Foreword

This book, *Type Theory and Formal Proof: An Introduction*, is a gentle, yet profound, introduction to systems of types and their inhabiting lambda-terms. The book appears shortly after *Lambda Calculus with Types* (Barendregt *et al.*, 2013). Although these books have a partial overlap, they have very different goals. The latter book studies the mathematical properties of some formalisms of types and lambda-terms. The book in your hands is focused on the use of types and lambda-terms for the complete formalisation of mathematics. For this reason it also treats higher order and dependent types. The act of defining new concepts, essential for mathematical reasoning, forms an integral part of the book. Formalising makes it possible that arbitrary mathematical concepts and proofs be represented on a computer and enables a machine verification of the well-formedness of definitions and of the correctness of proofs. The resulting technology elevates the subject of mathematics and its applications to its maximally complete and reliable form.

The endeavour to reach this level of precision was started by Aristotle, by his introduction of the axiomatic method and quest for logical rules. For classical logic Frege completed this quest (and Heyting for the intuitionistic logic of Brouwer). Frege did not get far with his intended formalisation of mathematics: he used an inconsistent foundation. In 1910 Whitehead and Russell introduced types to remedy this. These authors made proofs largely formal, except that substitutions still had to be understood and performed by the reader. In 1940 Church published a system with types, based on a variant of those of Whitehead and Russell, in which the mechanism of substitution was captured by lambda-terms and conversion. Around 1970 de Bruijn essentially extended the formalism of types by introducing *dependent types* with the explicit goal to formalise and verify mathematics. By 2004 this technique was perfected and George Gonthier established, using the mathematical assistant Coq, a full formalisation of the Four Colour Theorem.

The learning curve to formalise remains steep, however. One still needs to be an expert in a mathematical assistant in order to apply the technique. I hope and expect that this book will contribute to the familiarisation of formalising mathematical proofs and to improvements in the mathematical assistants, bringing this technique within the reach of the working mathematician and computer scientist.

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