Block Preconditioners for Magnetohydrodynamics with Mixed Discretizations*

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Abstract

The magnetohydrodynamics (MHD) model describes the dynamics of charged fluids in the presence of electromagnetic fields. The continuum model of MHD is composed of a non-self-adjoint, nonlinear system of partial differential equations (PDEs) describing conservation of mass, momentum, and energy, augmented by the Maxwell equations. This PDE system can be strongly coupled and may span a very large range of length- and time-scales. For effective time integration of these systems some form of implicitness is required, resulting in the need to solve large linear systems in which discrete hydrodynamics are electromagnetics are coupled. In the context of finite element spatial discretization, mixed integration, stabilized methods, and structure-preserving (physics compatible) approaches have been employed, resulting in wide variation of the block structure of the implicit system. Consequently, strong coupling and disparate spatial discretizations make the scalable and robust iterative solution of MHD systems extremely challenging, and represent principle difficulties of general multiphysics systems.

In this talk we discuss approximate block factorization (ABF) and physics-based block preconditioning approaches that have been applied to various MHD formulations. These methods decompose the coupled systems into block components so that multilevel methods can be applied to operators associated with individual unknowns. A critical aspect of these methods is the development of approximate Schur complement operators that encode the stiff cross-coupling physics of the system. To demonstrate the flexibility and performance of these methods we consider application of these techniques to resistive, incompressuble MHD applications. In this context, we focus on the robustness, efficiency, and parallel and algorithmic scaling of the preconditioning methods.

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