A new ray tracing method based on phase space

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Outline

Phase space based ray tracing

• Introduction of ray tracing
  – A two-dimensional optical system: the TIR-collimator
  – Physical background
  – Issues with ray tracing

• The idea: phase space based ray tracing
  – How to improve ray tracing
  – Boundaries are mapped onto boundaries

• Implementation
  – Refinement procedure
  – Boundary detection
  – Calculation of intensity

• Results
  – Speed of convergence

• Conclusion and outlook
Ray tracing is like playing billiards
Ray tracing for a TIR-collimator
Background

Law of reflection:

\[ t_2 = t_1 - 2(t_1, n)n \]

Law of refraction:

\[
\frac{s_2}{n_2} = \frac{n_1}{n_1} s_1 + \left[ \sqrt{1 - \left( \frac{n_1}{n_2} \right)^2 + \left( \frac{n_1}{n_2} \right)^2 (n, s_1)^2} - \left( \frac{n_1}{n_2} \right) (n, s_1)n \right] n
\]

Flux: Measure of the perceived power of light by the human eye, \( (\phi = [lm]) \).

Intensity:

\[ I = \frac{d\phi}{dt} \quad [cd = lm/rad] \]

Luminance:

\[ L = \frac{d^2\phi}{\cos(t) da dt} \quad [cd/m] \]

Étendue:

\[ dE = n da \cos(t) dt \quad [m \text{ rad}] \]
What is ray tracing?

- Shoot rays from source to target randomly
- Intensity calculated by counting the rays in bins

Our model

Convergence
(Compared to a simulation with a large number of rays)
How to improve ray tracing?

- Full system is characterized by phase space
  \( \mathcal{P}_s: S \times [-1,1] \) (position \( x \in S \) and angle \( \tau = \sin t \))
  \( \mathcal{P}_t: T \times [-1,1] \) (position \( q \in T \) and angle \( \eta = \sin \theta \)).
- The optical map \( \mathcal{M}: \mathcal{P}_s \to \mathcal{P}_t \) maps region to region and boundary to boundary (Riesz)
Implementation: How to find the boundaries on source phase space?

- On source phase space: take a coarse grid
- If paths are different refine unless already fine enough
Implementation: How to find the boundaries on target phase space?

Method 1: use alpha-shapes

For a cloud of points, like eating stracciatella ice-cream with a spoon without eating the chocolate

Issue: how to choose the size of your spoon? (We used an Étendue argument)

Implementation: How to find the boundaries on target phase space?

Method 2: use that boundary is mapped on boundary for regions of phase space \(^1\)

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Implementation: How to find the boundaries on target phase space?

Method 2: use that boundary is mapped on boundary for regions of phase space

More points around the boundaries

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Implementation: How to find the boundaries on target phase space?

To calculate intensity: integrate along a line

\[ I_{PS}(\eta) = \sum_{i,j} \int_{q_{\Pi_{j,2i-1}(\eta)}}^{q_{\Pi_{j,2i}(\eta)}} L_t(q, \eta) dq \]

If we assume constant brightness (Lambertian source)

\[ L_S(x, \tau) = L_t(q, \eta) = 1 \]

We obtain:

\[ I_{PS}(\eta) = \sum_{i,j} \left( q_{\Pi_{j,2i}(\eta)} - q_{\Pi_{j,2i-1}(\eta)} \right) \]
Results

- Method calculates intensity correctly
- Convergence is $1/N$ instead of $1/\sqrt{N}$

PS intensity with around $10^5$ rays.
Reference intensity: MC ray tracing with $10^8$ rays.

Using ray tracing on PS far fewer rays need to be traced to obtain the same accuracy.
Conclusions and outlook

- This method is 2D only (meridional plane) but can be extended to 3D as well
- More rapid convergence $1/N$ versus $1/\sqrt{N}$
- Future work: include Fresnel reflection and scattering phenomena

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How to choose the parameter $\alpha$ for the $\alpha$-shapes method
Stopping criterion for the triangulation refinement