Application of inner-iteration preconditioning to singular linear systems

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Several steps of stationary iterative methods serve as inner-iteration preconditioning for linear systems of equations $A\boldsymbol{x} = \boldsymbol{b}$, where $A \in \mathbb{R}^{n\times n}$ may be singular and $\boldsymbol{b} \in \mathcal{R}(A)$, the range of A. We apply the preconditioner to the generalized minimal residual (GMRES) method and present theoretical justifications for using, this approach including the singular case, based on a necessary and sufficient condition such that GMRES gives a solution of linear systems of equations without breakdown with an arbitrary index, $\min\{d \in \mathbb{N}_0 | \operatorname{rank} A^d = \operatorname{rank} A^{d+1}\}$, $A^0 = \mathrm{I}$, and with an initial guess and a right-hand side in certain subspaces. We show how the subspaces and the solution determined by GMRES depend on the index of the (preconditioned) coefficient matrix. Thus, we characterize classes of stationary iterative methods that are feasible as inner-iteration preconditioning. Numerical experiments show that the successive overrelaxation (SOR) inner-iteration preconditioning is more robust and efficient compared to diagonal scaling for some test problems of large sparse singular linear systems.

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