Algorithms for Model Checking (2IW55)

Lecture 2

Fairness & Basic Model Checking Algorithm for CTL and fair CTL
– based on strongly connected components –
Chapter 4.1, 4.2 + SIAM Journal of Computing 1(2), 1972

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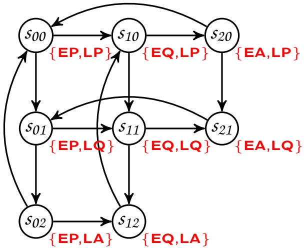
Outline

Temporal Logics: Fairness



- Atomic Propositions: EP, EQ, EA, LP, LQ, LA
- Intended meaning: Linus or Emma is either Playing, posing Questions, getting Answers
- To exclude runs in which one child gets all attention, we want that both $\neg\mathcal{E}Q$ as well as $\neg\mathcal{L}Q$ hold infinitely often
- fairness constraints ensuring this: $\mathcal{F} = \{\{s_{00}, s_{01}, s_{02}, s_{20}, s_{21}\}, \{s_{00}, s_{10}, s_{20}, s_{02}, s_{12}\}\}$

Temporal Logics: Fairness

Sometimes properties are violated by "unrealistic" paths only, for instance due to a scheduler. In this case, one may restrict to fair paths.

A Kripke Structure over *AP* with fairness constraints is a structure $\mathcal{M} = \langle \mathcal{S}, \mathcal{R}, \mathcal{L}, \mathcal{F} \rangle$, where:

- $\langle \mathcal{S}, \mathcal{R}, \mathcal{L} \rangle$ is an "ordinary" Kripke Structure as before
- $\mathcal{F} \subseteq 2^{\mathcal{S}}$ is a set of fairness constraints

A path is fair if it "hits" each fairness constraint infinitely often:

$$\mathsf{fair}(\pi) \quad \textit{iff} \quad \forall \mathcal{C} \in \mathcal{F}.\ \{i \mid \pi(i) \in \mathcal{C}\} \ \text{ is an infinite set}$$

Temporal Logics: Fairness

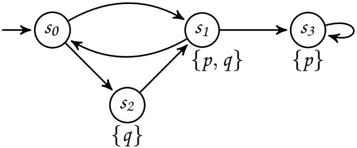
In CTL* with fairness semantics ($\models_{\mathcal{F}}$), only fair paths will be considered.

Given a fixed Kripke Structure with fairness constraints $\mathcal{M} = \langle \mathcal{S}, \mathcal{R}, \mathcal{L}, \mathcal{F} \rangle$, $s \models_{\mathcal{F}} f$ means: formula f holds in state s in the fair CTL* semantics.

The definition of $\models_{\mathcal{F}}$ coincides with $\models$ except for the following four clauses:

$s \models_{\mathcal{F}} \text{true}$	iff	there is some fair path starting in s
$s \models_{\mathcal{F}} p$	iff	$p \in \mathcal{L}(s)$ and there is some fair path starting in s
$s \models_{\mathcal{F}} \mathsf{A}\, f$	iff	for all fair paths π starting in s, we have $\pi \models_{\mathcal{F}} f$
$s \models_{\mathcal{F}} \mathsf{E}\, f$	iff	for some fair path π starting in s, we have $\pi \models_{\mathcal{F}} f$

Temporal Logics: Fairness



Note that $s_0 \models$ E F G p, but $s_0 \not\models$ A F G p

- First, consider as Fairness constraint: $\mathcal{F} = \{ \{s_3\} \}$
  - then all fair paths contain s_3 infinitely often
  - we have $s_0 \models_{\mathcal{F}}$ A F G p
- Next, consider as Fairness constraint: $\mathcal{F} = \{ \{s_2\} \}$
  - then all fair paths contain s_2 infinitely often
  - in particular, fair paths cannot contain s_3
  - so $s_0 \not\models_{\mathcal{F}}$ E F G p

