Boundary Layer - Acoustic Liner Instabilities

Mirela Darau

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Outline

1. Where It All Started..
2. The Model
3. Stability Analysis
4. Piecewise Linear Shear Layer
5. Current and Future Work
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4. Piecewise Linear Shear Layer
5. Current and Future Work
Turbofan Aircraft Engines

- aircraft certification includes complying to noise regulation → aircraft cannot be sold if they make too much noise
- principal noise source in an aircraft: the engine
- significant noise sources: interaction noise of the fan via the rotor-stator interaction (dominant with landing), the exhaust jet (dominant at take off), and the compressor and turbine of the core engine
- methods of noise suppression: by basic design and by use of acoustically absorbent linings
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The Acoustic Liner
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Mass-Spring-Damper

- assumptions: wall behaves like a mass-spring-damper
- so at \( y = 0 \):
  \[
  m \frac{\partial^2 v}{\partial t^2} + R \frac{\partial v}{\partial t} + Kv = -\frac{\partial p}{\partial t}
  \]
- for time-harmonic motion \( (\sim e^{i\omega t}) \)
  \[
  \frac{\hat{p}}{-\hat{v}} = i\omega m + R + \frac{K}{i\omega} =: Z(\omega).
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Ingard-Myers Boundary Condition

- for the **mean flow** the wall is solid: \(((U_0, 0) \cdot n) = 0\) at \(y = 0\)
- for the **acoustic field** the wall is soft
- at the wall:

\[ \hat{p} = Z((\hat{u}, \hat{v}) \cdot n), \quad Z \in \mathbb{C}. \]

- the limit \(h \downarrow 0\) is usually taken for a point near the wall but still (just) inside the mean flow

\[ (i\omega + (U_0, 0) \cdot \nabla)\hat{p} = i\omega Z((\hat{u}, \hat{v}) \cdot n), \quad \text{at} \quad y = 0^{+}. \]
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The Problem: Instability in the Model
Conclusions and Conjectures

- no instabilities seen in practice (except for two isolated cases: Auregan, Ronneberger)
- conjecture: the flow - stable for small but finite $h$
- the critical $h$ depends on the impedance $Z$, since there is no other length scale in the problem
- we scale $h$ versus $\frac{B}{\rho_0 U}$, $\frac{m}{\rho_0}$, $\frac{\rho_0 U^2}{K}$,

\[ Z(\omega) = i\omega m + R + \frac{K}{i\omega} \]
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Euler Equations

The governing model equations:

\[
\rho \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} \right) + \frac{\partial p}{\partial x} = 0
\]

\[
\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + u \frac{\partial v}{\partial x} \right) + \frac{\partial p}{\partial y} = 0
\]

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho u)}{\partial x} = 0.
\]

Assume a mean flow with small perturbations:

\[
u(x, y, t) = U_0(y) + \tilde{u}(x, y, t)
\]

\[
v(x, y, t) = \tilde{v}(x, y, t)
\]

\[
p(x, y, t) = p_0 + \tilde{p}(x, y, t)
\]

\[
\rho(x, y, t) = \rho_0 + \tilde{\rho}(x, y, t),
\]
**Linearized Euler Equations**

**Assumptions**: perfect gas with constant heat capacities $C_p$ and $C_v$; inviscid, non-heat conducting, with a uniform mean flow entropy $\Rightarrow \frac{d}{dt} \tilde{\rho} = c_0^2 \frac{d}{dt} \tilde{\rho}$.

Linearizing around $(U_0(y), 0, \rho_0, \rho_0)$ and eliminating $\tilde{\rho}$:

\[
\frac{1}{\rho_0 c_0^2} \left( \frac{\partial \tilde{p}}{\partial t} + U_0 \frac{\partial \tilde{p}}{\partial x} \right) + \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0
\]

\[
\frac{\partial \tilde{u}}{\partial t} + U_0 \frac{\partial \tilde{u}}{\partial x} + \tilde{v} U_0 + \frac{1}{\rho_0} \frac{\partial \tilde{p}}{\partial x} = 0
\]

\[
\frac{\partial \tilde{v}}{\partial t} + U_0 \frac{\partial \tilde{v}}{\partial x} + \frac{1}{\rho_0} \frac{\partial \tilde{p}}{\partial y} = 0.
\]
Modes

We consider waves of the type:

\[ \tilde{p} = e^{i\omega t - i\alpha x} \hat{p}(y), \quad \tilde{u} = e^{i\omega t - i\alpha x} \hat{u}(y), \quad \tilde{v} = e^{i\omega t - i\alpha x} \hat{v}(y). \]

The equations reduce to:

\[ \frac{i(\omega - \alpha U_0)\hat{p}}{\rho_0 c_0^2} - i\alpha \hat{u} + \frac{d\hat{v}}{dy} = 0 \]

\[ i(\omega - \alpha U_0)\hat{u} + U_0' \hat{v} - \frac{i\alpha}{\rho_0} \hat{p} = 0 \]

\[ i(\omega - \alpha U_0)\hat{v} + \frac{1}{\rho_0} \frac{d\hat{p}}{dy} = 0. \]

Substituting \( \hat{v} \) and \( \hat{u} \) we come to:

\[ \frac{d^2\hat{p}}{dy^2} + \frac{2\alpha U_0'}{\omega - \alpha U_0} \frac{d\hat{p}}{dy} + \left[ \frac{(\omega - \alpha U_0)^2}{c_0^2} - \alpha^2 \right] \hat{p} = 0. \]
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$$

