Extending the pivoting strategy of Backward IJK version of Gaussian Elimination to an IUL preconditioner

A. Rafiei*, M. Bollhöfer †

Abstract

Consider the linear system of equations of the form $Ax = b$ where the coefficient matrix $A \in \mathbb{R}^{n\times n}$ is nonsingular, large, sparse and nonsymmetric and also $x, b \in \mathbb{R}^n$. We refer to this system as the original system.

An explicit preconditioner M for this system is an approximation of matrix A^{-1}. In [1], Lou presented the **B**ackward **F**actored **INV**erse or $BFINV$ algorithm which computes the inverse factorization of A in the form of $A^{-1} = ZD^{-1}W$ where W and Z^T are unit upper triangular matrices and D is a diagonal matrix. By applying a dropping rule for the entries of W and Z, the explicit preconditioner M is computed as $A^{-1} \approx M = ZD^{-1}W$ and the process is termed the **B**ackward **F**actored **AP**proximate **INV**erse or $BFAPINV$ [3].

An implicit preconditioner for the original system is an approximation of matrix A. In [2], we could compute an implicit preconditioner M as the by-product of the $BFAPINV$ process. This preconditioner is in the form of $A \approx M = UDL$ where U and L^T are unit upper triangular matrices and D is a diagonal matrix. After merging the factors D and L, we have termed M as the $IULBF$ (**IUL** factorization obtained from **B**ackward **F**actored approximate inverse process).

The IJK version of Gaussian Elimination process has two Forward and Backward versions. We can obtain two implicit preconditioners for the original system from these two versions. By applying the dropping strategy in the Forward and Backward versions of this process, one can compute an implicit ILU preconditioner $\bar{M}$ as $A \approx \bar{M} = LDU$ and an implicit IUL preconditioner M as $A \approx M = UDL$, respectively. As in the Forward version of Gaussian Elimination, the whole parts of the Schur-Complement matrices are also explicitly available in the Backward version of this process. Therefore, it is possible to apply the complete pivoting strategy in the Backward version of Gaussian Elimination process.

In this talk, we show how one can extend the complete pivoting strategy of the Backward IJK version of Gaussian Elimination process to compute the $IULBF$ preconditioner which is coupled with the complete pivoting strategy. There is a parameter α to control the complete pivoting process. We have studied the effect of different values of α on the quality of the computed preconditioner.

References

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*Department of Applied Mathematics, Hakim Sabzevari University, Sabzevar, Iran. Email: rafiei.am@gmail.com, a.rafiei@hsu.ac.ir. This work was partially supported by a grant from the Niels Henrik Abel Board.

†Institute computational Mathematics, Technische Universität Braunschweig, D-38106 Braunschweig, Germany. E-mail: m.bollhoefer@tu-bs.de.