Outline

Strongly Connected Components

Given a directed graph $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$

- let $s \rightarrow^*_{\mathcal{G}} t$ mean that there is a path from node s to t in $\mathcal{G}$
- a strongly connected component (SCC) is a maximal subgraph $\mathcal{S}$ of $\mathcal{G}$, such that for all $s, t \in \mathcal{S}$, $s \rightarrow^*_{\mathcal{G}} t$ and $t \rightarrow^*_{\mathcal{G}} s$
- an SCC is non-trivial if it contains at least one edge

The SCCs of a graph (e.g. a Kripke Structure) can be computed in $\mathcal{O}(|\mathcal{V}| + |\mathcal{E}|)$ time with an algorithm based on depth-first search:

- Text book version (see Introduction to Algorithms, Corben *et al*)
- Tarjan's original algorithm (see SIAM Journal on Computing 1(2), 1972)

The second algorithm is useful in model checking contexts

Strongly Connected Components

Idea behind Tarjan's SCC algorithm
Given is a directed graph $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$

- compute spanning trees by depth-first search; number the nodes in the order they are visited
- the other, non-tree edges are either:
  - forward edges (can be ignored)
  - backward edges (to an ancestor)
  - cross edges (to another subtree)

  backward and cross edges lead to nodes with smaller numbers
- nodes are kept on a stack; the nodes of a discovered SCC will be popped immediately from this stack
- compute $root[v]$: the smallest node which is:
  - reachable from v by a sequence of tree-edges followed by at most one non-tree edge; and
  - if $root[v] = v$, the root of a new SCC is found, and the whole SCC is popped from the stack

Strongly Connected Components

Procedure find_scc applies a repeated depth-first search on yet unprocessed nodes of the input graph $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$
The depth-first search is delegated to the procedure dfs_scc.

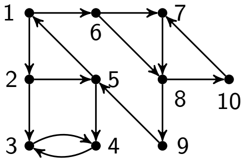
procedure find_scc
    $i := 0$;
    empty the stack;
    leave all nodes unnumbered;
    for vertices $w \in \mathcal{V}$ **do**
        if w is not yet numbered **then**
            dfs_scc(w);
        end if
    end for
end procedure

Strongly Connected Components

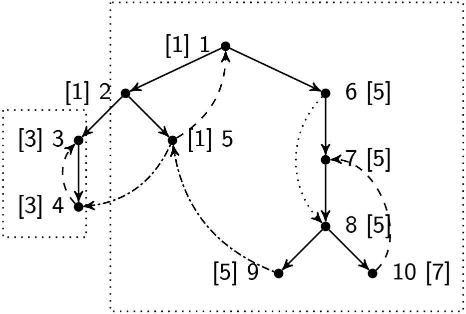
procedure dfs_scc(v)
    $root[v] := number[v] := i := i + 1$;
    push v on the stack;
    for successor w of v **do**
        if w is not yet numbered **then** {tree edge}
            dfs_scc(w);
            $root[v] := \min(root[v], root[w])$;
        else if $number[w] < number[v]$ and w on the stack **then** {cross/back edge}
            $root[v] := \min(root[v], number[w])$;
        end if
    end for
    if $root[v] = number[v]$ **then** {start new SCC}
        while top w of stack satisfies $number(w) \geq number(v)$ **do**
            pop w from stack;
        end while
    end if
end procedure

Strongly Connected Components

Example: SCC algorithm



A possible run of the SCC algorithm, with DFS node numbers, final root-values (in square brackets), tree edges (plain arrow), forward edges (dotted), back edges (dashed), cross edges (dash/dot). Two SCCs are found: number and root value are equal



Strongly Connected Components

We analyse the space and time requirements for running find_scc on a graph $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$:

- for every node:
  - dfs_scc is called exactly once
  - all its outgoing edges are explored exactly once
- each node is pushed and popped from the stack exactly once
- checking whether a node is on the stack can be done in constant time, for instance by maintaining a Boolean array

Conclusion: Tarjan's algorithm for finding strongly connected components runs in time and space $\mathcal{O}(|\mathcal{V}| + |\mathcal{E}|)$

Outline

CTL Model Checking Algorithm

Recall that CTL has the following ten temporal operators:

- A X and E X : for all/some next state
- A F and E F : inevitably and potentially
- A G and E G : invariantly and potentially always
- A [U] and E [U]: for all/some paths, until
- A [R] and E [R]: for all/some paths, releases

Besides atomic propositions (AP), the constant true and the Boolean connectives ($\neg, \vee$), the following temporal operators are sufficient: E X , E G , E [U].

