Single- and two-phase flows are highly encountered in several industrial applications, such as drilling for oil and gas. Isothermal Euler equations and Drift Flux Model (DFM) are commonly used to mathematically describe the behavior of single- and two-phase flows inside a pipe. These two sets of equations are as follows (Left: Isothermal Euler, Right: DFM):

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} &= 0, \\
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} &= S,
\end{align*}
\]

\[
\begin{align*}
\frac{\partial (\alpha_l \rho_l)}{\partial t} + \frac{\partial (\alpha_l \rho_l u_l)}{\partial x} &= 0, \\
\frac{\partial (\alpha_g \rho_g)}{\partial t} + \frac{\partial (\alpha_g \rho_g u_g)}{\partial x} &= 0, \\
\frac{\partial (\alpha_l \rho_l u_l + \alpha_g \rho_g u_g)}{\partial t} + \frac{\partial (\alpha_l \rho_l u_l^2 + \alpha_g \rho_g u_g^2 + p)}{\partial x} &= S,
\end{align*}
\]

where \(\rho, u, p\) and \(\alpha\) are density, velocity, pressure and volume fraction of each phase, respectively. Subscripts \(l\) and \(g\) stand for liquid and gas phase, respectively. In addition, \(S\) represents the source terms. To put it in a nutshell, these two models describe the conservation of mass and momentum in the system. Energy-based modeling of such systems encodes the conservation laws directly into the structure of the model. Moreover, this kind of modelling is beneficial to preserve conservative variables at all levels, including the modeling level and the discretization level.

Systems of conservation laws with boundary inputs might be cast into **port-Hamiltonian (pH)** formulation. This formulation implies passivity, i.e., energy is not generated inside the system. Isothermal Euler equations admit such a formulation at the modelling and the discretization level, i.e., it can be formulated as follows:

\[
\begin{align*}
\dot{x} &= (I - R) \nabla_x \mathcal{H}(x) + (B - P) u, \\
y &= (B + P)^T \nabla_x \mathcal{H}(x) + (S + N) u,
\end{align*}
\]

where the function \(\mathcal{H}(x)\) is the Hamiltonian function describing the distribution of the energy in the system and \(J, R, B, P, S\) and \(N\) are system matrices with specific properties. Most importantly, \(W = \begin{bmatrix} R & P \\ p^T & S \end{bmatrix}\) should be positive definite.

To the best of our knowledge, no research has focused on pH formulation of the DFM at any level. Thus, the perspective for the future research would be casting the DFM into pH framework encompassing the modeling and discretization level.

**Tasks**
- Literature review (provided by the supervisors);
- Implementing pH for isothermal Euler equations in Matlab;
- Assessing the possibility of casting DFM into pH framework;
- Gaining knowledge on mixed finite element method, as a tentative approach for the discretization;
- Implementing pH formulation for the DFM of in Matlab.

**Requirements**
- Basic programming skills in Matlab;
- Knowledge of Partial differential equations.

**Supervisor**

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