p \vee q)$, using the techniques for symbolic model checking.

4 Equivalences and Pre-Orders

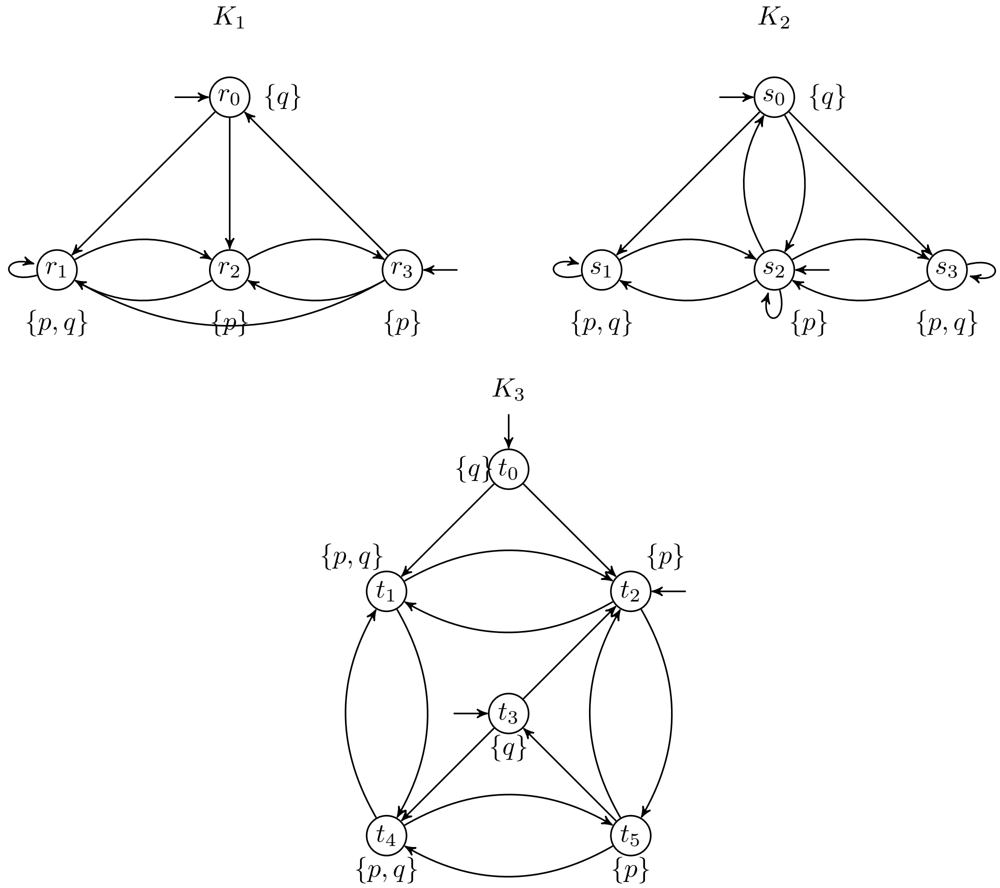

Figure 4:

1. Let K_1 and K_2 be arbitrary Kripke Structures. Let f be an arbitrary ACTL^* formula such that $K_1 \models f$ and $K_2 \not\models f$. Prove that there must be a formula g in positive form such that $K_1 \not\models g$ and $K_2 \models g$, and all path quantifiers in g are existential path quantifiers.

2. For each pair of Kripke Structures K_i, K_j in Fig. 4, prove or disprove $K_i \sqsubseteq K_j$, either by providing a simulation relation, or by providing a distinguishing ACTL-formula f (i.e., $K_i \models f$ and $K_j \not\models f$).

3. For each pair of Kripke Structures K_i, K_j in Fig. 4, prove or disprove $K_i \equiv K_j$, either by computing a bisimulation relation, or by providing a distinguishing CTL-formula f (i.e., $K_i \models f \Leftrightarrow K_j \not\models f$).