Hence: only algorithms for computing formulae of the above form are needed.

CTL Model Checking Algorithm

Main loop of model checking CTL: check formula f on a Kripke Structure $\langle \mathcal{S}, \mathcal{R}, \mathcal{L} \rangle$.

By recursion on f, algorithm mc_ctl(f) computes $label(s)$ for all states $s \in \mathcal{S}$, where $label(s)$ shall contain those subformulae of f that hold in s.

Algorithm mc_ctl(f) employs a case distinction on the structure of f:

$f = p$	add p to $label(s)$ for those states s with $p \in \mathcal{L}(s)$
$f = g_0 \vee g_1$	mc_ctl(g_0); mc_ctl(g_1); add f to all states labelled with g_0 or g_1
$f = \neg g$	mc_ctl(g); add f to all states not labelled with g
$f = \mathsf{E}\ \mathsf{X}\ g$	mc_ctl(g); add f to all states with an $\mathcal{R}$-successor labelled by g
$f = \mathsf{E}\ [g_0\ \mathsf{U}\ g_1]$	mc_ctl(g_0); mc_ctl(g_1); check_eu(g_0, g_1)
$f = \mathsf{E}\ \mathsf{G}\ g$	mc_ctl(g); check_eg(g)

Upon termination, $s \models f$ if and only if $f \in label(s)$

CTL Model Checking Algorithm

procedure check_eu(f,g)
    $\mathcal{T} := \{s \mid g \in label(s)\}$;
    for all $s \in \mathcal{T}$ **do** $label(s) := label(s) \cup \{\mathsf{E}\ [f\ \mathsf{U}\ g]\}$;
    end for
    while $\mathcal{T} \neq \emptyset$ **do**
        choose $s \in \mathcal{T}$;
        $\mathcal{T} := \mathcal{T} \setminus \{s\}$;
        for all t satisfying $t\ \mathcal{R}\ s$ **do**
            if $\mathsf{E}\ [f\ \mathsf{U}\ g] \notin label(t)$ and $f \in label(t)$ **then**
                $label(t) := label(t) \cup \mathsf{E}\ [f\ \mathsf{U}\ g]$;
                $\mathcal{T} := \mathcal{T} \cup \{t\}$;
            end if
        end for
    end while
end procedure

Observations:

- label all states where g holds
- search backwards over states where f holds

CTL Model Checking Algorithm

procedure check_eg(f)
  $S' := \{s \mid f \in label(s)\}$;
  $SCC := \{C \mid C$ is a nontrivial SCC of $S'\}$;
  $T := \bigcup_{C \in SCC} \{s \mid s \in C\}$;
  for all $s \in T$ **do** $label(s) := label(s) \cup \{\mathrm{E\ G}\ f\}$;
  end for
  while $T \neq \emptyset$ **do**
    choose $s \in T$;
    $T := T \setminus \{s\}$;
    for all t satisfying $t \in S'$ and $t\ R\ s$ **do**
      if $\mathrm{E\ G}\ f \notin label(t)$ **then**
        $label(t) := label(t) \cup \{\mathrm{E\ G}\ f\}$;
        $T := T \cup \{t\}$;
      end if
    end for
  end while
end procedure

Observations:

- restrict attention to subgraph where f holds
- an infinite path in a finite graph eventually reaches a non-trivial SCC

CTL Model Checking Algorithm

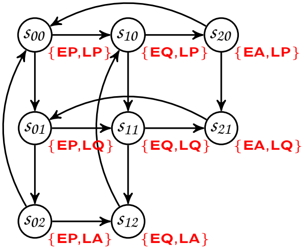
We analyse the time complexity for the standard CTL model checking algorithm of formula f (with $|f|$ the number of subformulae) on Kripke Structure $\mathcal{M} = \langle \mathcal{S}, \mathcal{R}, \mathcal{L} \rangle$.

