Ternary Blending Operations

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Abstract

We discuss new analytical formulations for localized and controllable blending operations in the function-based solid modeling. The blending set operations are defined using R-functions and displacement functions with the localized area of influence. The shape and location of the blend are controlled by an additional bounding solid thus turning the operation into a ternary one. We also describe a new approach to solving the problem of shape metamorphosis between k-dimensional shapes by applying space-time bounded blending to the specially constructed (k+1)-dimensional half-cylinders and making cross-sections for getting intermediate shapes under the transformation.

1 Blending in solid modeling

Blending operations in solid modeling generate smooth transitions between two or several surfaces. Blending is also considered a natural property of implicit surfaces, where the basic operation is an algebraic sum (or difference) between skeleton-based scalar fields. Blending operations are typically used in computer-aided design for modeling fillets and chamfers. These operations are usually smooth versions of set-theoretic operations on solids (intersection, union, and difference), which approximate exact results of these operations by rounding sharp edges and vertices.

The major requirements to blending operations [1] are tangency of the blend surface with the initial surfaces, automatic clipping of unwanted parts of the blending surface, C^1 continuity of the blending function everywhere in the domain, support of added and subtracted material blends. Special attention is paid to the intuitive control of the blend shape and position: the construction of the blend and its parameters should have clear geometric interpretation.

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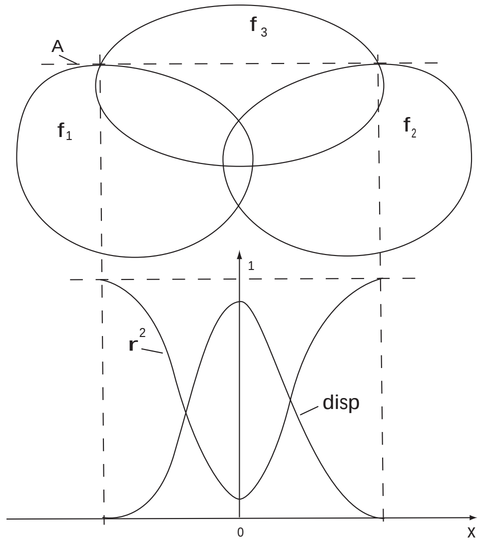

Figure 1: Components of the definition of the ternary bounded blending union operation.

2 Bounded blending

To satisfy most of the above requirements for solids exactly represented by continuous real functions (with implicit surfaces as boundaries), we introduce bounded blending operations defined using R-functions [2] and displacement functions with the localized area of influence. The shape and location of the blend is defined by an additional bounding solid thus making the ternary blending operation (having three solids as arguments).

Let two initial solids be described by the inequalities $f_1(X) \geq 0$ and $f_2(X) \geq 0$, and the bounding solid be described as $f_3(X) \geq 0$, where f_i are continuous real functions, and X is a vector of point coordinates. The bounded blending operation can be defined as

$$F_b(f_1, f_2, f_3) = R(f_1, f_2) + disp_b(f_1, f_2, f_3)$$

where R stands for an R-function defining one of the set-theoretic operations, and $disp_b$ is a displacement function. For example, a union operation can be exactly defined by the following R-function:

$$R(f_1, f_2) = f_1 + f_2 + \sqrt{{f_1}^2 + {f_2}^2}$$

The relatioships between the components of the definition of the ternary bounded blending union operation are shown in Fig. 1. In the upper part of the figure two solids to be blended (f_1 and f_2) and a bounding solid f_3 are shown. The lower part illustrates the behavior of the functions $disp_b$ and r in the cross-section A of the bounding solid.

It is required that the blending surface exists only inside the bounding solid, and only initial surfaces exist outside the bounding solid. Therefore, the displacement function $disp_b(r)$, where $r = r(f_1, f_2, f_3)$ is a generalized distance from the initial surfaces, has to satisfy the following conditions:

1) $disp_b(r) \geq 0$, it takes the maximal value for $r = 0$, and the displacement is symmetric in respect to the initial defining functions;

2) $disp_b(r) = 0, r \geq 1$, is a condition of the blend localization inside the bounding solid;

3) $\partial disp_b/\partial r = 0, r = 1$, means the curve $disp_b(r)$ tangentially approaches the horizontal axis at $r = 1$ and accordingly the blend tangentially approaches initial surfaces.

