Comparison of preconditioning strategies for time domain photonic crystal modeling with Krylov subspace exponential methods

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Time domain photonic crystal modeling typically involves solution of the Maxwell equation in a spatial domain with space-dependent permittivity and special non-reflecting boundary conditions, e.g., Perfectly Matched Layers (PMLs). After a spatial discretization with a suitable finite difference or finite element scheme (such as Yee staggered finite differences or Nedelec vector finite elements), we are left with a large initial value problem

$$y'(t) = Ay(t) + g(t), \qquad y(0) \text{ given}, \tag{1}$$

where $A \neq A^T$ is a large square matrix representing the discretized Maxwell operator, $y(t)$ contains the values of both magnetic and electric fields on the spatial mesh and some additional variables, and $g(t)$ is a given function representing sources and possibly boundary conditions. The popular Yee finite difference method for solving the Maxwell equation (well known as the Finite Difference Time Domain method, FDTD) can be obtained in this setting when (1) is solved in time with a staggered leapfrog scheme.

Time integration schemes based on the actions of the matrix exponential $\exp(tA)$ with rational block Krylov subspace methods allow to obtain a significant gain with respect to standard time integration methods as FDTD and alike. A prerequisite for this is that a fast linear solver is available for systems with a matrix $I - \gamma A$, where $\gamma > 0$ is a pole usually related to the time t.

In this talk we compare several preconditioning strategies for iterative solution of the shifted linear systems with the matrix $I - \gamma A$. Due to the special PML boundary conditions, this matrix is not skew-symmetric and the linear system can not be brought to a saddle point form. Furthermore, the standard incomplete LU preconditioners do not perform well for this system. Therefore, we focus on preconditioners based on directional and field (magnetic and electric) splittings.