- There are at most $|f|$ calls to mc_ctl
- Backward reachability and detecting strongly connected components can be done in time linear to the Kripke Structure: $\mathcal{O}(|\mathcal{S}| + |\mathcal{R}|)$
- Hence, each recursive call takes at most $\mathcal{O}(|\mathcal{S}| + |\mathcal{R}|)$ time

So, the complexity of this CTL model checking algorithm is $\mathcal{O}(|f| \cdot (|\mathcal{S}| + |\mathcal{R}|))$, which is linear in both the formula and the state space.

Outline

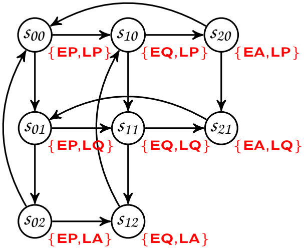
Example: demanding children



- Atomic Propositions: EP, EQ, EA, LP, LQ, LA
- Intended meaning: Linus or Emma is either Playing, posing Questions, getting Answers

Requirement: Whenever Linus asks a question, he eventually gets an answer
Formula: A G $(\mathcal{LQ} \rightarrow$ A F $\mathcal{LA})$

Example: demanding children

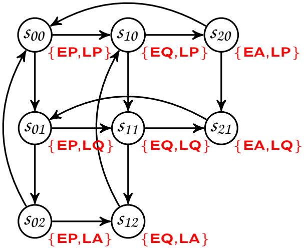


- Atomic Propositions: EP, EQ, EA, LP, LQ, LA
- Intended meaning: Linus or Emma is either Playing, posing Questions, getting Answers

- *Step 1: express using basic operators*

$$
\begin{array}{ll}
 & \mathsf{A}\ \mathsf{G}\ (\mathcal{LQ} \rightarrow \mathsf{A}\ \mathsf{F}\ \mathcal{LA}) \\
\equiv & \\
 & \neg \mathsf{E}\ [\mathsf{true}\ \mathsf{U}\ \neg(\neg \mathcal{LQ} \vee \neg \mathsf{E}\ \mathsf{G}\ \neg \mathcal{LA})]
\end{array}
$$

Example: demanding children



- *Step 2: treat* E G $\neg\mathcal{LA}$
    - Restrict to the subgraph where $\neg\mathcal{LA}$ holds
    - Find non-trivial SCCs
    - Backward reachability

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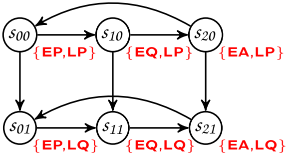

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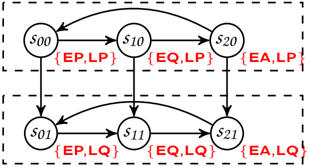

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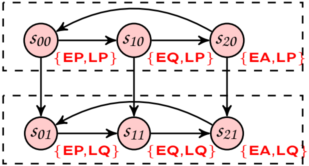
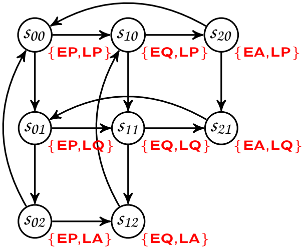


- *Step 2: treat* $\mathsf{E}\ \mathsf{G}\ \neg\mathcal{LA}$
  - Restrict to the subgraph where $\neg\mathcal{LA}$ holds
  - Find non-trivial SCCs
  - Backward reachability

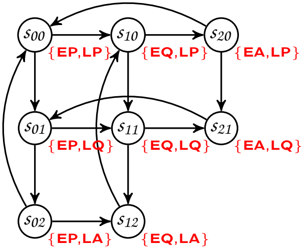
No new states are found. So, $\mathsf{E}\ \mathsf{G}\ \neg\mathcal{LA}$ holds in the states $\{s_{00}, s_{10}, s_{20}, s_{01}, s_{11}, s_{21}\}$;