There are many different functions satisfying the above requirements. Here, we use the polynomial function of lowest order:

$$disp_b(r) = 2r^3 - 3r^2 + 1$$

A different displacement function and details of the generalized distance $r(f_1, f_2, f_3)$ function construction can be found in [3].

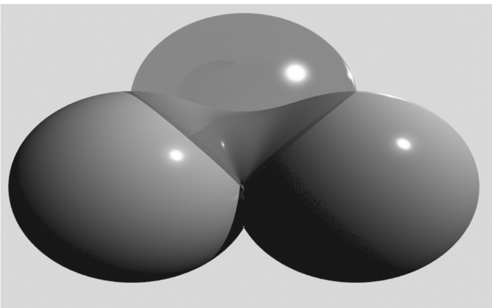

Figure 2: Ternary blending operation: two ellipsoids to be blended and the boundung ellipsoid (transparent).

Let us illustrate such properties of the proposed bounded blending operation as its local character and intuitive control of blend shape and position. In Fig. 2 the pure union of two ellipsoids is changed to the bounded blending union using the third ellipsoid (transparent shape). The resulting blend is located strictly inside the bounding ellipsoid, which produces an unusual blending shape localized at the top part of the initial union of ellipsoids. At the next step (Fig. 3), we increase the size of the bounding ellipsoid and correspondingly change the shape of the blend, which stretches out to the lower part of the initial shape.

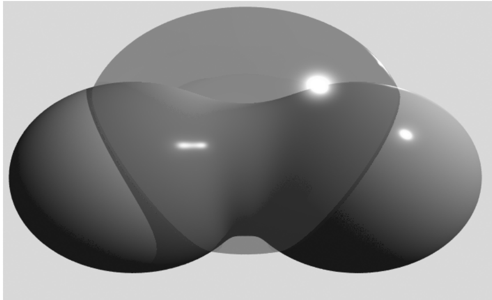

Figure 3: Control of blending by changing the bounding solid.

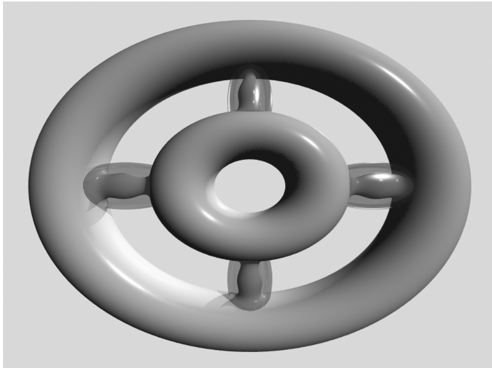

Figure 4: Multiple blending controlled by a bounding solid consisting of four disjoint components.

The definition of the bounding solid by a single function allows for unusual operations such as multiple blending. As it is shown in Fig. 4, the bounding solid can be constructed using arbitrary primitives and operations. In this example, the bounding solid controls the blending union of two non-intersecting tori. The bounding solid is described using R-functions by a single function f_3 as union of four ellipsoids. The result of the bounded blending operation is a single connected solid with multiple blending components located inside the disjoint components of the bounding solid.

The proposed bounded blending operations can replace pure set-theoretic operations in the construction of a solid without rebuilding the entire construction tree data structure. Note that the ternary bounded blending operations have three solids as their arguments and hence require ternary nodes in the construction tree in comparison with binary nodes typical for the traditional Constructive Solid Geometry (CSG).

3 Space-time blending

Shape transformation between given objects (metamorphosis) is one of typical space-time modeling operations. The existing approaches to metamorphosis are based on one or several of the following assumptions: equivalent topology (mainly topological disks or balls are considered), polygonal shape representation, shape alignment (shapes have common coordinate origin and significantly overlap), possibility of shape matching (establishing of shape vertex-vertex, control points or other features correspondence), the resulting transformation should be close to the motion of an articulated figure.

