Block Preconditioning and Multigrid Methods for Coupled Systems

Scott MacLachlan
Memorial University of Newfoundland

This talk focuses on the solution of saddle-point problems of the form

$$\begin{bmatrix} A & B \\ B^T & 0 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix}.$$

When A is a standard discretization of a diffusion or convection-diffusion operator and B is a discrete gradient, this is the well-studied case of a stable discretization of the incompressible Stokes or (linearized) Navier-Stokes Equations. In this case, many optimal preconditioning strategies are known, including block-factorization preconditioners (using multigrid to solve systems with the A block) and monolithic multigrid methods with appropriately chosen relaxation. This focus of this talk is the case where A is more complex, such as in DG discretizations of the Stokes equations, or mixed FEM discretizations of magnetohydrodynamics. Both block preconditioning and monolithic multigrid can be naturally extended to these cases, but work is needed to ensure optimality of the resulting preconditioners. In this talk, we consider the development of effective multigrid methods for the resulting A blocks, as well as the extension of monolithic relaxation schemes for the coupled systems. This is joint work with James Adler and Thomas Benson from Tufts University, and Eric Cyr and Ray Tuminaro from Sandia National Laboratories.