Example: demanding children



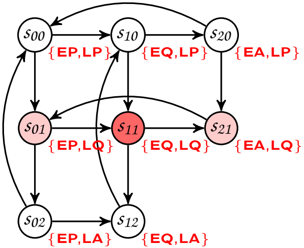
- *Step 3: treat* $\neg$E G $\neg\mathcal{LA}$
  - E G $\neg\mathcal{LA}$ holds in $\{s_{00}, s_{10}, s_{20}, s_{01}, s_{11}, s_{21}\}$, so $\neg$E G $\neg\mathcal{LA}$ holds in $\{s_{02}, s_{12}\}$
- *Step 4: treat* $\neg\mathcal{LQ}$
  - $\neg\mathcal{LQ}$ holds in $\{s_{00}, s_{10}, s_{20}, s_{02}, s_{12}\}$
- *Step 5: treat* $\neg\mathcal{LQ} \vee \neg$E G $\neg\mathcal{LA}$
  - $\neg\mathcal{LQ} \vee \neg$E G $\neg\mathcal{LA}$ holds in $\{s_{00}, s_{10}, s_{20}, s_{02}, s_{12}\} \cup \{s_{02}, s_{12}\} = \{s_{00}, s_{10}, s_{20}, s_{02}, s_{12}\}$

Example: demanding children



- *Step 6: treat* $\neg(\neg\mathcal{LQ} \vee \neg \mathsf{E}\ \mathsf{G}\ \neg\mathcal{LA})$
  - $\neg\mathcal{LQ} \vee \neg \mathsf{E}\ \mathsf{G}\ \neg\mathcal{LA}$ holds in $\{s_{00}, s_{10}, s_{20}, s_{02}, s_{12}\}$, so $\neg(\neg\mathcal{LQ} \vee \neg \mathsf{E}\ \mathsf{G}\ \neg\mathcal{LA})$ holds in $\{s_{01}, s_{11}, s_{21}\}$
- *Step 7: compute* $\mathsf{E}\ [\mathit{true}\ \mathsf{U}\ \neg(\neg\mathcal{LQ} \vee \neg \mathsf{E}\ \mathsf{G}\ \neg\mathcal{LA})]$
  - Start in $\{s_{01}, s_{11}, s_{21}\}$
  - Perform a backward reachability analysis over states for which true holds

Example: demanding children



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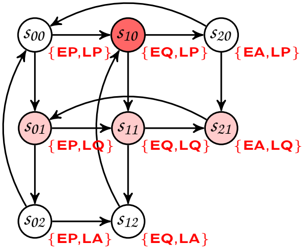

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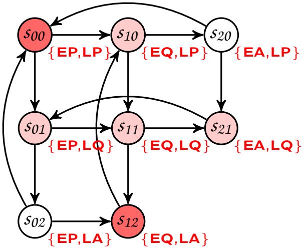

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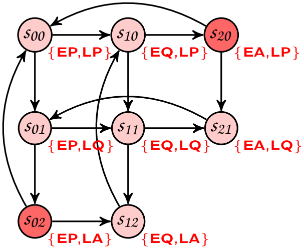

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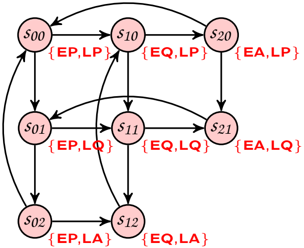

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Example: demanding children

Conclusion:

- So, E $[\text{true U } \neg(\neg\mathcal{LQ} \vee \neg\text{E G } \neg\mathcal{LA})]$ holds in all states
- Hence, its negation A G $(\mathcal{LQ} \rightarrow \text{A F } \mathcal{LA})$ holds in no state
- The requirement does not hold for the full Kripke Structure
- Why? Because in this case, there is a path in which only Emma progresses while Linus is not being served.
- Next, we look at the Kripke Structure with Fairness Constraints

