Limited Memory Preconditioners for symmetric indefinite problems with application to structural mechanics

Sylvain Mercier*, Serge Gratton†, Nicolas Tardieu‡, and Xavier Vasseur§

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This talk will focus on preconditioning techniques for solving sequences of symmetric indefinite systems $A_i x_i = b_i$, where the matrices A_i are supposed to slightly change. This case often arises in Computational Science and Engineering, for example when a Newton-type method is used to solve a nonlinear problem. With this aim in mind, we study the case of a sequence with a given matrix $A \in \mathbb{R}^{n\times n}$ and multiple right-hand sides that are not known simultaneously. The idea is to generalize the class of *Limited Memory Preconditioners*, initially motivated by the BFGS method, analysed in [1] for A symmetric positive definite. The extension to the symmetric indefinite case leads us to study the following preconditioner H, constructed from a small set of k linearly independent vectors forming the columns of a matrix S:

$$H = (I_n - S(S^T AS)^{-1}S^T A)(I_n - AS(S^T AS)^{-1}S^T) + S(S^T AS)^{-1}S^T.$$

Several theoretical results will be presented, including the characterization of the spectrum of AH and the sign of its eigenvalues. Moreover, under some assumptions about the choice of S, we can show an interlacing property of the eigenvalues of AH with respect to the eigenvalues of A. In practice, the key of this preconditioning technique is to recover some relevant directions gained during the solution of the first system $Ax = b_1$ and construct S from this information in order to improve the convergence for the remaining linear systems. In this context, we analyse the use of Ritz vectors as columns of S and we show a closeness property of the spectra of AH and A restricted to $range(S)^{\perp}$, with a bound depending quadratically on the norm of the Ritz residual.

Then, we illustrate the numerical behaviour of this class of preconditioners on two realistic applications in structural mechanics problems that require the solution of sequences of large-scale symmetric saddle-point systems. These industrial studies are performed within the framework of the open-source software *Code_Aster* which is a general purpose finite element code developed at EDF. The first simulation deals with the mechanical properties of a containment building of a nuclear reactor, while the second addresses the solution of a polycrystalline aggregate problem used to obtain macroscopic constitutive laws of a material from microscopic considerations only. Both numerical experiments show the relevance of the proposed preconditioner leading to a significant decrease in terms of computational operations when solving such sequences of linear systems.

References

[1] S. Gratton, A. Sartenaer, and J. Tshimanga. On a class of limited memory preconditioners for large scale linear systems with multiple right-hand sides. *SIAM Journal on Optimization*, 21(3):912–935, 2011.

*EDF R&D, 1 Avenue du Général De Gaulle, F-92141 Clamart, France and CERFACS, 42 Avenue Gaspard Coriolis, F-31057 Toulouse Cedex 1, France

†INPT-IRIT, University of Toulouse and ENSEEIHT, 2 Rue Camichel, BP 7122, F-31071 Toulouse Cedex 7, France

‡EDF R&D, 1 Avenue Du Général De Gaulle, F-92141 Clamart, France

§CERFACS, 42 Avenue Gaspard Coriolis, F-31057 Toulouse Cedex 1, France