FIELD OF VALUES AND PSEUDOSPECTRA OF BLOCK PRECONDITIONED LINEAR SYSTEMS

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In this work we consider the field of values and pseudospectra of matrices of the form

$$\mathcal{A} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{1}$$

where each block comes from the discretization of a system of PDEs. The *H-field of values* of a matrix is the set in the complex plane given by

$$\mathcal{W}_H(M) = \left\{ z \in \mathbb{C} : z = \frac{\mathbf{x}^* H M \mathbf{x}}{\mathbf{x}^* H \mathbf{x}}, \mathbf{x} \neq \mathbf{0} \right\}$$

where H is a symmetric, positive definite matrix. Amongst other things, the field of values provides well-known bounds for the convergence rate of iterative methods such as GMRES and MINRES. In [1], the authors consider several preconditioners to saddle-point problems and prove that several block-style preconditioners are H-FOV-equivalent to the original saddle-point system.

Another tool for establishing GMRES convergence bounds is the pseudospectrum. The ϵ pseudospectrum of a matrix M is the set in the complex plane given by

$$\sigma_\epsilon(M) = \left\{ z \in \mathbb{C} : \|(zI - M)^{-1}\| > 1/\epsilon \right\}.$$

In [3], the authors characterize the pseudospectra of saddle point systems preconditioned by Murphy-Golub-Wathen [2] style block preconditioners.

In this work we generalize the results of [1, 3] to linear systems of the form (1) and state under what additional conditions we maintain H-FOV equivalence and what new pseudospectral bounds are obtained.

Computational examples from a wide range of applications are provided. Field of values and pseudoespectra plots are provided by eigtool.

References

[1] D. Loghin and A. Wathen, *Analysis of preconditioners for saddle-point problems*, SIAM J. Sci. Comput., 25 (2004), pp. 2029–2049.

[2] M. F. Murphy, G. H. Golub, and A. J. Wathen, *A note on preconditioning for indefinite linear systems*, SIAM Journal on Scientific Computing, 21 (2000), pp. 1969–1972.

[3] G. Ymbert, M. Embree, and J. Sifuentes, *Approximate Murphy-Golub-Wathen Preconditioning for Saddle Point Problems*, In Preparation, (2015).