Outline

CTL Model Checking with Fairness

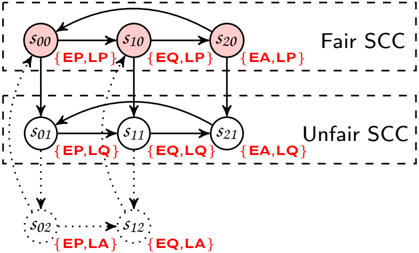
Recall: Kripke Structure $\mathcal{M} = \langle \mathcal{S}, \mathcal{R}, \mathcal{L}, \mathcal{F} \rangle$ with fairness constraints $\mathcal{F} \subseteq 2^{\mathcal{S}}$.

- A path is fair if it "hits" each fairness constraint infinitely often
- A fair SCC is an SCC that contains an element from each constraint $\mathcal{C} \in \mathcal{F}$

Main idea of fair model checking for CTL:

- Special treatment for $s \models_{\mathcal{F}} \mathsf{E\ G}\ f$: check_fair_eg
  - Restrict attention to $\mathcal{S}' \subseteq \mathcal{S}$ where f holds
  - Find a path to a fair non-trivial SCC in $\mathcal{S}'$
- Label states where E G true fairly holds with a new proposition symbol *fair*
- Treat the other operators using the original "unfair" procedures:
  - $s \models_{\mathcal{F}} p$ .... $s \models p \wedge \mathit{fair}$
  - $s \models_{\mathcal{F}} \mathsf{E\ X}\ f$ .... $s \models \mathsf{E\ X}\ (f \wedge \mathit{fair})$
  - $s \models_{\mathcal{F}} \mathsf{E}\ [f\ \mathsf{U}\ g]$ .... $s \models \mathsf{E}\ [f\ \mathsf{U}\ (g \wedge \mathit{fair})]$

CTL Model Checking with Fairness



- Assume fairness constraints $\neg\mathcal{EQ}$ and $\neg\mathcal{LQ}$.
- Remark: full graph is one big fair SCC, so E G true holds everywhere

- E G $\neg\mathcal{LA}$:
  - Restrict to subgraph with $\neg\mathcal{LA}$
  - Find fair non-trivial SCCs
  - Do backward reachability
- Hence: $\mathcal{LQ} \wedge$ E G $\neg\mathcal{LA}$ holds fairly in NO state
- Hence E F $(\mathcal{LQ} \wedge$ E G $\neg\mathcal{LA})$ holds nowhere fairly
- Hence, its negation, the requirement A G $(\mathcal{LQ} \rightarrow$ A F $\mathcal{LA})$ fairly holds everywhere!

Outline

Summary

CTL model checking:

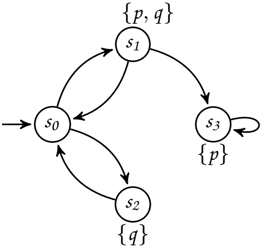
- SCC algorithm is used
- Tarjan's SCC algorithm runs one depth-first search, computing SCCs on-the-fly. Time complexity is linear
- CTL model checking can be done in time linear in the size of the formula as well as in the Kripke Structure
- Extension with Fairness Constraints is straightforward and is useful in practice
- Why not treat fairness in formulae?

$$\mathsf{A}\ [(\mathsf{G}\ \mathsf{F}\ \mathcal{C}_1 \wedge \mathsf{G}\ \mathsf{F}\ \mathcal{C}_2) \rightarrow \mathit{Requirement}]$$

  - fairness cannot be expressed in CTL
  - for LTL all known algorithms are exponential in the size of the formula

Outline

Exercise



CTL formulae: p, E $[q$ R $p]$, A G E F p, A G $p \vee$ A F q

- Determine for each formula in which states of the above Kripke Structure it holds; use both the semantics and use the appropriate algorithms
- Extend the Kripke structure with the Fairness constraints $\mathcal{F} = \{\ \{s_1\}, \{s_2\}\ \}$. In which states do the above formulae *fairly* hold?
- Similarly for the Fairness constraint $\mathcal{F} = \{\ \{s_3\}\ \}$