

Matrix balancing for field of values type inclusion regions

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The field of values may be an excellent tool for generating a spectral inclusion region: it is easy to approximate numerically, and for many matrices this convex region fits relatively tightly around the eigenvalues. However, for some matrices the field of values may be a poor eigenvalue inclusion region (which happens, more precisely, if the numerical radius is much larger than the spectral radius). In this note, we show that balancing (or scaling) the matrix may be very helpful for generating a quality inclusion region based on the field of values. We review some known balancing techniques, present an implementation for the balancing of sparse matrices, and introduce a new scaling method by scaling the Hessenberg matrix resulting from a Krylov process. Moreover, several interesting connections with nonnormality of matrices are pointed out. We believe that the sparse balancing and the “Krylov and balance” technique, combined with a projected field of values, render spectral inclusion regions that may be hard to beat in both quality and efficiency.