A matching-based heuristic approach to Vehicle Routing Problem with Time Windows

Murat Fırat

Post-doc researcher under DAIPEX Project
Mathematics & Computer Science, TU/e

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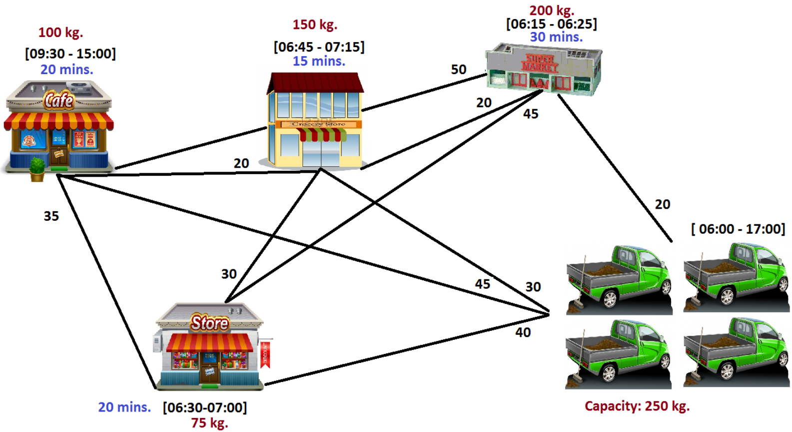
A milk Company



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  - every arc $(i, j) \in A$ has cost c_{ij} and travel time t_{ij}.

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- Objectives:
  - Minimize (1) number of vehicles used, (2) total travel distance.

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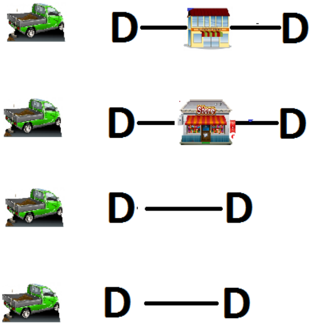
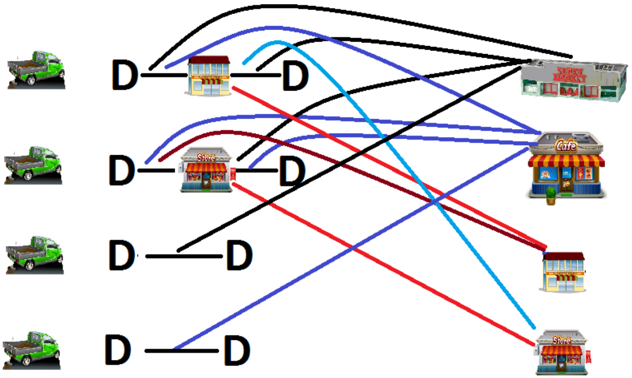
  - existing customers may be swapped among routes.

Intialization



Bipartite graph of insertions



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- Objective: Minimize the extra travel due to insertion of customers.
- Assigning a customer to an empty vehicle has high cost.

Solomon instances: 100 clustered customers, short routes

Instance	$\#Routes(B*)$	Travel dist.(B)	$\#Routes(H**)$	Travel dist.(H)
c101	10	828.94	10	828.94
c102	10	828.94	10	1036.83
c103	10	828.06	10	1119.47
c104	10	824.78	10	1312.11
c105	10	828.94	10	860.79
c106	10	828.94	10	900.64
c107	10	828.94	10	995.32
c108	10	828.94	10	912.73
c109	10	828.94	10	1111.33

$*$ Best known solution values.

$**$ Solution values of our heuristic.

Coded in JAVA, LP Solver: CPLEX 12.0, Processor: 2.7 GHz, RAM: 8GB.

Solomon instances: 100 clustered customers, long routes

Instance	$\#Routes(B*)$	Travel dist.(B)	$\#Routes(H**)$	Travel dist.(H)
c201	3	591.56	3	624.38
c202	3	591.56	3	908.14
c203	3	591.17	3	939.50
c204	3	590.6	3	1032.29
c205	3	588.88	3	731.37
c206	3	588.49	3	846.00
c207	3	588.29	4	749.532
c208	3	588.32	3	792.77

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Solomon instances: 100 randomly located customers

Instance	$\#Routes(B*)$	Travel dist.(B)	$\#Routes(H**)$	Travel dist.(H)
r101	19	1650.8	19	1680.46
r102	17	1486.12	17	1577.41
r103	13	1292.68	13	1362.71
r104	9	1007.31	10	1173.78
r105	14	1377.11	14	1568.73
r106	12	1252.03	12	1568.73
r107	10	1104.66	11	1248.55
r108	9	960.88	10	1200.33
r109	11	1194.73	12	1372.47
r110	10	1118.84	11	1309.13
r111	10	1096.72	11	1246.59
r112	9	982.14	10	1129.07

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- We construct good-quality solutions w.r.t number of vehicles.
- There is (always) more room to improve this heuristic.
- We will adapt this heuristic to VRPTW with stochastic travel times.
- We will use this heuristic in a Branch-and-Price method for VRPTW to find good routes.

Thanks for your attention