Building Verification Condition Generators by Compositional Extension

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Why Build Verification Condition Generators Compositionally?

We want mechanized tool support for reasoning over languages, but:

- Languages change
- Logics for languages must change along
- Mechanized tools for these logics must change along
- Keeping trust in tools is hard
- Changing such tools diminishes trust
- Compositionality improves clarity, ease of correct modification and therefore trust

Oh, and it’s fun too!
Consider language $L$:

$$Stmt \rightarrow Variable := Expr$$

| if Expr then \{ Stmt \} else \{ Stmt \} |
| inv Expr while Expr do \{ Stmt \} |
| Stmt; Stmt |

with these derivation rules for sufficient precondition predicate transformer semantics:

$$pre (x := e) q = q[e/x]$$

$$pre (S_1; S_2) q = pre (S_1) (pre (S_2) q)$$

$$pre (if g then \{ S_1 \} else \{ S_2 \}) q =$$

$$\begin{cases} 
\text{if } g \text{ then } pre (S_1) q & \text{ else } pre (S_2) q \\
\end{cases}$$

$$p = pre (S) i$$

$$i \land \neg g \Rightarrow q$$

$$i \land g \Rightarrow p$$

$$pre (\text{inv } i \text{ while } g \text{ do } \{ S \}) q = i$$
Function: $pvcg :: Stmt \rightarrow Expr \rightarrow Expr$
Function: \texttt{pvcg :: Stmt} \rightarrow (\texttt{Expr}, [\texttt{Expr}]) \rightarrow (\texttt{Expr}, [\texttt{Expr}])
Function: \textit{pvcg} :: \textit{Stmt} \rightarrow m \textit{Expr} \rightarrow m \textit{Expr}
Function: $pvcg :: Stmt \rightarrow Expr \rightarrow m\ Expr$

Decouple from recursion, abstract away gathering:

\[
pvcg\_Assign\ x\ e = \lambda q \rightarrow return\ (subst\ (x, e)\ q)
\]

\[
pvcg\_Seq\ p\_s1\ p\_s2 = \lambda q \rightarrow p\_s2\ q \ggg \lambda p' \rightarrow\ p\_s1\ p' \ggg \lambda p \rightarrow return\ p
\]

\[
pvcg\_IfThenElse\ g\ s1\ s2 = --\ similar\ to\ others
\]

\[
pvcg\_InvWhile\ i\ g\ p\_body = \lambda q \rightarrow p\_body\ i \ggg \lambda p \rightarrow\ record\ (i \land \neg\ g \rightarrow q) \ggg \lambda_\rightarrow\ record\ (i \land g \rightarrow p) \ggg \lambda_\rightarrow return\ i
\]

\[
pvcg = foldStmt\ (pvcg\_Assign,\ pvcg\_Seq,\ pvcg\_IfThenElse,\ pvcg\_InvWhile)
\]
Add exception handling constructs to the language:

\[
Stmt \rightarrow \quad -- \text{existing alternatives}
\]

\[
\begin{align*}
&\text{try } Stmt \ \text{catch } Stmt \\
&\text{raise}
\end{align*}
\]
Add exception handling constructs to the language:

\[
Stmt \rightarrow \quad \text{-- existing alternatives}
| \quad \text{try } Stmt \text{ catch } Stmt
| \quad \text{raise}
\]

\[
pvcg\_Raise = \lambda q \rightarrow \text{getPostE}
pvcg\_TryCatch p\_s1 p\_s2 = \lambda q \rightarrow \text{getPostE} \gg= \lambda q' \rightarrow p\_s2 q \gg= \lambda p' \rightarrow \text{setPostE} p' \gg= \lambda \_ \rightarrow p\_s1 q \gg= \lambda p \rightarrow \text{setPostE} q' \gg= \lambda \_ \rightarrow \text{return } p
\]

\[
pvcg = \text{foldStmt} (pvcg\_Assign, pvcg\_Seq, pvcg\_IfThenElse, pvcg\_InvWhile, pvcg\_Raise, pvcg\_TryCatch)
\]
Choosing the monad

- The monad is an unspecified parameter
- Specifying it directly is not very compositional
- We can use monad transformers, distributive laws or coproducts

- Monad transformers fix the order of composition
- Distributive laws cannot always be given
- Coproducts are more general than both
- Coproducts require a third monad to encapsulate semantics
- Given mapping from each element to the third monad, the mapping from the coproduct can be derived
We have introduced an approach to implementing verification condition generators that:

▶ Can deal with changes to the language
▶ Can deal with changes to the underlying domain
▶ Without having to change things that have already been specified