Boundary conditions:
- exponential decay for $y \to \infty$
- impedance boundary condition at $y = 0$:

$$
- \frac{\hat{p}(0)}{\hat{v}(0)} = Z(\omega).
$$
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**Spatio-Temporal Instabilities**

- **absolute instability**: affects the whole flow
- **convective instability**: observed only when traveling with a certain velocity along the flow
The Impulse Response

Impulse response of the system - a $\omega$-$\alpha$ integral:

$$\Psi(x, y, t) = \frac{1}{(2\pi)^2} \int_{F_\alpha} \int_{L_\omega} \frac{\varphi(y)}{D(\alpha, \omega)} e^{i\omega t - i\alpha x} d\omega d\alpha,$$

Integration contours:

- in domains of absolute convergence in the complex $\omega$- and $\alpha$-planes
- for the Fourier integral: a strip along the real axis
- for the Laplace integral: a part of the lower half-plane.
Integration Contours

Paths of integration: in the complex-frequency ($\omega$) and in the complex-wavenumber ($\alpha$) plane.
Summary

IDEA: to lift the integration contour in the $\omega$-plane such that all the poles move to the upper half plane.

METHOD:

- find $\omega_{min} = \min_{\alpha \in \mathbb{R}}(\omega_i)$; if
  - $\omega_{imin} > 0$ then the flow is stable;
  - $\omega_{imin} < 0$ then the flow is unstable $\rightarrow$ continue

- find the lines of constant $\omega_i$, starting with $\omega_{imin}$ and plot them in the $\alpha$-plane: as $\omega_i$ is increased, the $\alpha^+$ and $\alpha^-$ (corresponding to different $\alpha$-half-planes) approach each other, and eventually collide $\rightarrow \alpha^*$, where the $F_{\alpha}$-integration contour is pinched, and allows no further deformations

- if $\text{Im}(\omega(\alpha^*)) < 0$ then the instability is absolute
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The Model Equations

Incompressible limit: Mach number $M_0 = U_\infty / c_0$ is much smaller than unity.

The system reduces to:

$$\frac{d^2 \hat{p}}{dy^2} + \frac{2\alpha U_0'}{\omega - \alpha U_0} \frac{d\hat{p}}{dy} - \alpha^2 \hat{p} = 0.$$  

A piecewise linear velocity profile:

$$U_0(y) = \begin{cases} 
0 & \text{for } -\infty < y \leq 0 \\
\frac{y}{h} U_\infty & \text{for } 0 \leq y \leq h \\
U_\infty & \text{for } h \leq y < \infty
\end{cases}$$
The Dispersion Relation

The solution for $y \geq h$ (assuming exponential decay at $+\infty$):

$$\hat{\rho} = Ae^{-\alpha y}, \text{ assuming } \text{Re}(\alpha) > 0.$$ 

The solution in the shear layer region $(0, h)$ (due to Rayleigh):

$$\hat{\rho}(y) = C_1 e^{\alpha y} (h\omega - \alpha y U_\infty + U_\infty) + C_2 e^{-\alpha y} (h\omega - \alpha y U_\infty - U_\infty)$$

$$\hat{u}(y) = \frac{\alpha h}{\rho_0} (C_1 e^{\alpha y} + C_2 e^{-\alpha y})$$

$$\hat{v}(y) = \frac{i\alpha h}{\rho_0} (C_1 e^{\alpha y} - C_2 e^{-\alpha y}).$$
The Dispersion Relation

- at the interface \( y = h \): continuity of pressure and particle displacement

- continuity + boundary condition at \( y = 0 \) \( \Rightarrow \) the dispersion relation \( D(\alpha, \omega) \):

\[
Z(\omega) - \frac{e^{\alpha h}(2\omega h - 2\alpha hU_\infty + U_\infty)(\omega h - U_\infty) + e^{-\alpha h}(\omega h + U_\infty)}{i\alpha h(e^{\alpha h}(2\omega h - 2\alpha hU_\infty + U_\infty) - e^{-\alpha h})} = 0
\]
A Briggs-Bers Analysis Example

The impedance:

$$Z(\omega) = \rho_0 U_\infty \left( i \frac{\omega m}{\rho_0 U_\infty} + \frac{R}{\rho_0 U_\infty} - i \frac{K}{\omega \rho_0 U_\infty} \right)$$

We choose $h = \frac{1}{2}$, $\frac{m}{\rho_0} = \frac{\rho_0 U_\infty^2}{K} = 1$ and $\frac{R}{\rho_0 U_\infty} = 0.015$:

The flow is absolutely unstable!
## A Parameter Study

### Table 1: A Parameter Study

<table>
<thead>
<tr>
<th>$h$</th>
<th>$m/\rho_0$</th>
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### Table 2: A Parameter Study

<table>
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### Table 3: A Parameter Study

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</table>

The critical height $h_c$ is estimated by:

$$h_c \approx -0.76 \frac{R}{\rho_0 U_\infty} + 0.52 \frac{m}{\rho_0} + 0.32 \frac{\rho_0 U_\infty^2}{K}.$$
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Current and Future Work

- investigation of the instability of the incompressible flow for a more general velocity profile using numerical tools
- instability analysis for compressible flows
- an improvement to the Ingard Myers boundary condition leaving the flow stable
Thank you for your attention!