Homogenization of models involving moving interfaces at the microscale

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Introduction

Flows through the porous media (soil, bone, concrete, etc.) are basically flows through complex geometries (pores, grains, aggregates, etc.) and are often accompanied by microscopic physical-chemical processes. These processes often lead to changes in the pore-scale geometry. To understand the effect of these changes on the macroscale behavior, following issues are of great practical importance:

- How to scale-up processes involving diffusion and reactions?
- Rigorous evaluation of the quality of averaging
- Appropriate multiscale numerical methods

Case studies

- 3D solid state batteries
- Biofilm growth

Schematic representation of a two-dimensional, thin strip with a biofilm attached to the strip boundary.

Approaches

- Asymptotic homogenization
- Multiscale computations using domain decomposition methods
- Boundary homogenization
- Rigorous averaging approaches, 2-scale convergence, periodic unfolding techniques

Results and discussions

- Rigorous existence and uniqueness results in 1-D
- Formal asymptotic upscaling for multi-dimensional case
- Numerical evidence for quality of averaging
- Convergence proof for multiscale computations using domain decomposition techniques
- Effective boundary laws for domains involving geometry changes (moving rough boundaries)
- Effective dispersion equations for reactive flows in a thin strip
- Effective model for bio-film growth in a porous medium
- Effective models involving nonlinear transmission conditions (fractured medium)

Cooperations

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