Linear interpolation between functionally defined shapes have proven to solve some of the above problems for computer animation and artistic applications. The problem which remains open is a transformation between non-overlapping shapes, which combines metamorphosis and non-linear motion. We develop a new approach to shape metamorphosis using blending operations in space-time. The key steps of the metamorphosis algorithm are: dimension increase by converting two input kD shapes into half-cylinders in $(k+1)D$ space-time, applying bounded blending union to the half-cylinders, and making cross-sections for getting intermediate shapes [4].

The bounded space-time blending procedure for 2D shapes consists of the following steps:
1) two initial 2D shapes are given on a 2D plane;
2) each shape is considered as a 2D cross-section of a half-cylinder (a semi-infinite cylinder bounded by a plane from one side along the time axis) defined in 3D space-time;
3) two half-cylinders are placed at some distance along time axis to provide a time interval for making the blend;
4) the bounded blending union operation with added material is applied to the 3D half-cylinders with two planar half-spaces orthogonal to the time axis forming a bounding 3D object (a slab between two planes);
5) consecutive cross-sections of the blend along the time axis are combined into a 2D animation.

As no critical assumptions were made in the proposed approach about the dimensionality of the initial shapes, we can apply it to 3D objects. In this case, each shape is considered as a 3D cross-section of a half-cylinder defined in 4D space-time. Note that in both 2D and 3D cases topological changes of objects are handled automatically as shown in Fig. 5.

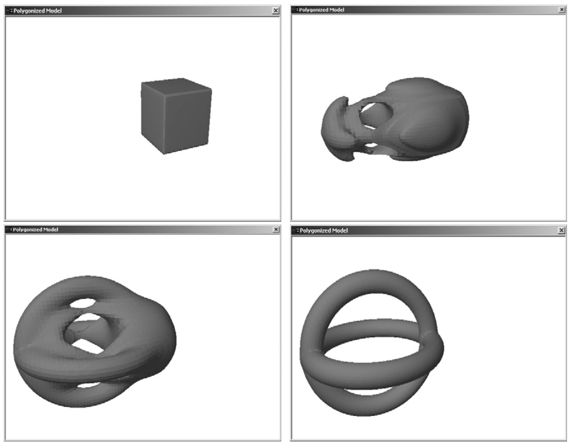

Figure 5: Metamorphosis of a cube into the union of two tori using space-time bounded blending.

4 Conclusion

We proposed an original solution for a long-standing problem of the blend localization and control. The main idea is to apply localized displacements to the standard R-functions describing pure set-theoretic operations. This allows for support of several unusual operations such as multiple blending or partial edge blending, which hardly can be supported by other modeling techniques.

We discribed a new approach to shape metamorphosis on the basis of the dimension increase, bounded blending between higher-dimensional space-time objects, and cross-sectioning the blend area for getting frames of the animation. The space-time blending operation with such simple bounds as two planes satisfies the condition of the localization of the shape transformation at some predefined time interval. The proposed approach can handle non-overlapping 2D and 3D shapes with arbitrary topology.

The described bounded blending operations have three objects as their arguments. This brings a new requirement for a functionally based modeling system to support n-ary nodes in the construction tree.

References

[1] J. Woodwark. Blends in geometric modeling. In *Proc. The Mathematics of Surfaces II*, pages 255–297., Clarendon Press, 1987.

[2] V. Rvachev. *Theory of R-functions and Some Applications*, Naukova Dumka, Kiev, USSR, 1987 (in Russian).

[3] G. Pasko, A. Pasko, T. L. Kunii. Bounded blending for the function-based shape modeling. *IEEE Computer Graphics and Applications*, 2005 (to appear)

[4] G. Pasko, A. Pasko A., T. L. Kunii. Space-time blending, *Journal of Computer Animation and Virtual Worlds*, vol. 15, No. 2, pages 109–121, John Wiley, 2004.