Multilevel Sequentially Semiseparable Preconditioners and its Convergence Analysis

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In this talk, we will introduce the multilevel sequentially semiseparable (MSSS) preconditioning technique for a general class of linear systems from the discretization of partial differential equations (PDEs). MSSS preconditioners are originally developed for PDE-constrained optimization problems [1], and then have been extended to the computational fluid dynamics problems [2]. This type of preconditioner exploits the global multilevel sequentially semiseparable structure of the system matrix and computes an approximate factorization with prescribed accuracy in linear complexity. This factorization is performed via approximating the Schur complements by matrices with low off-diagonal rank using MSSS matrix computations. Our results in [1, 2] illustrate the superiority of such preconditioning technique over the state-of-the-art preconditioning techniques.

Next, we will investigate the convergence property of the MSSS preconditioning technique. Our analysis shows that the preconditioned spectrum is contained in a circle centered at $(1, 0)$ and we give an analytic bound of the radius of this circle. It is shown that this radius can be made arbitrarily small by properly setting a parameter in the MSSS preconditioner. This in turn gives super-fast and mesh size independent convergence for a wide class of Krylov solvers. Our results suit for a general class of linear systems, which can be either symmetric or unsymmetric, definite or indefinite.

References

[1] Y. Qiu, M.B. van Gijzen, J.W. van Wingerden, M. Verhaegen, C. Vuik. *Efficient Preconditioners for PDE-Constrained Optimization Problems with a Multilevel Sequentially SemiSeparable Matrix Structure*. To appear in *Electronic Transactions on Numerical Analysis*, 2015. Currently available at http://ta.twi.tudelft.nl/nw/users/yueqiu/publications.html

[2] Y. Qiu, M.B. van Gijzen, J.W. van Wingerden, M. Verhaegen, C. Vuik. *Evaluation of Multilevel Sequentially Semiseparable Preconditioners on CFD Benchmark Problems Using Incompressible Flow and Iterative Solver Software*. *Mathematical Methods in the Applied Sciences*, doi:10.1002/mma.3416, Vol. 38, 2015.