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Professor N. G. deBruijn
Combinatorial Conference
c/o Faculty of Mathematics
University of Waterloo
Waterloo, Ontario
Canada

Dear Dick:

I think I can demonstrate the usefulness of that idea of "subsorts" which I mentioned to you last week.

Enclosed is a proof that equivalence relations determine a partition, written in the extension of your language which I am proposing. The proof has three parts: Chapter 1 introduces the boolean operations and quantifiers; Chapter 2 introduces some aspects of set theory; and Chapter 3 is the proof itself.

When I write

$$\alpha := \text{PN} \quad \underline{\text{sort}}(\xi)$$

I mean α is a subsort of ξ . Then if y is of sort α , and if f is a "function" $[x \xi] \xi_1$, I am allowed to write $\{y\}f$ and the latter expression is of sort ξ_1 .

Furthermore constructions such as

$$\begin{array}{l} \boxed{x \text{ --- } \xi} \\ \quad \theta := \Sigma(x) \quad \xi_1 \\ \\ \boxed{x \text{ --- } \alpha} \\ \quad \theta := \Gamma(x) \quad \xi_2 \end{array}$$

may be used; in these circumstances $\theta(y)$ is defined to be $\Gamma(y)$, of sort ξ_2 . In other words I allow the symbol θ to be defined twice, both for sort ξ and its subsort α ; the definition of $\theta(y)$ which uses the smallest sort containing y is always used.*

* Or maybe it is better to let either definition be used

I know your initial reaction to this idea will be somewhat skeptical, but I hope you will have time to study the example anyway since I believe it is an effective demonstration of the usefulness of subsorts. My claim is that no matter how much you increase the size of Chapter 1 and 2 you will not be able to prove the results about equivalence relations without using about 5 times as much space and effort in Chapter 3, if you work entirely in your language as it is now defined. The use of subsorts makes it possible for me to cut through most of the red tape and the circumlocutions which seem to be inevitable without subsorts. Furthermore the enclosed solution seems to mirror quite nicely a formal proof that a working mathematician would give. And I don't think the subsort extension puts much of an extra burden on the computer program which processes texts in the language.

Best regards,

Donald E. Knuth

DM:ld

P.S. Thanks very much for the letter you wrote me from O'Hare field, which I just received. I am glad to have had this chance to learn so many things from you during the last month.